Theory of relaxation of metastable states

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(Submitted 20 February 1979; resubmitted 3 May 1979)
Zh. Eksp. Teor. Fiz. 77, 1417–1431 (October 1979)

The nucleation in the metastable phase near the critical point of a thermodynamic system is described as a relaxation of a metastable state of the order-parameter field. For cases with conserved and nonconserved scalar order parameters, equations are obtained that describe the random change of the dimension of the nucleus of the new phase. On the basis of the random-process equation a closed scheme is constructed for a statistical description of the metastable phase. Equations are obtained for the transition probability and for the distribution function of the sizes of the nuclei. The lifetime t_m of the metastable state and its variance D_m are calculated. The scaling behavior of t_m in the region of strong fluctuations is discussed.

PACS numbers: 05.70.Fh

1. FORMULATION OF PROBLEM

It is known that a metastable state of matter can be obtained in a first-order phase transition.[1] This state is thermodynamically stable to small perturbations. Its relaxation into a thermodynamically stable state is accompanied by formation of a critical nucleus of a new phase, suggesting the surmounting of an energy barrier. The classical theory of Zel'dovich and Vollmer (ZV)[2-4] considers the relaxation of a metastable state as relaxation of the distribution of the nuclei in size. An equation is postulated for the flux J of the nuclei along the phase axes in a region of large dimensions. The result obtained in the theory, $J=J_0\exp(-\Delta F_c/T)$ (where ΔF_c is the minimal work of production of the critical nucleus) is universal, apart from a pre-exponential factor, and is determined by the equilibrium thermodynamics of the system. The quantity J_0 is different for systems with different relaxation thermodynamics and is not calculated within the framework of the ZV theory.

The dynamics of the nucleation lends itself to observation, particularly near critical points of systems (for example, near critical stratification points). The purpose of the present paper is to investigate the dynamics of the relaxation of the metastable state of a system near a critical point. The theory developed below starts from the relaxation equation for the field (fields) of the order parameter. This makes it unnecessary to make additional assumptions concerning the properties of the nucleus (for example, concerning the dependence of the nucleus energy on the dimension, concerning the conditions on its boundary, etc.). These assumptions are not always obvious, particularly in a system with a large correlation radius. The method makes it possible to take into account in regular fashion the influence of various effects (fluctuations of the shape of the nucleus, coalescence processes, etc.) on the relaxation of the metastable state. The proposed theory is general in character and the ZV approximation can be obtained within its framework.

The dynamics of a system near a critical point is connected with the relaxation and fluctuations of the hydrodynamic modes—fields of the order parameter $\varphi(\mathbf{x}, t)$, energy density $\varepsilon(\mathbf{x}, t)$, etc. The slowness of their relaxation makes it possible to exclude other degrees of freedom that manage to reach local equilibrium. The form of the relaxation equations for the hydrodynamic modes is determined mainly by the conservation properties. We consider a system described by a scalar field $\varphi(\mathbf{x}, t)$; we assume that the energy of this field is not conserved.[5] For systems with a conserved parameter $\varphi(\mathbf{x}, t)$, the relaxation equation for $\varphi(\mathbf{x}, t)$ takes the form

$$\frac{1}{\Gamma_c}\frac{\partial\varphi}{\partial t}=-\Delta\left(-\frac{\delta H}{\delta\varphi}+f_{\rm extr}\right), \tag{1}$$

while for systems without conservation of $\varphi(\mathbf{x}, t)$ we have

$$\frac{1}{\Gamma_n}\frac{\partial\varphi}{\partial t}=-\frac{\delta H}{\delta\varphi}+f_{\rm extr}, \tag{2}$$

where $H\{\varphi\}$ is the effective Hamiltonian, which we take in the Landau form

$$H\{\varphi\}=\frac{1}{2}\int\left\{c(\nabla\varphi)^2+\mu\varphi^2+\frac{1}{2}g\varphi^4-2h\varphi\right\}d\mathbf{x}. \tag{3}$$

Here Γ_c and Γ_n are kinetic coefficients, and $f_{\rm extr}$ is an extraneous force that imitates a thermal ensemble.

The scheme used by us to describe the dynamics of the field of an order parameter is standard.[5] Near the critical point, the properties of the system with Hamiltonian (3) are well known.[6] At $h=0$ and $\mu=\mu_c<0$ there is a critical point in the system; the line $h=0$, $\mu<\mu_c$ is a line of first-order phase transitions. The metastable state of the field $\varphi(\mathbf{x}, t)$ is obtained by intersecting the line $h=0$ at $\mu<\mu_c$ in the thermodynamic plane. We shall investigate its relaxation for cases (1) and (2). The smearing of the interphase boundary does not make it possible in our case to use the illustrative concepts of particle absorption by the nucleus, and the mechanism of nucleus formation must be determined.

In the absence of an extraneous force $f_{\rm extr}$, Eqs. (1) and (2) have stationary homogeneous solutions $\varphi=\varphi_{1,2}(h)$, which are stable in the small and can be obtained from the condition

$$\mu\varphi+g\varphi^3=h,\quad \mu<0,\ g>0. \tag{4}$$

There is also a class of quasistationary solutions, for which $\varphi(\mathbf{x}, t)$ is almost everywhere close to the values

$\varphi_{1,2}$ with the exception of boundary regions of thickness $\delta \sim (c/|\mu|)^{1/2}$. The evolution of the quasistatic solutions reduces to motion of the boundaries. For a small extraneous force $f_{\text{extr}}(\mathbf{x}, t)$ the solutions of Eqs. (1), (2) at each instant of time constitute small fluctuations about the solutions at $f_{\text{extr}}=0$. The quantity $f_{\text{extr}}(\mathbf{x}, t)$ can be assumed small if the resultant amplitude of the fluctuations of the field $\varphi(\mathbf{x}, t)$ is small compared with the quantity $\varphi_s = (|\mu|/g)^{1/2}$ in a region with dimension $\lambda \lesssim r_c \sim (c/|\mu|)^{1/2}$. This condition is satisfied for systems that are in the region of applicability of the Landau theory[6]

$$1 \gg |\mu| \gg \text{Gi} = g^2T^2/c^3. \tag{5}$$

We consider first this case of small fluctuations.

2. NUCLEUS DYNAMICS FOR A SYSTEM WITH A NONCONSERVATIVE PARAMETER $\varphi(x,t)$

The relaxation of the system is described by Eq. (2). We consider first a spherical nucleus with center at the origin ($h=0, f_{\text{extr}}=0$). We introduce the following dimensionless variables: the radius ξ measured in units of $(2c/|\mu|)^{1/2}$, the time measured in units of $2/\Gamma_n|\mu|$, and the transition parameter

$$\psi(\xi, t) = \varphi(\xi, t)/\varphi_s, \quad \varphi_s = (|\mu|/g)^{1/2}.$$

For the angle-independent configuration $\psi(\xi, t)$ in n-dimensional space ($n = 1, 2, 3, \ldots$) we obtain

$$\frac{\partial\psi}{\partial t} = \frac{\partial^2\psi}{\partial\xi^2} + \frac{n-1}{\xi}\frac{\partial\psi}{\partial\xi} + 2(\psi-\psi^3). \tag{6}$$

The solution of Eq. (6) of the form of interest to us is well known for the one-dimensional case:

$$\psi(\xi, t) = \pm\text{th}(\xi-\xi_0), \quad \xi_0 = \text{const}. \tag{7}$$

We expect that in the case $n>1$ the solution will be similar to (7). At $\xi_0 \gg 1$ the term

$$\frac{n-1}{\xi}\frac{\partial\psi}{\partial\xi}$$

in (6) must be taken into account only in the region of the boundary of the nucleus $\xi \sim \xi_0(t)$, the position of which depends on the time t. At other values of ξ this term is relatively small. If the width of the boundary is small compared with the dimension of the nucleus, then Eq. (6) can be written in the form:

$$\frac{\partial\psi}{\partial t} = \frac{\partial^2\psi}{\partial\xi^2} + \frac{n-1}{\xi_0(t)}\frac{\partial\psi}{\partial\xi} + 2(\psi-\psi^3). \tag{8}$$

The solution of Eq. (8), which describes the relaxation of a spherical nucleus, is

$$\psi(\xi, t) = \pm\text{th}(\xi-\xi_0(t)), \quad \frac{d\xi_0}{dt} = -\frac{n-1}{\xi_0(t)}. \tag{9}$$

The quantity $\xi_0(t)$ is the effective radius of the nucleus and decreases with the time like

$$\xi_0(t) = \xi_0(0)\,(1-2(n-1)t/\xi_0^2(0))^{1/2}. \tag{10}$$

The solution (9) is valid if $\xi_0(t) \gg 1$. Nuclei with smaller dimensions $\xi_0(t) \sim 1$ attenuate to a solution that is homogeneous in space. Both phases $\langle\psi\rangle = \pm 1$ turn out to be stable at $h=0$ relative to formation of a nucleus of arbitrarily large size. The relaxation equation at $h \neq 0$ (h is measured in units of $\frac{1}{2}|\mu|\varphi_s$) has two homogeneous and stable (in the small) solutions only if $|h| \ll h_c = 4/3^{3/2}$. We shall consider weakly metastable states, when $|h| \ll h_c < 1$, so that it suffices to obtain the solution of the relaxation equation accurate to terms linear in h.

Real nuclei in a system have a shape close to spherical if their dimension is large compared with the correlation radius $\xi_{\text{cor}} = \frac{1}{2}$. We shall assume therefore that in the expansion of the nucleus radius $\xi_0(\theta, \varphi, t)$ in the spherical functions

$$\xi_0(\theta, \varphi, t) = \sum_l \xi_0^{lm}(t)\, Y_{lm}(\theta, \varphi) \tag{11}$$

the quantities $\xi^{lm}(t) (l \geq 1)$ are small compared with $\xi_0^0(t)$ which is the radius of the nucleus averaged over the angles. The calculations here and below are carried out for three-dimensional space, $n=3$. The solution of the relaxation equation

$$\frac{\partial\psi}{\partial t} = \frac{\partial^2\psi}{\partial\xi^2} + \frac{2}{\xi_0(\theta, \varphi, t)}\frac{\partial\psi}{\partial\xi} - \frac{\hat{l}^2\psi}{\xi_0^2(\theta, \varphi, t)} + 2(\psi-\psi^3) + h,$$
$$\hat{l}^2 = -\left[\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}\right] \tag{12}$$

accurate to terms of higher order in the quantities $\xi_0^l (l \geq 1)$ and h, is

$$\psi(\xi, t) = \text{th}[\xi-\xi_0(\theta, \varphi, t)] + \frac{1}{4}h; \quad -\frac{d\xi_0^0}{dt} = \frac{2}{\xi_0^0} + \frac{3h}{2},$$
$$-\frac{d\xi_0^l}{dt} = \frac{l(l+1)-2}{(\xi_0^0)^2}\xi_0^l, \quad l \geq 1. \tag{13}$$

As seen from (13), the amplitudes of the spherical harmonics $\xi_0^l (l \geq 2)$ attenuate in the course of relaxation quite rapidly:

$$\xi_0^l(t) = \xi_0^l(0)\exp\left\{-\int_0^t \frac{l(l+1)-2}{(\xi_0^0(t'))^2}dt'\right\}, \tag{14}$$

i.e., the nucleus becomes spherical. The undamped harmonic $\xi_0^1(t)$ describes the displacement of the nucleus as a unit in space. In a reference frame connected with the center of the nucleus we have $\xi_0^1(t) = 0$. The solution (13) describes a nucleus of a phase $\langle\psi\rangle \approx -1$ in a phase $\langle\psi\rangle \approx 1$. At $h<0$ the nuclei with angle-averaged radius $\xi_0^0 < \xi_c = 4/3|h|$ attenuate, while the nuclei with $\xi_0^0(0) > \xi_c$ increase. The phase $\langle\psi\rangle \approx 1$ at $h<0$ is metastable—unstable to formation of a nucleus with radius $\xi_0^0 > \xi_c$. At $h>0$ the phase $\langle\psi\rangle \approx 1$ is stable. A similar solution is obtained for the nucleus of the phase $\langle\psi\rangle \approx 1$ in the phase $\langle\psi\rangle \approx -1$.

So far we have disregarded the random force $f_{\text{extr}}(\mathbf{x}, t)$ due to the small-scale fluctuations. The properties of the random force f_{extr} were defined under the assumption that the small-scale fluctuations are in equilibrium. The random field $f_{\text{extr}}(\mathbf{x}, t)$ is Gaussian with a 0 mean value. The force $f_{\text{extr}}(\mathbf{x}, \text{t})$ is assumed to be weak, so that the amplitude of the response of the field $\varphi(\mathbf{x}, t)$ to the action of the force $f_{\text{extra}}(\mathbf{x}, t)$ is small compared with φ_s. We seek the solution of the relaxation equation with account taken of the force $f(\xi, t)$ measured in units of $|\mu|\varphi_s/2$

$$\frac{\partial\psi}{\partial t} = \Delta\psi + 2(\psi-\psi^3) + h + f_{\text{extr}}(\xi, t) \tag{15}$$

in the form

$$\psi(\xi,t)=\text{th}\,[\xi-\xi_0(\theta,\varphi,t)]+{}^1/_4h+v(\xi,t),$$
$$-d\xi_0{}^0/dt=2/\xi_0{}^0+{}^3/_2h+6v^0(\xi_0{}^0(t),t), \qquad (16)$$
$$-d\xi_0{}^l/dt=[l(l+1)-2]\xi_0{}^l/(\xi_0{}^0)^2+6v^l(\xi_0{}^0(t),t),$$

where the function $v(\xi,t)$ describes the response of the field $\psi(\xi,t)$ to the action of the force $f(\xi,t)$ and is a solution of the equation

$$\frac{\partial v}{\partial t}=\Delta v-4v+f(\xi,t), \qquad (17)$$

and

$$v(\xi,t)=\frac{1}{(2\pi)^3}\int\exp[ik\xi-(k^2+4)t]\left\{\int_0^t\exp[(k^2+4)t']f_k(t')dt'\right\}dk,$$

$$f_k(t)=\int f(\xi,t)\exp(-ik\xi)d\xi. \qquad (18)$$

The quantities $v^l(\xi,t)$ are the amplitudes of the expansion of the function $v(\xi,t)$ in spherical functions:

$$v(\xi,t)=\sum v^{lm}(\xi,t)Y_{lm}(\theta,\varphi). \qquad (19)$$

The quantity $v(\xi,t)$ is a linear function of the force $f(\xi,t)$, and if the force is weak enough we have $|v|<1$. We investigate now the correlation properties of the field $v(\xi,t)$. Assuming the random force $f(\xi,t)$ to be δ-correlated:

$$\langle f(\xi_1,t_1)f(\xi_2,t_2)\rangle=\alpha\delta(t_1-t_2)\delta(\xi_1-\xi_2),$$

we obtain

$$\langle v(\xi_1,t_1)v(\xi_2,t_2)\rangle=\frac{\alpha}{(2\pi)^3}\int\frac{\exp[-(k^2+4)|\delta t|]}{(k^2+4)}\exp[ik(\xi_1-\xi_2)]dk. \qquad (20)$$

It is seen from (20) that the field $v(\xi,t)$ is correlated in the space ($\xi_{\text{cor}}=\frac{1}{2}$) and in time ($t_{\text{cr}}=\frac{1}{4}$). The assumption that the field $f(\xi,t)$ is δ-correlated can always be made if its proper correlation radii are small compared with those obtained for the field $v(\xi,t)$.

We turn now to Eq. (15). Substituting in it the solution (16) and retaining only the terms linear in $v(\xi,t)$, we find that the equation is satisfied accurate to the term

$$[v(\xi,t)-v(\xi_0(\theta,\varphi,t),t)]\,\text{ch}^{-2}[\xi-\xi_0(\theta,\varphi,t)].$$

At $|\xi-\xi_0(\theta,\varphi t)|>1$ this term is exponentially small. For $\xi\sim\xi_0(\theta,\varphi,t)$ we expand $v(\xi,t)$ in a series near $\xi_0(\theta,\varphi,t)$. Since the correlation radius of the field $v(\xi,t)$ is of the order of the width of the boundary of the nucleus, we can confine ourselves in the expansion to the first term. In this approximation, the solution (16) satisfies Eq. (15). Allowance for their random force $f(\xi,t)$ leads to small fluctuations of the amplitude $\psi(\xi,t)$ and of the nucleus boundary, described by Eq. (16). The amplitudes of spherical harmonics of radius $\xi_0^l(t)$ fluctuate under the influence of the corresponding harmonics of the effective random force $v(\xi,t)$, which is a functional of the field $f(\xi,t)$.

3. NUCLEUS DYNAMICS FOR S SYSTEM WITH A CONSERVATIVE PARAMETER $\varphi(x,t)$

We write down Eq. (1), using the dimensionless radius ξ measured in units of $(2c/|\mu|)^{1/2}$, the time t measured in units of $4c/\Gamma_c|\mu|^2$, and the transition parameter $\psi=\varphi/\varphi_s$. The field h and the extraneous force f_{extr} are measured in units of $|\mu|\varphi_s/2$,

$$\frac{\partial\psi}{\partial t}=-\Delta[\Delta\psi+2(\psi-\psi^3)+f(\xi,t)]. \qquad (21)$$

Just as in the case considered above, we obtain in the absence of a random force f_{extr} a quasistatic solution that describes a spherical nucleus.

The homogeneous solutions of Eq. (21) ($f_{\text{extra}}=0$) satisfy the condition

$$\psi^3-\psi-h/2=0 \qquad (22)$$

and describe the two phases of the system. The field h in the investigated case is the analog of the chemical potential of the system. For example, for the binary mixture $h\sim\mu_c-\mu_i$, where μ_c is the chemical potential on the stratification line and μ_i is the chemical potential of the initial phase.

We consider the nucleus of the phase $\langle\psi\rangle\approx-1$ in the metastable phase $\langle\psi\rangle\approx1$, corresponding to $h<0$ and $|h|\ll1$. An arbitrary configuration of the field $\psi(\xi,t)$ should satisfy in this case, in accord with the conservation law, the condition $\psi(\infty,t)=1-|h|/4$. The stationary solution of (21) ($f_{\text{ext}}=0$) corresponding to a nucleus of radius $\xi_0\gg1$ is

$$\psi(\xi,t)=\text{th}\,(\xi-\xi_0)-1/3\xi_0,\quad \xi_0=\text{const}. \qquad (23)$$

It satisfies the boundary condition only in the case when $\xi_0=4/3|h|=\xi_c$. A nucleus of this size is critical.

For a nucleus with dimension $\xi_0\neq\xi_c$ it is necessary to find that the correction to the solution (23). The correction $\delta\psi=u(\xi,t)$ at $\xi\gg\xi_c$ tends to the limit $1/3\xi_0-|h|/4$. Inside the nucleus $\xi<\xi_0$ the correction $u(\xi,t)$ tends to zero. Outside the nucleus there exists a nonzero flux ($\nabla u\neq0$), and the dimension ξ_0 of the nucleus depends on the time t. Retaining in (21) terms up to those linear in $u(\xi,t)$, we get

$$\frac{\partial u}{\partial t}-\frac{d\xi_0}{dt}\text{ch}^{-2}(\xi-\xi_0)=-\Delta[\Delta u-4u+6u\,\text{ch}^{-2}(\xi-\xi_0)]. \qquad (24)$$

Since the relaxation of the nucleus is quasistatic, the quantity $\partial u/\partial t$ is of higher order of smallness than the other terms of (24). The terms containing the factor $\cosh^{-2}(\xi-\xi_0)$ must be taken into account only in the region of the boundary of the nucleus $\xi_0\sim\xi_0(t)$. Outside the boundary ($\xi\gg\xi_0$ or $\xi\ll\xi_0$) we obtain the equation

$$\Delta(\Delta u-4u)=0. \qquad (25)$$

We take the solution of (25) in the form $u_i=\beta_i+\alpha_i/\xi$ ($i=1,2$). Inside the nucleus ($0<\xi<\xi_0$) we have $\alpha_1=0$ and $\beta_1=0$. The solution outside the nucleus should go over into the solution inside the nucleus in the region $\xi\sim\xi_0$, therefore $\alpha_2=-\beta_2\xi^*$, where $\xi^*\sim\xi_0$. We use the boundary condition

$$u(\infty,t)={}^1/_3(1/\xi_0-1/\xi_c),\quad \xi_c=4/3|h|$$

and obtain

$$u(\xi,t)=\begin{cases}0, & \xi\leq\xi^*\\ {}^1/_3(1/\xi_0-1/\xi_c)(1-\xi^*/\xi), & \xi>\xi^*.\end{cases} \qquad (26)$$

We used the law of conservation of the parameter $\psi(\xi,t)$

to determine the dependence of ξ_0 on t. We integrate Eq. (24) over the region $0 < \xi < \bar{\xi}(\bar{\xi} \gg \xi_0)$. The integral of the right-hand side of the equation will be transformed into a surface integral over a sphere of radius $\bar{\xi}$. As a result we obtain

$$d\xi_0/dt = -{}^2/_3(1/\xi_0 - 1/\xi_c)\xi^*/\xi_0^2. \tag{27}$$

We require that the solution (26), (27) satisfy the Eq. (25) also in the nucleus boundary region $\xi \sim \xi_0$. We obtain $\xi^* = \xi_0$. Thus, the relaxation of a spherical nucleus is described by the solution

$$\begin{gathered}\psi(\xi, t) = \mathrm{th}\,[\xi - \xi_0(t)] - 1/3\xi_0 + u(\xi, t),\\ d\xi_0/dt = -{}^2/_3(1/\xi_0 - 1/\xi_c)/\xi_0.\end{gathered} \tag{28}$$

The generalization to the case of an arbitrary form of the nucleus and the allowance for the weak random force f_{extra} are carried out in the same manner as in the preceding case. In the principal order in the force f_{extra} and in the harmonics of the radius $\xi_0^l (l \geq 1)$ the solution of (21) is

$$\begin{gathered}\psi(\xi, t) = \mathrm{th}[\xi - \xi_0(\theta, \varphi, t)] - 1/3\xi_0 + u(\xi, t) + v(\xi, t),\\ -d\xi_0^0/dt = {}^2/_3(1/\xi_0^0 - 1/\xi_c)/\xi_0^0 + 12v^0(\xi_0^0(t), t),\\ -d\xi_0^l/dt = 2[l(l+1) - 2]\xi_0^l/(\xi_0^0)^3 + 12v^l(\xi_0^0(t), t).\end{gathered} \tag{29}$$

The function $u(\xi, t)$ coincides here with the function defined by (26), with $\xi_0(t)$ replaced by the nucleus radius $\xi_0^0(t)$ averaged over the angles ($\xi^* = \xi_0^0$). The function $v(\xi, t)$ describes the response of the field $\psi(\xi, t)$ on the action of the random force $f(\xi, t)$ and is a solution of the equation

$$\begin{gathered}\partial v/\partial t = -\Delta[\Delta v - 4v + f(\xi, t)],\\ v(\xi, t) = \frac{1}{(2\pi)^3}\int \exp(i\mathbf{k}\xi - k^2(k^2+4)t)\left\{\int_0^t \exp(k^2(k^2+4)t')f_{\mathbf{k}}(t')\,dt'\right\}d\mathbf{k}.\end{gathered} \tag{30}$$

The quantities $v^l(\xi, t)$ are spherical harmonics of the field $v(\xi, t)$.

Eqs. (29) for the quantities $\xi_0^l(t)$ $(l \geq 2)$ describe the fluctuations of the shape of the nucleus. The shape of the nucleus remains close to spherical if $\xi_0^0 \gg 1$. The random deviations of the nucleus as a unit are described by the equation for $\xi_0^1(t)$. The solution (29) describes the metastable phase $\langle\psi\rangle \approx 1$. At $h > 0$, the phase $\langle\psi\rangle \approx 1$ is stable and the metastable phase is $\langle\psi\rangle \approx -1$.

4. CASE OF STRONG FLUCTUATIONS

In the case $\mathrm{Gi} \gtrsim |\mu|$ the amplitude of the fluctuations of the field $\varphi(\mathbf{x}, t)$ with scales $\lambda \sim r_c$ is not small compared with the difference between the mean values of the field in the two phases. In this case the state with the nucleus is not described by a specific configuration of the field $\varphi(\mathbf{x}, t)$, and has an appreciable probability of containing a set of strongly differing configurations. The field $\varphi(\mathbf{x}, t)$, averaged over the fluctuations of the scales $\lambda \lesssim R$ $(R \gg r_c)$, fluctuates weakly. For a field smoothed out to such a scale we can determine the configuration that describes the nucleus. The relaxation equation for the smoothed field is obtained by the renormalization group method (see, e.g., Ref. 5).

For scales $R \gg r_c$ the result in the thermodynamic-equilibrium state is determined by the fact that the renormalized Hamiltonian lands in the vicinity of a Gaussian immobile point. The equations of motion and the Hamiltonian take the form (1), (2), (3) but with renormalized coefficients: $\Gamma, c, \mu, g \to \Gamma^*, c^*, \mu^*, g^*$. To determine the procedure of smoothing in the metastable state it is necessary to know the probability distribution of the configurations of the field $\varphi(\mathbf{x}, t)$ in this state. This distribution will be given in the next section, and $R < R_c$ it coincides for scales with the given distribution. Consequently, for small deviations of the smoothed field $\varphi(\mathbf{x}, t)$ from an equilibrium from the metastable mean value, the Hamiltonian takes the form of the Landau Hamiltonian

$$H\{\varphi\} = \frac{1}{2}\int\left\{c^*(\nabla\varphi)^2 + \mu^*\varphi^2 + \frac{1}{2}g^*\varphi^4 - 2h\varphi\right\}d\mathbf{x}.$$

For large deviations of the smoothed field from the mean values $|\delta\varphi| \sim \varphi_s$ the explicit form of the Hamiltonian is not determined from general considerations. If the Hamiltonian $H\{\varphi\}$ differs from the Landau Hamiltonian only by terms that are independent of $\nabla\varphi$, then the dynamics of the nucleus coincides with that considered in Secs. 2 and 3, except that the quantities Γ, c, μ, and g are replaced by their renormalized quantities Γ^*, c^*, μ^*, and g^*, which are defined in terms of measureable characteristics of the system (the ordering φ_s, the susceptibility χ, and the correlation radius r_c) by means of the formulas

$$\varphi_s = (|\mu^*|/g^*)^{1/2}, \quad \chi = 1/2|\mu^*|, \quad r_c^2 = c^*/2|\mu^*|. \tag{31}$$

Let the relaxation of the configuration of the smoothed field $\varphi(\mathbf{x}, t)$ be described by the equation

$$\frac{1}{\Gamma_n^*}\frac{\partial\varphi}{\partial t} = -\frac{\delta H\{\varphi\}}{\delta\varphi}.$$

The change of the field $\varphi(\mathbf{x}, t)$ at the point $\mathbf{x}$ during the time dt is

$$\delta\varphi(\mathbf{x}, t) = \frac{\partial\varphi}{\partial t}dt.$$

The total change of the energy of the configuration of the field $\varphi(\mathbf{x}, t)$ is

$$\delta H = \int\frac{\delta H}{\delta\varphi}\delta\varphi\,d\mathbf{x} = -\left\{\frac{1}{\Gamma_n^*}\int\left(\frac{\partial\varphi}{\partial t}\right)^2 d\mathbf{x}\right\}dt. \tag{32}$$

For configurations describing a nucleus, the quantity $\partial\varphi/\partial t$ deviates noticeably from zero only in the region of the boundary of the nucleus. Recognizing that

$$\partial\varphi/\partial t = -\dot{r}\nabla\varphi,$$

we get

$$\left\{\frac{1}{\Gamma_n^*}\int\left(\frac{\partial\varphi}{\partial t}\right)^2 d\mathbf{x}\right\}dt = \dot{r}^2 S a^2\,dt, \quad a^2 = \frac{1}{\Gamma_n^* S}\int(\nabla\varphi)^2\,d\mathbf{x}, \tag{33}$$

where the quantity a^2 does not depend on the surface area S of the nucleus. On the other hand

$$\delta H = \delta H_V + \delta H_S = 2h\varphi_s\dot{r}S\,dt + \frac{2\alpha}{r}\dot{r}S\,dt, \tag{34}$$

where $2\varphi_s$ is the difference between the mean values of the field in the two phases; α is by definition the effective surface tension. From (33) and (34) we get

$$\dot r=-\frac{2\alpha}{a^2}\left(\frac{1}{r}-\frac{1}{R_c}\right),\quad R_c=\frac{\alpha}{|h|\varphi_0}. \tag{35}$$

For the system (1) with conserved transition $\varphi(\mathbf{x}, t)$ parameter the equation of motion can be written in the form

$$\frac{\delta H}{\delta\varphi}=-\frac{1}{4\pi}\int\frac{\partial\varphi(\mathbf{x}',t)}{\partial t}|\mathbf{x}-\mathbf{x}'|^{-1}d\mathbf{x}'. \tag{36}$$

Repeating the arguments presented above, we obtain

$$\dot r=-\frac{2\alpha}{b^2}\left(\frac{1}{r}-\frac{1}{R_c}\right)\frac{1}{r},\quad b^2=\frac{a^2}{4}\frac{\Gamma_n^*}{\Gamma_c^*}. \tag{37}$$

The properties of the random force that causes a change of the radius of the nucleus coincide in the case of weak and strong fluctuations. We note that when the system is described by the averaged field $\varphi(\mathbf{x}, t)$, the lower-bound of the correlation radius is lifted, so that the description is applicable also far from the critical point. Formulas (35) and (37) can be expressed in the same form as the formulas of the preceding section, by introducing the effective quantites Γ^*, μ^*, etc. In this case the scale dimensionality of these quantities, as can be readily verified, coincides with the scale dimensionality that follows for these quantities from the theory of equilibrium fluctuations.[5]

5. STATISTICAL DESCRIPTION OF THE METASTABLE STATE AND OF ITS RELAXATION

The metastable state produced when the first-order phase transition lines are crossed with finite velocity is a state of incomplete equilibrium. In such a state, the distribution of a small scale ($\lambda \ll R_c$) of the degrees of freedom corresponds to local equilibrium for slowly varying large-scale degrees of freedom ($\lambda \gtrsim R_c$) and is close to their distribution in the initial phase. The critical dimension R_c depends on the depth of penetration into the region of metastability of the initial phase. In the case of weak metastability, the critical dimension is by definition large compared with the correlation radius r_c. On the phase-transition line $R_c=\infty$, whereas the correlation radius is finite.

The considered systems with correlation radii larger than the interatomic dimension are described by the transition-parameter field $\varphi(\mathbf{x}, t)$. We introduce the transition-parameter field smoothed out to a scale $\lambda(r_c \ll \lambda < R_c)$

$$\varphi_\lambda(\mathbf{x},t)=\frac{1}{V}\int I_\lambda(\mathbf{x}-\mathbf{x}')\varphi(\mathbf{x}',t)d\mathbf{x}', \tag{38}$$

where $I_\lambda(\mathbf{x})$ is a certain smoothing function with characteristic dimension λ, for example, $I_\lambda(\mathbf{x})=\exp(-x^2/\lambda^2)$. The fluctuations of the field $\varphi(\mathbf{x}, t)$ with scales $R>\lambda$ are small, therefore the quantity

$$|\varphi_\lambda-\varphi^+|\ll\varphi^+-\varphi^-,$$

(where $\varphi^\pm$ is the mean value of the field $\varphi(\mathbf{x}, t)$ in the initial (+) and final (−) phases, respectively), provided only that the averaging region is not occupied by a nucleus of a new phase with dimension $R \gtrsim \lambda$.

The nuclei of the dimensions $R > R_c$ increase with overwhelming probability. Their presence in the system denotes that a transition into the heterophase state has taken place. In a homogeneous metastable state there should be no such nuclei ($R \gtrsim R_c$). These arguments allow us to propose a distribution of the probabilities of the configurations of the field $\varphi(\mathbf{x}, t)$ also in the metastable phase.[7]

We introduce the functional $\rho^+\{\varphi_\lambda\}$ of the smoothed field $\varphi_\lambda(\mathbf{x})$ with the following properties: $\rho^+\{\varphi_\lambda\}=1$ if everywhere

$$\varphi_\lambda(\mathbf{x})>\varphi^+-{}^1/_2(\varphi^+-\varphi^-),$$

and $\rho^+\{\varphi_\lambda\}=0$ if these conditions are violated even at one point. The probability density of the configurations of the fields $\varphi_+(\mathbf{x})$ in a state that is a metastable continuation of the phase φ^+ is by assumption

$$W_m{}^+\{\varphi\}=\exp\left[\frac{F-H\{\varphi\}}{T}\right]\rho^+\{\varphi_\lambda\}. \tag{39}$$

The probability of configurations containing at least one nucleus of a new phase with dimension $R>\lambda$ is equal to zero. On the set of the remaining configurations, the distribution $W_m^+\{\varphi\}$ coincides with the Gibbs distribution. If $\lambda \gg r_c$, then the distribution W_m^+ differs from the Gibbs distribution only for configurations of the field $\varphi(\mathbf{x})$ that are extremely improbable in the region of the stability of the phase φ^+. Consequently, in this region, these distributions are thermodynamically identical. The distribution $W_m^+\{\varphi\}$ describes the stable states of the phase φ^+, and also states that are metastable continuations of the phase φ^+ and in which $R_c>\lambda$. We can construct analogously the ensemble $W_m-\{\varphi\}$, which describes the metastable continuation of the phase φ^-. The ensembles $W_m^\pm$ are not stationary and should be used as initial distributions when solving the problem of the relaxation of the corresponding metastable states.

The relaxation of the metastable state will be described as the relaxation of the distribution of the nuclei of the new phase. The distribution of the nuclei at instant $t=0$, namely $W(r,0)=W_m$, is determined by the ensemble (39). The evolution of each nucleus is described by either Eq. (16) or (29), depending on whether the transition parameter is not conserved on conserved, respectively. We note that both in the system (16) and in (29) the quantities $\xi_0^l(l\geq 1)$ do not enter in the equation for the radius $\xi_0^0(t)$ averaged over the angles. In the approximation considered, the distribution $W(r, t)$ in the values of the radius r turns out to be independent of the distribution in the deviations from the spherical form of $\xi_0^l(l\geq 1)$.

We consider now nuclei with dimensions $r\gg\lambda_0$, where λ_0 is a scale whose fluctuation amplitude is comparable with the value of the spontaneous ordering. The concentration of these nuclei is small, and the probability of their collision can be neglected. The interaction of the nuclei ($r\lesssim\lambda_0$) with the fluctuations of the scales ($r\lesssim\lambda_0$) is taken into account by the effective random force $v(\xi, t)$. In this approximation, the change of the nucleus radius $\xi_0^0(t)$ averaged over the angles is described in both considered cases by the equation

$$d\xi_0{}^0/dt=-[F(\xi_0{}^0(t))+cv^0(\xi_0{}^0(t),\ t)]. \tag{40}$$

The regular "force" $F(\xi_0^0)$ is determined by formulas (13) or (27); $v^0(\xi_0^0, t)$ is the value, averaged over the sphere $\xi_0^0(t)$ of the effective random force $v(\xi, t)$ determined by formulas (18) and (30) respectively; c is equal to 6 or 12.

The field $v^0(\xi_0^0(t), t)$ by virtue of the relative slowness of the change of the dimension of the nucleus, can be regarded as δ-correlated in time

$$\langle v^0(\xi_0{}^0(t_1), t_1) v^0(\xi_0{}^0(t_2), t_2)\rangle = 2D(\xi_0{}^0)\delta(t_1-t_2), \tag{41}$$

where $D(\xi_0^0) = D/(\xi_0^0)^2$ in the case of a system with a nonconserved transition parameter and $D(\xi_0^0) = D/(\xi_0^0)^3$ in the case of a system with a conserved transition parameter.

Following the theory of homogeneous random processes,[8] we introduce the transition probability

$$P(r, r_0, t) = \langle\delta(r-\xi_0{}^0(t))\rangle, \quad r_0 = \xi_0{}^0(0), \tag{42}$$

where the averaging is over all the realizations of the random force $v^0(\xi_0^0, t)$. The quantity $P(r, r_0, t)$ is the probability that at the instant of time t the dimension of the nucleus is equal to r if $r = r_0$ at $t = 0$. The distribution of the nuclei at the instant of time t is

$$W(r,t) = \int P(r, r_0, t) W_m(r_0)\, dr_0. \tag{43}$$

The distribution (43) satisfies the Kolmogorov equation determined with the aid of (40) and (41)

$$\frac{\partial W}{\partial t} - \frac{\partial}{\partial r}\{F(r)W\} = \frac{\partial}{\partial r}\left\{D(r)\frac{\partial W}{\partial r}\right\}, \tag{44}$$

where

$$F(r) = 2(1/r + 3h/4), \quad D(r) = D/r^2$$

in the case of a system with a nonconserved transition parameter and

$$F(r) = {}^2/_3(1/r + 3h/4)/r, \; D(r) = D/r^3$$

in the case of a system with a conserved transition parameter.

The stationary solution of the equation (44)

$$W(r) = V\omega \exp\left[-\frac{1}{D}\left(r^2 + \frac{h}{2}r^3\right)\right] \tag{45}$$

is, at $h \geq 0$, the distribution of the nuclei of the phase $\langle\psi\rangle \approx -1$ in the stable phase $\langle\psi\rangle \approx 1$. Comparing (45) with the equilibrium distribution of the nuclei, we find $D = 3T/8\pi$ (where T is the temperature measured in units of $|\mu|\varphi_s^2 r_c^3$) and V is the volume of the system. The quantity ω does not depend on the dimension r and can be calculated by integrating the Gibbs distribution over all the configurations corresponding to a nucleus of dimension r, at $h = 0$.

When $h < 0$ the distribution (45) increases without limit as $r \to \infty$. This means that the most probable are states with a nucleus of infinite size, i.e., the system has gone over into a new phase state. The distribution of nuclei in the metastable phase $W_m(r)$, corresponding to (39), is leave

$$W_m(r) = V\omega \exp\left[-\frac{8\pi}{3T}\left(r^2 - \frac{2}{3}\frac{r^3}{R_c}\right)\right]\theta(r_0 - r), \tag{46}$$

where

$$R_c = 4/3|h| \gg 1, \; 1 \ll r_0 < R_c,$$

$$\theta(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}.$$

The relaxation of the distribution (46) is described by Eq. (44). During a time that is short compared with the lifetime t_m of the metastable state, a stationary relaxation regime is established—a small flux of nuclei J in a region of large dimensions

$$J = D(r)\, dW/dr + F(r)W. \tag{47}$$

The flux J is a constant. Equation (47) is postulated in the ZV theory (see e.g., Ref. 9).

The distribution of the nuclei of dimensions $r \ll R_c$, accurate to small J, coincides with $W_m(r)$. Solving (47) with the boundary conditions $W(1) = W_m(1), W(\infty) = 0$, we get

$$t_m = J^{-1} = \frac{1}{V\omega}\int_1^\infty \frac{1}{D(r)} \exp\left[\int_1^r \frac{F(r')}{D(r')}dr'\right] dr. \tag{48}$$

The quantity (48) is the average time of appearance of a transcritical nucleus in the volume V. This time is inversely proportional to the volume and for large V it can be small than the time of establishment of the stationary regime. It is reasonable to define the lifetime of the metastable state by assuming that the volume V is much smaller than the volume of the system but much larger than the dimension of the critical nucleus.

To study the statistics of formation of transcritical nuclei, we introduce the probability $P(r, t)$ that a nucleus having at $t = 0$ a dimension r acquires in a time t a dimension larger than R:

$$P(r,t) = \int_R^\infty P(r', r, t)\, dr'. \tag{49}$$

The quantity $P(r, t)$ satisfies the second Kolmogorov equation[8]

$$\partial P/\partial t = -(F(r) - dD/dr)\,\partial P/\partial r + D(r)\,\partial^2 P/\partial r^2. \tag{50}$$

Assume that the system contains centers of nucleus formation of dimension r_i with concentration ρ. The dimension of the nucleus produced on such a center is $r \geq r_i$. The probability that a nucleus larger than R is produced on the center during the time t is the solution of Eq. (50) with initial condition $P(r, 0) = 0(r_i < r < R)$ and boundary conditions

$$P(R,t) = 1, \quad \left.\frac{\partial P}{\partial r}\right|_{r \to r_i} = 0.$$

We calculate the moments of the distribution $P(r, t)$:

$$T_n(r) = \int_0^\infty t^n \frac{\partial P}{\partial t} dt. \tag{51}$$

The quantity $\partial P/\partial t$ is the probability of production of a nucleus with dimension larger than R in unit time, if the dimension was r at $t = 0$. Consequently, $T_1(r)$ is the mean expectation time of such an event, $[T_2(r) - T_1^2(r)]^{1/2}$ is the variance of the expectation time, etc. Applying the operator

$$\hat{T}_n = \int_0^\infty t^n \frac{\partial}{\partial t} dt$$

to Eq. (50) we obtain equations for the moments:

$$D(r)\,d^2T_n/dr^2-(F(r)-dD/dr)\,dT_n/dr=-nT_{n-1}(r)\,. \tag{52}$$

For $T_0(r)=P(r,\infty)=1$.

Solving (52) with the boundary conditions

$$T_n(R)=0,\quad \left.\frac{dT_n}{dr}\right|_{r=r_i}=0,\quad n\geq 1,$$

we obtain

$$T_n(r)=n\int\limits_r^R\frac{1}{D(r')}\exp\left[\int\limits_{r_i}^{r'}\frac{F(r'')}{D(r'')}dr''\right]\times\left\{\int\limits_{r_i}^{r'}\exp\left[-\int\limits_{r_i}^{r''}\frac{F(r''')}{D(r''')}dr'''\right]T_{n-1}(r'')\,dr''\right\}dr'. \tag{53}$$

At $t=0$, the dimensions of the nuclei produced on the centers are $r\sim r_i$. In the case $r_i\ll R\sim R_c$, we calculate the integral in (53) by the saddle-point method and find that $T_1(r)$ does not depend on r. This means that, just as in the case of homogeneous nucleation, a stationary relaxation regime is established. The expression for the flux J_i differs from the expression for J only in normalization. The lifetime of the metastable state is $t_m=(J+J_i)^{-1}$. In the case $r_i\gtrsim R_c$, the relaxation of the distribution of the nuclei reduces to its displacement along the dimension axis into the region of large r at a rate $F(r)$, and to a spreading as a result of diffusion $D(r)$. If we assume that at $t=0$ we have

$$W_i(r)=V\rho\delta(r-r_i),$$

then the moments (53) describe the subsequent distribution W_i.

The equations for the dynamics of the nucleus (16) and (29), describe also the changes in the form ($l\geq 2$) and position of the center of gravity ($l=1$). The analysis of the ($l\geq 1$) equations is simple if it is recognized that the nucleus radius $\xi_0^0(t)$ averaged over the angles varies slowly. The amplitudes of the probable fluctuations of the quantities $\xi_0^l(l\geq 2)$ are of the order of the width of the boundary of the nucleus, i.e., the deviation of the shape of the nuclei with dimensions $\xi_0^0\gg 1$ from spherical is small. If necessary, the influence of the fluctuations of the shape on the nucleus growth process can be taken into account in the next order of the theory. The equation for ξ_0^1 describes the increase of the nucleus as a whole, in which case the diffusion coefficient $\bar{D}$ in space is described in terms of the thickness D determined in formulas (41) and (45).

Writing down the results in dimensional units, we obtain

$$\bar{D}=\frac{3}{16\pi}\frac{T}{|\mu|\varphi_s^2r_c^3}\left(\frac{r_c}{r}\right)^2=\frac{3}{16\pi}\left(\frac{\mathrm{Gi}}{|\mu|}\right)^{1/2}\left(\frac{r_c}{r}\right)^2. \tag{54}$$

In the weak-fluctuation region the diffusion coefficient is small. In the strong-fluctuation region, where $\mathrm{Gi}|\mu|\gtrsim 1$, the formula (54) is valid for nuclei with dimensions $r\gg\lambda$, where λ is the smoothing scale, chosen such that

$$\frac{T}{|\mu|\varphi_s^2r_c^3}\left(\frac{r_c}{\lambda}\right)^2<1.$$

In the region of strong fluctuations, the diffusion coefficient of the considered nuclei is also small. The smallness of the diffusion coefficient and of the concentration of the nuclei with dimensions $r\gg r_c$ justifies the neglect of the probability of their coalescence in the course of the growth. The mobility of the nucleus, according to the Einstein formula, is $b=\bar{D}/T$.

6. DISCUSSION OF RESULTS

Substituting in (48) the functions $F(r)$ and $D(r)$, we obtain the lifetime of the metastable state. Using dimensional quantities, we obtain in the case of a system without conservation of the transition parameter

$$t_m^n=\frac{16\pi}{V\omega}\frac{1}{\Gamma_n|\mu|}\left(\frac{2}{3}\frac{|\mu|\varphi_s^2r_c^3}{T}\right)^{1/2}\left(\frac{R_c}{r_c}\right)^2\exp\left[\frac{128\pi}{9}\frac{|\mu|\varphi_s^2r_c^3}{T}\left(\frac{R_c}{r_c}\right)^2\right], \tag{55}$$

and in the case with conservation

$$t_m^c=\frac{32\pi}{V\omega}\frac{c}{\Gamma_c|\mu|^2}\left(2\frac{|\mu|\varphi_s^2r_c^3}{T}\right)^{1/2}\left(\frac{R_c}{r_c}\right)^3\exp\left[\frac{128\pi}{9}\frac{|\mu|\varphi_s^2r_c^3}{T}\left(\frac{R_c}{r_c}\right)^2\right]. \tag{56}$$

In either case, the critical dimension is

$$R_c=\frac{2}{3}\frac{|\mu|\varphi_s}{|h|}r_c, \tag{57}$$

where r_c is the correlation radius.

Formulas (55) and (56) are applicable for weakly metastable states, when $R_c/r_c\gg 1$, which coincides with the usual weak-field condition of the theory of phase transitions. In the case of weak fluctuations, the quantity $|\mu|\varphi_s^2r_c^3/T$ separated by us is large. It is expressed in terms of the Ginzburg number

$$\frac{|\mu|\varphi_s^2r_c^3}{T}=\left(\frac{|\tau|}{\mathrm{Gi}}\right)^{1/2},\quad \tau=\frac{T-T_c}{T_c}. \tag{58}$$

Weak fluctuations are described by the Landau theory, in which case $\mu=\mu_0\tau$. The other coefficients g and c of the effective Hamiltonian and the kinetic coefficient $\Gamma_{n(c)}$ are slowly varying functions of the temperature: $\varphi_s=(|\mu|/g)^{1/2}$ and $r_c=(c/2|\mu|)^{1/2}$.

As the critical point is approached $\tau\to 0$ the condition $|\tau|/\mathrm{Gi}\gg 1$ is violated, and the fluctuations are no longer weak. As shown in the analysis of the strong fluctuations, the obtained formulas (55) and (56) can be retained by replacing in them the quantities Γ, c, μ, and g by the renormalized values that have definite scale dimensionalities. The fluctuations have the similarity property, as is well known,[6] at scales $r\lesssim r_c$. In the metastable state the distribution of such fluctuations is determined by the same effective Hamiltonian as the stable phase, therefore the behavior of the quantities under scale transformations and their critical exponents are the same in both cases. The critical dimension R_c has the same scale dimensionality as the correlation radius r_c, since the combination $|\mu^*|\varphi_s/|h|$ is scale-invariant. The ratio $R_c/r_c=\mathrm{const}\cdot s$, where $s=|\tau|^{\gamma+\beta}/|h|$ is scale-dimensionless. The quantity

$$|\mu|\varphi_s^2r_c^3/T=p(s),\quad p(\infty)=p_\infty, \tag{59}$$

is a function of a scale-invariant parameter. It is therefore convenient to use as the coordinates the lines $|\tau|=\mathrm{const}$ and $s=\mathrm{const}$. The change of t_m when moving along the line $|\tau|=\mathrm{const}$ is connected with the function $R_c(h)$:

$$t_m{}^n=\frac{16\pi}{V\omega}\frac{1}{\Gamma_n{}^*|\mu^*|}\left(\frac{2}{3}\frac{|\mu^*|\varphi_s{}^2r_c{}^3}{T}\right)^{1/2}\left(\frac{2}{3}\frac{|\mu^*|\varphi_s}{|h|}\right)^2 \times\exp\left[\frac{128\pi}{9}\frac{|\mu^*|\varphi_s{}^2r_c{}^3}{T}\left(\frac{2}{3}\frac{|\mu^*|\varphi_s}{|h|}\right)^2\right],$$
$$t_m{}^c=\frac{32\pi}{V\omega}\frac{c^*}{\Gamma_c{}^*|\mu^*|^2}\left(2\frac{|\mu^*|\varphi_s{}^2r_c{}^3}{T}\right)^{1/2}\left(\frac{2}{3}\frac{|\mu^*|\varphi_s}{|h|}\right)^3 \times\exp\left[\frac{128\pi}{9}\frac{|\mu^*|\varphi_s{}^2r_c{}^3}{T}\left(\frac{2}{3}\frac{|\mu^*|\varphi_s}{|h|}\right)^2\right]. \qquad (60)$$

When moving along the line $s=\text{const}$, the scale-invariant argument of the exponential remains unchanged. The change of t_m is determined by the scale non-invariant factor in the preexponential multiplier. In the case of a system with nonconserving parameter, this is the factor $\Gamma_n^*|\mu^*|$ ($|\mu^*|\sim|\tau|^\gamma$, the renormalized kinetic coefficient is $\Gamma_n^*\sim|\tau|^{\Delta_\Gamma}$), and then

$$t_m{}^n=\text{const}\cdot|\tau|^{-(\gamma+\Delta_\Gamma)}p^{1/2}(s)\left(\frac{R_c}{r_c}\right)^2\exp\left[\frac{128\pi}{9}p(s)\left(\frac{R_c}{r_c}\right)^2\right], \qquad (61)$$
$$\Delta_\Gamma=\frac{6\eta\ln{}^4/_3}{\nu}.$$

In the case of a conserved transition parameter the kinetic coefficient Γ_c^* is not renormalized, and $c^*\sim|\tau|^{\gamma-2\nu}$; this yields

$$t_m{}^c=\text{const}\cdot|\tau|^{-(\gamma+2\nu)}p^{1/2}(s)\left(\frac{R_c}{r_c}\right)^3\exp\left[\frac{128\pi}{9}p(s)\left(\frac{R_c}{r_c}\right)^2\right]. \qquad (62)$$

In the strong-fluctuation region, the average lifetime of the metastable state has a definite scale dimensionality that depends onthe conservation properties of the relaxing system, and the critical exponent t_m is determined by formulas (61) and (62).

A variant of nucleation theory, based on the ZV ideas, was developed by Langer,[10] who made concrete assumptions concerning the form of the coefficients in expressions of the type (47). These assumptions, which do not influence the form of the universal exponential factor, yield for the pre-exponential factor expressions that differ from those obtained in the present paper.

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Translated by J. G. Adashko

Microscopic-theory equations of the dynamics of an electron–ion system of a metal

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(Submitted 27 February 1979)
Zh. Eksp. Teor. Fiz. 77, 1432–1449 (October 1979)

The equations of the dynamics of an electron–ion system in a nontransition metal with a simple anisotropic lattice are derived on the basis of the electron and ion Hamiltonian and with account taken of the scattering of the electrons by the impurities. In the quasiclassical long-wave approximation the equations reduce to the elasticity equations for the lattice and to the kinetic equation for the electrons. Microscopic expressions are derived in terms of the pseudopotential of the deformation-potential tensor and the bare elastic moduli of the lattice. It is shown that under adiabatic and neutrality conditions the long-wave oscillations in the metal can be described by the Fröhlich Hamiltonian.

PACS numbers: 62.20.Dc

1. INTRODUCTION

Two essentially different approaches are presently used for the theoretical description of those electronic properties of metals which are connected with deformations of the crystal lattice. One of them, most widely used in the theory of metals, is in essence phenomenological. It is based, on the one hand, on the notion that electrons are quasiparticles with a complicated dispersion law[1-3] that applies to the particular crystal lattice. On the other hand, this approach postulates the existence in the metal of "bare" phonons that do not interact with the electrons, and of corresponding "bare" elastic moduli of the metal λ_{iklm}. The interaction of the electrons with the phonons is the result of the change of the electron energy under the influence of the lattice deformation. This interaction is described with the aid of a deformation potential first introduced by Akhiezer.[4] In a strong magnetic field, an induction interaction exists besides the deformation interaction.[5,6] The the-

0038-5646/79/100719-10$02.40