Phase transition in a spin glass

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We construct a diagram technique for a spin glass in the vicinity of its transition without using the replica method. We observe a strong interaction of the large-scale longitudinal gapless modes in the low-temperature phase. The lower critical dimensionality of the theory is $d_c = 4$. When $4 < d < 6$ the critical indices determine the magnetic susceptibility and the specific heat [A. B. Harris, T. C. Lubensky, and J. H. Chen, Phys. Rev. Lett. 36, 415 (1976)]. We show that there is no transverse spin stiffness in a Heisenberg spin glass.

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1. INTRODUCTION

The problem of the existence and the properties of a phase transition in the spin glass state has in recent years been studied intensively both theoretically and experimentally. Recent experiments and computer simulations show that the kink in the magnetic susceptibility observed by Canelli and Mydosh depends on the time of observation and does, therefore, not correspond, apparently, to a phase transition being present. Nonetheless, something like a phase transition is going on in the systems considered and a theoretical study of phase transitions in model spin glasses is therefore of interest, at least as a first approximation to an understanding of the crux of the matter.

We give a survey of the main theoretical papers on phase transitions in a spin glass. Edwards and Anderson observed a phase transition in the framework of a self-consistent field theory in a system of spins with random alternating exchange interactions. The phase transition is connected with the occurrence of average coordinate-dependent spin values $\langle S_i \rangle = 0$ while the average magnetic moment vanishes: $\langle S \rangle = 0$ (here and henceforth pointed brackets indicate thermodynamics averages and a bar averaging over inhomogeneities). They proposed as an order parameter, characterizing the transition, the quantity

$$\lim_{T \to 0} S(0)(0) = -\infty S(0),$$

which characterizes the extent to which the spins are frozen in. Edwards and Anderson used the so-called replica method which makes it possible formally to average over random parameters of the system before taking the thermodynamic average. Using the same method, Harris, Lubensky, and Chen showed that the mean field theory for a spin glass is valid only when the spatial dimensionality $d > 6$, and they evaluated critical indices in a $6 - c$-expansion. Harris and Fish, and also Reed using an analysis of high-temperature series expansions, showed that the phase transition in the Edwards-Anderson model with nearest-neighbor interactions vanishes when $d = 4$, i.e., the lower critical (marginal) dimensionality of the theory is $d_c = 4$. Unfortunately, this method does not enable us to explain the physical reason for the disappearance of the phase transition; in particular, the magnitude of $d_c$ remains unknown for a real spin glass with RKKY exchange. To elucidate these problems it is necessary to study the behavior of the correlation functions in the low-temperature phase, in a similar way as was done for ordered systems.

A study of the low-temperature phase in the framework of the replica method has met with serious difficulties; it turned out that the equation of state obtained by Sherrington and Kirkpatrick which corresponds to the mean field approximation is unstable [one of the correlation functions in their solution has a negative gap $(T - T_c)^2$]; moreover, the instability occurs already in first order in $|T - T_c|$ when fluctuations are taken into account. Bray and Moore observed that the Hamiltonian of the replica method has a solution different from the one given in Ref. 9, and obtained a solution stable to order $(T - T_c)^1$ and in first order in the fluctuations. The most important singularity of this solution is the presence of a longitudinal gapless mode of fluctuations which leads to a divergence of the first correction to the order parameter when $d < 4$.

It is apparently extremely difficult to obtain an exact proof of the existence of a gapless mode to all orders in $|T - T_c|$ and in the fluctuations in the framework of the replica method. Bray and Moore used the non-averaged self-consistent field equations of Thouless, Anderson, and Palmer (TAP) to show that the presence of local soft modes in an Ising spin glass is uniquely connected with the existence of the gapless correlation function found in Ref. 11. Using a numerical solution of the TAP equations they showed in the same paper the existence of the soft modes. The existence of gapless longitudinal fluctuations in a spin glass is thus firmly established (at least in the self-consistent field approximation).

In the present paper we consider the Edwards-Anderson model with a Gaussian random distribution of the exchange integrals. We construct (without using the replica method) a diagram technique which describes an Ising spin glass in the vicinity of a phase transition—section 3 of the paper is devoted to an exposition of this technique. Section 2 also contains an evaluation of the critical indices in the paramagnetic phase. The results are the same as the ones obtained in Ref. 5 by the replica method.

In section 3 we consider in the self-consistent field method framework the low-temperature phase. We
show that the standard method of introducing condensate averages leads to the same difficulties as in Refs. 9-11; we propose a method which gives a correct expansion of the Green function in terms of the order parameter $Q$. Together with the condition $G(\tau=0)=0$ for there being no gap this expansion determines the equation of state which is the same as the one obtained by Bray and Moore.\textsuperscript{12}

In section 4 we study the effect of fluctuations on the behavior of the low-temperature phase. The advantage of the proposed diagram technique manifests itself in that case in the fact that taking fluctuations into account does not lead in the leading order in $|T-T_c|$ to the appearance of a negative gap, even if we use the standard method for introducing the condensate. The lower critical dimensionality of the theory turns out to equal $d_c=4$, as in Bray and Moore's theory.\textsuperscript{12}

We observe in 6-dimensional space a strong dependence of the long-wavelength ($q\ll q_c$) fluctuations leading to an increase of the effective charge at large distances ("asymptotic freedom"). Because of that we cannot determine exactly the form of the correlation functions at large distances. It turns out, however, that the temperature dependence of the order parameter $Q(\tau)$ and the specific heat are determined by the moments $q^{2n}Q$ and can thus be obtained from the critical indices found in the parametric phase.

We show in section 5 that there is no transverse spin stiffness in a Heisenberg spin glass (in the self-consistent field approximation); a detailed analysis of the Heisenberg glass will be given in subsequent papers.

Section 6 contains a discussion of the results; we show that the limitation to the Edwards-Anderson model is unimportant and all qualitative conclusions are retained for there being no gap this expansion determines the equation of state which is the same as the one obtained by Bray and Moore.\textsuperscript{12}

In the proposed diagram technique there is an infinite set of vertices and the $n$-th order vertex, into which $n$ lines $g$ converge is

\[ I = \frac{1}{n!} \int \cdots \int G(k_1) \cdots G(k_n) \]

We shall evaluate the functions

\[ G_n = G(k_1) \cdots G(k_n) \]

(3)

We must write down an expansion of the correlation functions for a given realization $\{J_{ij}\}$ and afterwards average them over the distribution (2). For a Gaussian distribution the average of any set of exchange integrals splits into a product of pair averages which will be indicated by a dash line in the diagrams.

The bare correlator $G_b$ is shown in Fig. 2— this is a double chain with successive averaging. Changing to the momentum representation we get

\[ G_b(k) = \frac{1}{Z(k)} \sum_{J_{ij}} \delta(k - |k|) \]

(4)

In the approximation $x \gg 1$ the transition temperature is thus $T_c = J_0 x^{\frac{1}{2}}$ (in the number of nearest neighbors). The exact correlator $G(k)$ can be expressed in terms of the self-energy part $\Sigma(k)$ which is not cut along a pair of lines $T_1$.

The diagrammatic series for $\Sigma$ when $T > T_c$ is shown in Fig. 3. The terms within the square brackets are of order of smallness $1/x$ and can be neglected in the self-consistent field approximation.

We note that we drop everywhere "finger" type diagrams—see Fig. 4, which cancel exactly when we sum. Diagrams $a, b$ of Fig. 4 may serve as an illustration of this statement, and an exact proof of it is the following one: all fingers are renormalized points on the line $\langle S_j S_k \rangle$, i.e., they are an extension of the correlator $\langle S_j S_k \rangle = 1 - \langle S_j \rangle^2$. Above the transition point $\langle S_j \rangle = 0$ and $\langle S_j S_k \rangle = 1$, i.e., the point is not renormalized. This means also that all fingers cancelled.

We note that in the diagram technique for a ferromagnetic in the renormalization of the transition point there are only graphs of the kind of Fig. 4a present so that the renormalized point is not $\langle S_j \rangle$.
FIG. 3.

It is clear from Fig. 3 that in our diagram technique there are triple vertices—this means that the theory will be logarithmic in 6-dimensional space and we apply the self-consistent field method for $d > 6$. An arbitrary graph for $N$ is obtained as follows: we select a line $g_{ab}$ from one correlator $(S_i S_j)$ and pair it with the same line from another core. One can easily formalize this process by taking the variation

$$\delta S_i S_j = -\frac{\partial}{\partial g_{ab}} \left( \frac{1}{2} \delta g_{ab} + \frac{1}{2} g_{ab} \right) \left( S_i S_j - \langle S_i S_j \rangle \right)$$

The quantity $\delta S_i S_j$ is graphically represented in Fig. 5. An open circle denotes an average spin and a hashed polygon irreducible correlators.

In the high-temperature phase $(S_i S_j = 0)$, the diagram of Fig. 5 goes into $G_0$, and, hence, there remain only the graphs of Fig. 5a, $b$ which give the diagrams of Fig. 6. Since for $T > T_c$

$$K = 6(S_i S_j) = 2G$$

(the hashed block in Fig. 6) reduces to the diagram of Fig. 5a with a minus sign.

For $T > T_c$, the diagram technique contains one vertex

$$W = 6(S_i S_j)(S_k S_l)$$

(triply connected vertex occurs in the diagram of Fig. 6a). The doubly connected vertex

$$r = 6(S_i S_j)$$

(see the diagram of Fig. 6b) is expressed at $T > T_c$ in terms of $W$.

In the 6-dimensional space the equations at the vertex have the following form:

$$dW/dq = -2W^2, \quad dS_i S_j (k) = \frac{1}{2} W^3 q, \quad dW^3 = -4W^2 q.$$  \hspace{1cm} (6)

We have used here the notation

$$L = \frac{1}{2} \left( \frac{\delta}{\delta q} \right) \left( S_i S_j \right), \quad \frac{\delta}{\delta q} = \frac{\partial}{\partial q} + \frac{1}{2} \delta q$$

which is a standard one for the theory of phase transitions. Performing the $\theta - \epsilon$-expansion we get the critical indices which are the same as those found in Ref. 5.

FIG. 4.

3. LOW-TEMPERATURE PHASE IN THE SELF-CONSISTENT FIELD APPROXIMATION

In the low-temperature phase the magnetic moments acquire average values $\langle S_i \rangle$ in which terms we can express measurable physical quantities (magnetic susceptibility, neutron elastic scattering cross section, and so on). In order to use the diagram technique to evaluate the order parameter we must determine the external field associated with it. It seems natural to do this as follows: we apply to the system a weak magnetic field $H_0$ depending on the site $k$ with a Gaussian correlation law:

$$H_0 = \langle S_i \rangle H, \quad H_0 = 0,$$

such a field leads to the appearance of randomly directed magnetic moments with a vanishing average moment while we have for its mean square $\langle S_i S_j \rangle$

$$\frac{\partial}{\partial H} = -\sum \delta E_{i,j} = G(q = 0).$$

The diagrammatic series for $G$ in the self-consistent field approximation is shown in Fig. 8 (the field $H$ is indicated by a cross, the average spin on a site by an open circle, and the order parameter $Q$ by two circles averaged together).

A similar diagram expression exists for the self-energy $\Sigma$ (see Fig. 9). Putting $H = H_0$, the field associated with the order parameter, we get for $Q$ and $\Sigma$ equations which are the same as those found in Refs. 9-11. Their graphical descriptions are given, respectively, in Figs. 10 and 11. The corresponding analytical expressions have the form

$$Q = k' + \frac{\gamma'}{\gamma''} - \frac{\gamma''}{\gamma''} Q + \frac{1}{2} \left( \frac{\partial}{\partial q} Q \right)^2 + \ldots.$$  \hspace{1cm} (8)

$$\Sigma = 1 - \frac{\gamma'}{\gamma''} \left( \frac{\partial}{\partial q} \right)^2 + \ldots$$

and lead to the meaningless result $G(q = 0) = 0$. (We remind ourselves that $G'' = \Sigma - \Sigma'/\Sigma''$.) The instability occurs in the second order in $\gamma''$.

The physical reason for this consists, apparently, in the fact that the mean square $H$ of the magnetic field is in actual fact not a true variable associated with the order parameter in the low-temperature phase. To check this we imagine a small fluctuation of the spins around $H_0$.
their average values: $\Delta y < \langle S^y \rangle$; it is clear that the change in the free energy for such a fluctuation depends on the correlation between $S^y$ and $S^z$ while our random field $h_0$ is not at all correlated with the quantities $\langle S^y \rangle$. In other words, the following happens: the free energy and the parameter $Q$ are complicated functionals of the distribution of the average moments $\langle S^y \rangle$ so that the change in the free energy depends not only on the magnitude of the change $\delta Q$ but also on the direction in space of the moment configurations $\langle S^y \rangle$ through which this change was reached. When we apply a random field $h_0$ to the system which is uncorrelated with the averages $\langle S^y \rangle$ we impose on the system a not completely correct direction of the change in the spins in configuration space; the deviation of this direction from the average one is small when the order parameter $Q$ is much less than the equilibrium value $Q(\epsilon)$ for a given temperature, but becomes appreciable when $Q(\epsilon) > Q(\epsilon)$.

It is thus completely natural that the formalism for introducing the condensate discussed here leads to an equation of state which is valid only as far as the main terms in the expansion in the order parameter are concerned. We note also that in the replica method $\bar{\sigma}$ is the exact field associated with the field variable $\bar{\sigma}_{apr}$ in principle it must thus be possible to give a correct evaluation with such a field which was demonstrated to lowest orders by Bray and Moore.11,12

In actual fact one can assume that, as was mentioned in the introduction, the absence of a gap in the correlation function $G$ is proven. It seems to us that one should take just the condition $G(Q = 0) = 0$ as the basic one for the description of a spin glass. On the other hand, Eqs. (8) and (9) must be considered to be expansions of $Q$ and $\Sigma$ in series in $T_1^2 / T^2$, where $Q$ in actual fact is not the same as $\bar{Q}$ because of the inexact choice of the associated field. To find the correct dependence $\Sigma(Q)$ we must thus eliminate the parameter $\bar{Q}$ from Eqs. (8) and (9) after which we get

$$\Sigma = 1 - 2|t|^{3/2} - 4|t|^2 + \cdots \quad (10)$$

The terms in (10) which are written down are the same as those obtained by Bray and Moore from other considerations. Together with the condition $G(Q = 0) = 0$ Eq. (10) determines the equation of state $Q(t)$:

$$Q(t) = e^{(1/r^2)^2} - 1 + \frac{4}{r^2} - 4t + \cdots \quad (11)$$

In Ref. 13 the same equation is written in terms of the variable $t = T / T_1 - 1$, $r = 6(2 - t)$.

At low temperatures we get

$$\begin{align*}
  \zeta = & \left[ \begin{array}{c}
    1 + 2t + 3t^2 + 4t^3 + \cdots \\
    12t + 24t^2 + 36t^3 + 48t^4 + \cdots \\
    120t + 240t^2 + 360t^3 + 480t^4 + \cdots
  \end{array} \right], \\
  \zeta = & \left[ \begin{array}{c}
    1 + 2t + 3t^2 + 4t^3 + \cdots \\
    12t + 24t^2 + 36t^3 + 48t^4 + \cdots \\
    120t + 240t^2 + 360t^3 + 480t^4 + \cdots
  \end{array} \right],
\end{align*}$$

In conclusion this section we show that there exists in the low-temperature phase still one gapless correlation function which is connected with the function $G$ and defined as follows:

$$D_{ij} = \delta_{ij} \langle \bar{\sigma}_{ij} \rangle$$

This correlator corresponds to small spin fluctuations matched to their average values

$$\tilde{D}_{ij} = \delta_{ij} \langle \bar{\sigma}_{ij} \rangle$$

where $\tilde{D}_{ij}$ is a smooth function of the coordinates. To evaluate the function $\tilde{D}$ it is convenient to use the identity

$$\tilde{D}_{ij} = \delta_{ij} \langle \bar{\sigma}_{ij} \rangle$$

which is graphically represented in Fig. 12. It is clear from the figure that

$$\tilde{D}_{ij} = \delta_{ij} \langle \bar{\sigma}_{ij} \rangle$$

(13)

(14)

(15)

For small momenta $Q < Q(0)$ the correlator $K$ has a finite gap: $-K^{-1}(Q = 0) = \chi(Q = 0)$. We note that the functions $G$ and $-\chi K$ correspond to the functions $G_\alpha$ and $G_\beta$ of Refs. 11 to 13.

The correlation function $K$ has the meaning of a susceptibility in relation to the field $\bar{\sigma}_{ij} = H$:

$$\chi = \sum_\alpha \langle \bar{\sigma}_{ij} \rangle H = \sum_\alpha \langle \bar{\sigma}_{ij} \rangle H = -\chi_1 K(0).$$

4. FLUCTUATIONS IN THE LOW-TEMPERATURE PHASE

In this section we consider fluctuations in the low-temperature phase of a spin glass. We shall then have in mind the immediate vicinity of the phase transition

$$\epsilon = \left[ \begin{array}{c}
    1 + 2t + 3t^2 + 4t^3 + \cdots \\
    12t + 24t^2 + 36t^3 + 48t^4 + \cdots \\
    120t + 240t^2 + 360t^3 + 480t^4 + \cdots
  \end{array} \right],$$

FIG. 9.

$$Q = \sum_\alpha \langle \bar{\sigma}_{ij} \rangle H = \sum_\alpha \langle \bar{\sigma}_{ij} \rangle H = -\chi_1 K(0).$$

FIG. 10.

FIG. 11.
and therefore we shall perform the calculations in the main order in $Q$ and $\tau$. In that approximation $Q = \bar{Q}$ and Eqs. (8) and (9) are
\[
Q = -\frac{\mu^2}{T^2} Q - 2 \left( \frac{\mu^2}{T^2} \right)^2, \quad \tau = 1 - 2 \left( \frac{\mu^2}{T^2} \right) \bar{Q},
\]
so that the equation of state can be written in the form
\[
Q(\omega = 0) = 0
\]
and leads to the correct result.

We shall now show that the equation of state retains its form (17) also when fluctuations are taken into account. We show in Fig. 13 the characteristic diagrams which represent the corrections to the equation of state (16). One sees easily that each of these diagrams differs from the corresponding diagram for $Q = \bar{Q}$ only by a factor $(ZJ/\tau)^3$ which is equivalent to (17). In our formalism the correlation function $G$ (and, hence, also $D$) is thus automatically gapless. It is clear from Fig. 13a that the first correction to the equation of state contains a diagram which diverges logarithmically when $d = 4$ (all such diagrams will be found below) so that there is no phase transition for $d < d_c = 4$.

It is convenient to use the equation $G''(\omega = 0) = 0$ for the derivation of the equation of state $Q(\tau)$. To do this we take its total differential along the trajectory $Q = Q(\tau)$. We get thus
\[
\frac{dQ}{d\tau} = \frac{\partial G''}{\partial \tau} \bigg|_{Q=\bar{Q}} = \frac{\partial G''}{\partial \tau} \bigg|_{\omega=0}.
\]
There are Ward identities for the partial derivatives of $G''$
\[
\frac{\partial G''}{\partial \tau} \bigg|_{\omega=0} = 2T, \quad \frac{\partial G''}{\partial \tau} \bigg|_{\omega=0} = \bar{F}.
\]
We get thus
\[
\frac{dQ}{d\tau} \bigg|_{Q=\bar{Q}} = \bar{F}(0)/\bar{F}(0),
\]
where $\bar{G}(0), \bar{F}(0)$ are renormalized vertices for zero momentum evaluated for $Q = \bar{Q}$, i.e., with the gapless functions $G$ and $D$.

The renormalizations of the vertices are, as usual, in the region of momenta $|\tau| \ll \omega < 1$ the same as those found in the high-temperature phase; however, in the small momentum region $\omega^2 \ll |\tau|$ there occur very unusual situations. In contrast to the usual Goldstone modes in degenerate systems the vertices of the interaction of gapless modes in a spin glass do not contain moments so that already in a 6-dimensional space there is a strong interaction of long-wavelength fluctuations. The parquet diagrams for small momenta for $\Sigma$ are shown in Fig. 14. The diagrams are constructed from the elements shown in Fig. 5. The graph of Fig. 5e gives the diagram of Fig. 14a; the diagrams of Figs. 14b, c, d are obtained from the elements of Figs. 5b, c, and the diagram of Fig. 14e from the element of 5d. The vertices $W$ enter into the diagram of Fig. 14a. A new vertex $X$, which we now define, participates in the other graphs.

We consider the identity for the correlator
\[
G(z) = \left[ -W(z) - \gamma(z) \right]^{-1}.
\]
Its diagrammatic expression is shown in Fig. 15. It is clear from the figure that the second term of the identity contains the vertex $X$ which we need; together with the earlier determined vertices $W$ and $\Gamma$ we get (20) in the form
\[
\Delta \Gamma = \beta W \Delta. \quad (21)
\]
In the region $\omega^2 \ll |\tau|$ we have $|\Sigma| \ll 1$ so that we get from (21) $X \rightarrow W$. After this we use (18) to get the parquet equations ($d = 6$):
\[
d \ln \Gamma(z) = -W(z), \quad d \ln \delta(z) = -W(z), \quad d \ln \bar{F}(z) = -W(z). \quad (22)
\]
The equation for the vertex $W$ has the form
\[
d W(z) = W(z) - 1/2. \quad (23)
\]
We can therefore not determine the behavior of the correlation functions for the smallest momenta
\[
\rho^2(0, t) = \left[ -W(z) - \gamma(z) \right]^{-1} \left[ -W(z) - \gamma(z) \right]^{-1} \left[ -W(z) - \gamma(z) \right]^{-1}.
\]
However, this does not prevent us from determining the temperature-dependence of the order parameter $Q(\tau)$ and the specific heat $C(\tau)$ as we shall show that they are determined by moments of dimensions $\omega^2 \ll |\tau|$.

The fact is that Eqs. (22), (23) for the vertices $\Gamma$ and $W$...
Although Eqs. (22) to (25) are derived in the main parquet approximation, Eq. (26) is more general and remains the same in all orders. The fact is that in the region $q^2 \ll T_1$ differentiating $C$ with respect to $Q$ and $T$ leads to the same parquet diagrams, since only the denominators of the Green functions $G$ are differentiated in which $Q$ and $T$ occur additively. In other words, although the partial derivatives $\partial G^{-1}/\partial T$, $\partial G^{-1}/\partial Q$ are renormalized for $q^2 \approx T_1$, the total derivative along the trajectory $(dG/d\Gamma)_{\text{parquet}}$ does not contain integrals which are singular for small momenta. Thus, $Q = \sqrt{1 - \left(\frac{C}{T}\right)}$, where $\square$ and $\Gamma$ are the renormalized vertices at the momentum $q^2 \approx T_1$, determined by scaling in the region $|\tau| = q^2 \ll 1$; the critical index is $\beta = 1 + \frac{1}{2}$ [see Ref. 5 and our Eq. (6)].

It is convenient for the determination of the specific heat to use the expression for the internal energy $E$:

$$E = \frac{1}{2} \sum \langle j_i, j_i \rangle - \frac{1}{2} \sum \langle \Delta^2 \rangle - \frac{1}{2} \sum \langle j_i, j_i \rangle + \frac{1}{2} \sum \langle \Delta^2 \rangle - \frac{1}{2} \sum \langle \Delta \rangle^2, $$

(28)

This equation is obtained by evaluating the variation $\delta S / \delta (d_i, d_i)$ for the system.

The last term in (28) gives the self-consistent field result, the second, third, and fourth correspond to fluctuation corrections.

The singular part of the derivative of the specific heat $dC/d\tau = dH/d\tau^2$ contains parquet diagrams obtained by differentiating the Green functions in (25). As was explained above, the total temperature derivative does not contain diagrams which are singular at small momenta so that the singularity of $dC/d\tau$ is also determined by scaling:

$$(dG/d\tau)_{\text{parquet}} = -\alpha \tau, \quad \alpha = \frac{1}{2}. $$

(29)

It was noted by Pytte and Rudnick [9] that $dC/d\tau$ contains a constant (non-singular) part so that the very weak singularity of (29) is practically unobservable.

The diagram equation for $D_m^m$ has the form shown in Fig. 17. We restrict ourselves here to the mean field approximation for the correlation functions. In that approximation

$$G_m^m = \delta_{m,0} G, $$

so that the analytical expression of Fig. 17 is

$$D_m^m = b_m G^{m-1} C^{m-1} G, $$

(31)

where $G(q) \sim 1/q^4$, as in the Ising model.

The fourth rank tensor $D_m^m$ has three linear invariants: $D_m^1$, $D_m^2$, $D_m^3$. The correlator corresponding to the transverse fluctuations is obtained by antisymmetrizing $D_m^m$ with respect to the indices pertaining to a single point:

$$D_m^m = b_m G^{m-1} C^{m-1} G, $$

(32)

The irreducible correlator $K_2^2$ is symmetric in the indices pertaining to a single point that in contrast to (33) the second term in (31) cancels out and we find

$$D_m^m = b_m G^{m-1} C^{m-1} G, $$

(33)

where $m$ is the number of spin components and $\frac{1}{2}m(m-1)$ the number of transverse modes. We have thus proved the absence of transverse spin stiffness in a spin glass. Taking the fluctuations completely into account can apparently not change this result.

6. DISCUSSION OF THE RESULTS

We have constructed a diagram technique for a spin glass in the vicinity of the transition in terms of the physical correlation functions without using the replica method. Together with the standard method of introducing the order parameter it allows us to take consistently into account fluctuations in the low-temperature phase in the main order in $|T - T_c|$. We have detected a strong interaction of long-wave-length fluctuations leading to an increase in the effective charge in the large size region. The graphs which lead to the increase of the charge are, as follows from Fig. 16, due to the inhomogeneity of the low-temperature phase:

$$(\Delta^2 G)_{\text{parquet}} = \Delta \Delta \Delta \Delta G, $$

The correlation function $G$ remains gapless by virtue

$$G(q) \sim 1/q^4, $$

(34)

where $G(q) \sim 1/q^4$

$$(\Delta^2 G)_{\text{parquet}} = \Delta \Delta \Delta \Delta G, $$

The correlation function $G$ remains gapless by virtue

$$G(q) \sim 1/q^4, $$

(34)
of the equation of state. The instability observed when one uses the replica method\cite{10,11} originates, apparently, from the fact that the quantity $Q$ is forced to play the role of a fluctuating variable which contradicts its physical meaning. The procedure of annihilating the "tadpoles" applied by Bray and Moore\cite{17,18} leads to a result which is obtained automatically in our case. However, another kind of instability which arises in terms of higher order in $|T-T_c|$ exists also in our formalism. As explained above (section 3) it is connected with the inexact definition of the field associated with the order parameter $Q$. We have suggested a method of obtaining a correct expansion of the self-energy in powers of $Q$. Together with the condition $G^0(q=0)=0$ for the absence of a gap this enables us to determine the function $Q(q)$ to any order in $|T-T_c|$. It seems to us that the existence of a longitudinal gapless mode is a basic and most characteristic property of a spin glass. It would be extremely interesting to explain whether this mode be connected with some powers of $Q$. Together with the condition $F(q=0)\neq 0$ this enables us to determine the function $G(q)$ in terms of the momentum $q$ which is the same as for the short range interaction:

$$\tilde{F}(q) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} \left(1 - \cos \varphi \right) - q^2.$$ 

The main qualitative conclusions—the existence of a longitudinal gapless mode, the absence of transverse stiffness in a Heisenberg spin glass, and the absence of a phase transition in a three-dimensional system—are retained therefore for real spin glasses with an RKKY interaction. The "kink" in the magnetic susceptibility which is observed in spin glasses to depend on the time of observation is, apparently, connected with the very slow relaxation of metastable states. Of most interest would be to find an explanation of the logarithmic dependence of the "transition" temperature on the time of observation.

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