Dynamic diffraction of x rays by a crystal with a periodic displacement field

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Resonant interaction of an x-ray wave field with the acoustic displacement field in a perfect crystal, which occurs under certain conditions, is considered. It is shown that the resonance effects correspond to self-intersection of the dispersion surface of the dynamic diffraction of the x rays; this surface is periodic in reciprocal space with a period equal to the wave vector of the acoustic wave. Under resonance conditions, a substantial change takes place in the x-ray absorption coefficient in the case of weak acoustic action, new Bloch-wave trajectories appear, and the oscillations of the reflected- and refracted-beam intensities are amplified.

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The high sensitivity of x-ray diffraction to crystal-lattice distortions makes it easy to observe the influence of ultrasonic oscillations on its intensity in both thin[1-3] crystals ($\mu t < 1$, where μ is the linear absorption coefficient and t is the thickness) and thick[4,5] ones ($\mu t \sim 10$, conditions for the appearance of the Borrmann effect[6]). In thin crystals, an increase of the integral reflection intensity takes place and corresponds to a transition to kinematic scattering with increasing ultrasonic deformation, whereas in thick crystals the anomalous passage of the x-rays becomes weaker. This influence is observed regardless of the wavelength of the ultrasound and increases with the deviations, due to acoustic displacements, from the Bragg condition. It was recently shown,[7-10] however, that ultrasound with a wave vector $\mathbf{K}_s$ lying in the reflecting plane causes resonant suppression of the Borrmann effect if its wavelength λ_s coincides with the x-ray extinction length τ. The theory[10] of this phenomenon, which made it possible to explain the exceedingly high sensitivity of the intensity of the anomalous passage to weak ultrasonic deformations (deformation amplitude $\varepsilon \sim 10^{-9}-10^{-8}$) was based on the Takagi system of equations,[11] which describes the diffraction of x-rays in distorted crystals. This method is effective only for a definite one-dimensional geometry of the problem. A more general analysis starts from the fact that the acoustic displacement field produces in the crystal an instantaneous superlattice with a period equal to the vibration wavelength, and uses the equations of the dynamic scattering theory for an ideal crystal. We note that in Ref. 12 these equations were used to solve the problem of thermal diffuse scattering in Bragg reflection of x-rays. The scattering of x-rays by a crystal with an ultrasonic deformation field was investigated in Ref. 13 in similar fashion at $\lambda_s \ll \tau$. In this case there are no resonance effects, so that the acoustic field influences the diffraction noticeably only at large values $\varepsilon \sim 10^{-4}$. The method used in Ref. 13 for a qualitative analysis of the general equations and for constructing the dispersion surface is not applicable when λ_s and τ are of the same order.

The purpose of the present study was to investigate the resonant interaction of an x-ray wave field in an absorbing crystal with the acoustic displacement field at an arbitrary geometry of the problem, on the basis of the dynamic scattering theory for an ideal crystal.

The reciprocal lattice of a crystal with a standing displacement wave

$$\mathbf{u}=\mathbf{U}\sin(2\pi\mathbf{K}_s\mathbf{r}),$$

(where $\mathbf{U}=\mathbf{U}_0\sin(2\pi v_s T)$ and v_s is the frequency of the ultrasound oscillations) contains in addition to the principal points $\mathbf{H}$ also satellites $\mathbf{H}\pm n\mathbf{K}_s$, which describe diffraction with absorption or emission of n phonons. The Fourier components of the polarizability corresponding to the point $\mathbf{H}+n\mathbf{K}_s$ is expressed in terms of a Bessel function of order n (Ref. 14):

$$\chi_{\mathbf{H}+n\mathbf{K}_s}=(-1)^n J_n\{2\pi(\mathbf{H}+n\mathbf{K}_s)\mathbf{U}\}\chi_{\mathbf{H}}^p,$$

$\chi_{\mathbf{H}}^p$ is the Fourier component of the polarizability of the undistorted crystal. Since U is a function of the time, the quantities $\chi_{\mathbf{H}\pm n\mathbf{K}_s}$ also depend on the time. Since, however, $v_s \ll v_x$ (v_x is the x-ray frequency), we can use a quasistatic approximation and neglect the transfer of energy from the phonon to the x-ray quantum. We are considering here diffraction by a static displacement wave, and the final result should be averaged over the period of the ultrasound oscillations. At $|\mathbf{H}\cdot\mathbf{U}_0| \ll 1$ it is necessary to neglect the coefficients $\chi_{\mathbf{H}\pm n\mathbf{K}_s}$ with $n>1$. In addition, $K_s \ll H$ and the coefficients $\chi_{\pm\mathbf{K}_s}$ are also negligible (they are exactly equal to zero for a transverse acoustic wave).

If the crystal is in a reflecting position, then simultaneously with the points 0 and $\mathbf{H}$ the points $\mathbf{H}\pm n\mathbf{K}_s$ and $\pm n\mathbf{K}_s$ are close to the Ewald sphere and the diffraction become multiwave. It is important that the number of plane waves that must be taken into account exceeds the number of nonzero structure factors. In fact, as shown by a consecutive calculation[12] of the Bragg scattering of waves corresponding to satellites of the principal points of the reciprocal lattice, the amplitudes of the waves $|\mathbf{K}_0+n\mathbf{K}_s\rangle$ ($\mathbf{K}_0$ is the wave vector of the refractive wave) and $|\mathbf{K}_0+\mathbf{H}+n\mathbf{K}_s\rangle$ are of the same order, regardless of the fact that $|\chi_{n\mathbf{K}_s}| \ll |\chi_{\mathbf{H}+n\mathbf{K}_s}|$. This follows from the structure of the system of equations

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describing multiwave diffraction[15]:

$$\frac{k^2-(\mathbf{K}_0+\mathbf{G})^2}{(\mathbf{K}_0+\mathbf{G})^2}\mathbf{d}_{\mathbf{K}_0+\mathbf{G}}+\sum_{\mathbf{G}\neq\mathbf{H}}\chi_{\mathbf{G}-\mathbf{H}}[\mathbf{d}_{\mathbf{K}_0+\mathbf{H}}]_{\mathbf{K}_0+\mathbf{G}}=0, \tag{1}$$

in which the amplitudes $\mathbf{d}_{\mathbf{K}_0+\mathbf{H}}$ of the plane waves $|\mathbf{K}_0+\mathbf{H}\rangle$ enter with coefficients $\chi_{\mathbf{G}-\mathbf{H}}$.

At $K_s \gg K|\chi_\mathrm{H}|$ the system (1) reduces to a set of independent systems of equations for the pairs of amplitudes $\mathbf{d}_{\mathbf{K}_0+n\mathbf{K}_s}$ and $\mathbf{d}_{\mathbf{K}_0+\mathbf{H}+n'\mathbf{K}_s}$, with arbitrary n and n'. The problem can be treated in analogy with the ordinary-wave case, and the dispersion surface is an aggregate of two-wave dispersion surfaces.[13]

Resonance effects are possible when K_s and $K|\chi_\mathrm{H}|$ are of the same order. The problem does not reduce here in a two-wave problem. Using the condition $|\mathbf{H}\cdot\mathbf{U}_0|\ll 1$, we confine ourselves only to the plane waves $|\mathbf{K}_0\pm\mathbf{K}_s\rangle$ and $|\mathbf{K}_0+\mathbf{H}+\mathbf{K}_s\rangle$, which are connected with the waves $|\mathbf{K}_0\rangle$ and $|\mathbf{K}_0+\mathbf{H}\rangle$ by polarizability coefficients with $n=1$ (this approximation will be justified below). Recognizing that in (1) we have $k=K(1+\chi_0/2)$ and that $[\mathbf{d}_{\mathbf{K}_0+\mathbf{H}}]_{\mathbf{K}_0+\mathbf{G}}$ is the component of $\mathbf{d}_{\mathbf{K}_0+\mathbf{H}}$ normal to $\mathbf{K}_0+\mathbf{G}$, and writing

$$\mathbf{K}_0=\mathbf{K}-q\mathbf{n},$$

where $\mathbf{K}$ is the wave vector of the incident radiation and $\mathbf{n}$ is the normal to the crystal surface, we obtain for the symmetrical Laue case and for a centrosymmetric absorbing crystal a system of six-wave diffraction equations:

$$(\hat{A}-\delta\hat{I})\mathbf{D}=0, \tag{2}$$

where $\hat{A}=\hat{A}_0+\hat{V}$,

$$\hat{A}_0=\begin{pmatrix} 0 & -A & 0 & 0 & 0 & 0\\ -A & r & 0 & 0 & 0 & 0\\ 0 & 0 & -s_0 & -A & 0 & 0\\ 0 & 0 & -A & r-s_\mathrm{H} & 0 & 0\\ 0 & 0 & 0 & 0 & s_0 & -A\\ 0 & 0 & 0 & 0 & -A & r+s_\mathrm{H}\end{pmatrix},$$

$$\hat{V}=\begin{pmatrix} 0 & 0 & 0 & a & 0 & b\\ 0 & 0 & b & 0 & a & 0\\ 0 & b & 0 & 0 & 0 & 0\\ a & 0 & 0 & 0 & 0 & 0\\ 0 & a & 0 & 0 & 0 & 0\\ b & 0 & 0 & 0 & 0 & 0\end{pmatrix},$$

$A=-CK\chi_\mathrm{H}/2\cos\theta_B$, θ_B is the Bragg angle, $C=1$ or $\cos 2\theta_B$ for radiation polarized perpendicular or parallel to the scattering plane

$$r=2K\sin\theta_B(\theta-\theta_B),\quad s_\mathrm{G}=\frac{\mathbf{K}_s(\mathbf{K}_0+\mathbf{G})}{K\cos\theta_B},$$

$$a=\frac{CK}{2\cos\theta_B}\chi_{\mathrm{H}+\mathrm{K}_s},\quad b=\frac{CK}{2\cos\theta_B}\chi_{\mathrm{H}-\mathrm{K}_s},$$

$\delta=-q-K\chi_0/2\cos\theta_B$, $\hat{I}$ is a unit matrix, and $\mathbf{D}$ is a generalized vector whose components are the wave amplitudes of the normalized Bloch wave:

$$\mathbf{D}=\begin{pmatrix} d_{\mathbf{K}_0}\\ d_{\mathbf{K}_0+\mathbf{H}}\\ d_{\mathbf{K}_0+\mathbf{K}_s}\\ d_{\mathbf{K}_0+\mathbf{H}+\mathbf{K}_s}\\ d_{\mathbf{K}_0-\mathbf{K}_s}\\ d_{\mathbf{K}_0+\mathbf{H}-\mathbf{K}_s}\end{pmatrix}.$$

Thus, the problem has been reduced to finding the eigenvalues and eigenvectors of a complex symmetrical matrix. In the limiting case $\hat{V}=\hat{0}$, $(U_0=0)$, $\hat{0}$ is a null matrix, we obtain three independent two-wave problems for the wave pairs $|\mathbf{K}_0\rangle$ and $|\mathbf{K}_0+\mathbf{H}\rangle$, $|\mathbf{K}_0+\mathbf{K}_s\rangle$ and $|\mathbf{K}_0+\mathbf{H}+\mathbf{K}_s\rangle$, $|\mathbf{K}_0-\mathbf{K}_s\rangle$ and $|\mathbf{K}_0+\mathbf{H}-\mathbf{K}_s\rangle$. Accordingly, the dispersion surface consists of three pairs of branches (for each polarization state): $\delta^0_{\mathbf{K}_0,i}$, $\delta^0_{\mathbf{K}_0+\mathbf{K}_s,i}$, $\delta^0_{\mathbf{K}_0-\mathbf{K}_s,i}$. The imaginary part of the eigenvalue with $i=1$ in each pair describes the anomalously weak absorption of one of the wave fields, the other wave field $(i=2)$ is absorbed anomalously strongly. Two additional pairs of branches $\delta^0_{\mathbf{K}_0\pm\mathbf{K}_s,i}$ are shifted by $\pm\mathbf{K}_s$ relative to $\delta^0_{\mathbf{K}_0,i}$. The form of the dispersion surface follows directly from the fact that the length of the reciprocal-lattice cell of the crystal with periodic displacement field in the $\mathbf{K}_s$ direction is equal to K_s, and the dispersion surface is periodic in reciprocal space with this period. Under definite conditions self-intersection of the dispersion surface is possible. If $\hat{V}\neq 0$, the eigenvalues and eigenvectors of the matrix equation (2) do not reduce to two-wave quantities. The largest changes occur when the dispersion branches intersect, and the perturbation $\hat{V}$ lifts (partially or fully) the degeneracy due to the self-intersection (a similar approach is used, for example, in the theory of helicoidal magnetic structures[16]). The substantial changes of the eigenvalues at small U_0 cause resonance effects.

Substituting the expressions for the eigenvector near the point of intersection of the dispersion branches $\delta^0_{\mathbf{K}_0,i}$ and $\delta^0_{\mathbf{K}_0-\mathbf{K}_s,j}$

$$\mathbf{D}_{\mathbf{K}_0}=c_{\mathbf{K}_0,i}\mathbf{D}^0_{\mathbf{K}_0,i}+c_{\mathbf{K}_0-\mathbf{K}_s,j}\mathbf{D}^0_{\mathbf{K}_0-\mathbf{K}_s,j},$$

where $\mathbf{D}^0_{\mathbf{K}_0,i}$ and $\mathbf{D}^0_{\mathbf{K}_0-s,j}$ are the eigenvectors of the matrix A_0, in Eq. (2) and multiplying the left in succession by $\mathbf{D}^0_{\mathbf{K}_0,i}$ and $\mathbf{D}^0_{\mathbf{K}_0-\mathbf{K}_s,j}$ we obtain the system of equations

$$\begin{aligned} &c_{\mathbf{K}_0,i}(\delta^0_{\mathbf{K}_0,i}-\delta)+c_{\mathbf{K}_0-\mathbf{K}_s,j}V^{ij}_{\mathbf{K}_s}=0,\\ &c_{\mathbf{K}_0,i}V^{ij}_{\mathbf{K}_s}+c_{\mathbf{K}_0-\mathbf{K}_s,j}(\delta^0_{\mathbf{K}_0-\mathbf{K}_s,j}-\delta)=0.\end{aligned} \tag{3}$$

We have used here the orthogonality property (relative to the simple scalar product rather than the Hermitian-scalar product) of the eigenvectors of a complex symmetrical matrix

$$V^{ij}_{\mathbf{K}_s}=\mathbf{D}^0_{\mathbf{K}_0,i}\hat{V}\mathbf{D}^0_{\mathbf{K}_0-\mathbf{K}_s,j}.$$

from the condition that the system (3) have a solution, we get

$$\begin{aligned}\delta^{\pm}=&{}^1/_2(\delta^0_{\mathbf{K}_0,i}+\delta^0_{\mathbf{K}_0-\mathbf{K}_s,j})\\ &\pm[{}^1/_4(\delta^0_{\mathbf{K}_0,i}-\delta^0_{\mathbf{K}_0-\mathbf{K}_s,j})^2+(V^{ij}_{\mathbf{K}_s})^2]^{1/2}.\end{aligned} \tag{4}$$

The form of the Bloch waves $\mathbf{D}$ can now be determined by solving (3) for c. From the boundary condition on the entry surface of the crystal

$$\sum_i\psi_i d^i_{\mathbf{K}_0}=1,\quad \sum_i\psi_i d^i_{\mathbf{K}_0+\mathbf{G}}=0\quad(\mathbf{G}\neq 0),$$

where ψ_i is the amplitude of the i-th Bloch wave, it follows[17] that $\psi_i=d^i_{\mathbf{K}_0}$. Therefore the Bloch waves $D_{\mathbf{K}_0\pm\mathbf{K}_s}$ have a noticeable amplitude only in the region of the intersection of the dispersion branches $\delta_{\mathbf{K}^0_0}$ and

$\delta^0_{\mathbf{K}_0 \pm \mathbf{K}_s}$ (at

$$|\delta^0_{\mathbf{K}_0} - \delta^0_{\mathbf{K}_0 \pm \mathbf{K}_s}| \gg |V_{\mathbf{K}_s}|$$

we have

$$\psi_{\mathbf{K}_0 \pm \mathbf{K}_s} = d^{\mathbf{K}_0 \pm \mathbf{K}_s}_{\mathbf{K}_0} \sim V_{\mathbf{K}_s} / (\delta^0_{\mathbf{K}_0} - \delta^0_{\mathbf{K}_0 \pm \mathbf{K}_s}),$$

The amplitudes of the satellites and of the Bloch waves $\mathbf{D}_{\mathbf{K}_0}$ are of the same order).

Before we discuss different concrete cases, let us examine the question of the validity of neglecting satellites with $n > 1$. The amplitudes of the latter are not small in the region of intersection of the dispersion branches $\delta^0_{\mathbf{K}0}$ and $\delta^0_{\mathbf{K}_0 \pm n\mathbf{K}_s}$. However, $V_{n\mathbf{K}_s} \sim (\mathbf{H} \cdot \mathbf{U})^n$, the corresponding interval of the incidence angle $\Delta\theta \sim V_{n\mathbf{K}_s} \chi_{\mathrm{H}}$ is negligibly narrow at $|\mathbf{H} \cdot \mathbf{U}_0| \ll 1$, and multiphonon processes can be disregarded.

Figure 1 corresponds to x-ray acoustic resonance.[7-10] An ultrasonic wave with $\mathbf{K}_s = \mathbf{K}_s n$ is excited in the crystal, and the polarization vector has a component along $\mathbf{H}$. The eigenvalues of $\hat{A}_0$ are

$$\delta^0_{\mathbf{K}_0,1} = -\frac{CK\chi_{\mathbf{H}}}{2\cos\theta_B} \operatorname{ctg}\frac{\beta}{2},$$

$$\delta^0_{\mathbf{K}_0,2} = \frac{CK\chi_{\mathbf{H}}}{2\cos\theta_B} \operatorname{tg}\frac{\beta}{2},$$

$$\delta^0_{\mathbf{K}_0 \pm \mathbf{K}_s, i} = \delta^0_{\mathbf{K}_0, i} \pm K_s,$$

where

$$\operatorname{ctg}\beta = \frac{r}{2A} = -\frac{\sin 2\theta_B (\theta - \theta_B)}{C\chi_{\mathbf{H}}}$$

and β is the complex argument. If

$$K_s = \operatorname{Re}(\Delta K_{\mathbf{H}}) > \operatorname{Re}(\Delta K_{\mathbf{H}}{}^0)$$

($\Delta K_{\mathrm{H}} = \delta^0_{\mathbf{K}_0,1} - \delta^0_{\mathbf{K}_0,2}$ is the splitting of the two-wave dispersion surface and ΔK^0_{H} is the gap at the boundary of the band), the branches $\delta^0_{\mathbf{K}_0,1}$ and $\delta^0_{\mathbf{K}_0+\mathbf{K}_s}$ intersect. As a result, the states with the eigenvectors

$$\mathbf{D}^0_{\mathbf{K}_0,1} = \begin{pmatrix} \sin(\beta^*/2) \\ -\cos(\beta^*/2) \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{D}^0_{\mathbf{K}_0+\mathbf{K}_s,2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \cos(\beta^*/2) \\ \sin(\beta^*/2) \end{pmatrix}$$

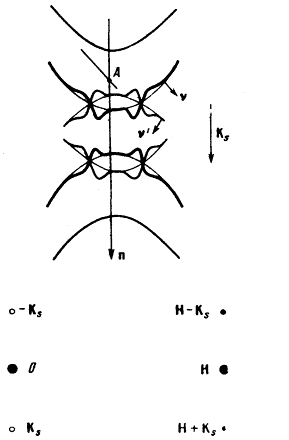

FIG. 1. Dispersion surface of the dynamic diffraction of x rays by a crystal with a periodic displacement field (for one of the polarization states), A—center of propagation of the plane wave incident on the crystal, $\mathbf{v}$—group-velocity vector, $\mathbf{K}_s = \mathbf{K}_s\mathbf{n}$.

become mixed, where β^* is the value of the argument at which the dispersion branches intersect. The matrix element is $V_{\mathbf{K}_s}{}^{12} = -a$. At

$$|V^{12}_{\mathbf{K}_s}| \ll |\delta^0_{\mathbf{K}_0,1} - \delta^0_{\mathbf{K}_0+\mathbf{K}_s,2}| \quad (|a| \ll \operatorname{Im}(\Delta K_{\mathbf{H}}{}^0))$$

we obtain from (4)

$$\delta^+ \approx \delta^0_{\mathbf{K}_0,1} + \frac{a^2}{\delta^0_{\mathbf{K}_0,1} - \delta^0_{\mathbf{K}_0+\mathbf{K}_s,2}}, \quad \delta^- \approx \delta^0_{\mathbf{K}_0+\mathbf{K}_s,2} - \frac{a^2}{\delta^0_{\mathbf{K}_0,1} - \delta^0_{\mathbf{K}_0+\mathbf{K}_s,2}}. \tag{5}$$

The intersection of $\delta^0_{\mathbf{K}_0,2}$ and $\delta^0_{\mathbf{K}_0-\mathbf{K}_s,1}$ yields two other eigenvalues $\delta^+ - K_s$ and $\delta^- - K_s$. In the case of a thick crystal, only fields that are weakly absorbed, with eigenvalues δ^+ and $\delta^+ - K_s$, are significant. The imaginary part of the correction to the eigenvalues, which depends resonantly on K_s, suppresses the anomalous passage of the x rays. The decrease of the intensity is described by the factor

$$\exp(-\Delta\mu t) \equiv \exp\{-4\pi \operatorname{Im}(\delta^0_{\mathbf{K}_0,1} - \delta^+) t\}.$$

It follows from (5) that

$$\Delta\mu \approx \frac{\pi^3 |\mathbf{HU}|^2 [\operatorname{Re}(\Delta K_{\mathbf{H}}{}^0)]^2 \operatorname{Im}(\Delta K_{\mathbf{H}})}{[\operatorname{Re}(\Delta K_{\mathbf{H}}) - K_s]^2 + [\operatorname{Im}(\Delta K_{\mathbf{H}})]^2}. \tag{6}$$

If $\lambda_s = \tau [K_s = \operatorname{Re}(\Delta K^0_{\mathrm{H}})$, tangency of the dispersion branches], the anomalous passage of the waves propagating along the Bragg planes ($\theta = \theta_B$) and making the largest contribution to the integrated (with respect to θ) intensity becomes suppressed. At $K_s = \operatorname{Re}(\Delta K_{\mathrm{H}})$, a resonant decrease takes place in the amplitude of the Bloch waves that correspond to a definite value of $|\theta - \theta_B|$ and propagate along normals to the dispersion surface at the corresponding propagation centers. The suppression factor should be averaged over the period of the ultrasonic oscillations.[10] Expression (6) coincides with that obtained in Ref. 10 by solving equations[17] that describe interband scattering by the displacement field. The real part of the correction to the eigenvalues gives rise to characteristic effects of the dispersion type (Fig. 1),[1] which lead to a change, that depends on the phase of the oscillations, in the period of the extinction beats in a thin crystal. When averaged over the period of the oscillations, this effect causes damping of the beats. At resonance, on the other hand, the depth of the oscillations, with a fundamental period equal to the wavelength λ_s of the ultrasound increases in the refracted and in the reflected beams that are due to the presence of two weakly absorbed Bloch waves with different eigenvalues δ^+ and $\delta^+ - K_s$, and are due also to the contributions from the satellites to each of these fields. A similar result was obtained in Refs. 8–10, while oscillations in a non-absorbing crystal with a superlattice were considered in Ref. 18.

Figure 2 shows the section of the dispersion surface in the case when the wave vector of the ultrasonic wave with longitudinal component is directed along the diffraction vector: $\mathbf{K}_s = -K_s\mathbf{H}/H$. Self-intersection of the dispersion branches is possible for any K_s. The complex gap, equal to

$$\pi \mathbf{HU} \Delta K_{\mathbf{H}}{}^0 \cos\beta^*,$$

lifts the degeneracy of both the real and imaginary parts of the eigenvalues on the boundary of the superband.

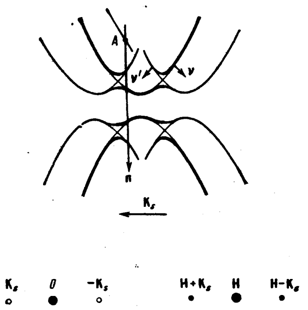

FIG. 2. The same as in Fig. 1 at $\mathbf{K}_s = K_3\mathbf{H}/H$.

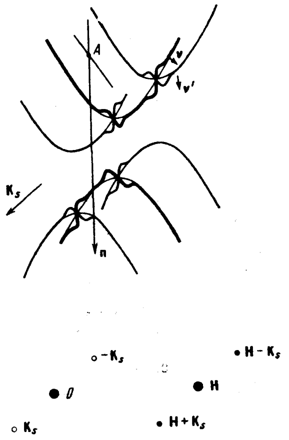

FIG. 3. The same as in Fig. 1 at $\mathbf{K}_s = K_n\mathbf{n} - K_H\mathbf{H}/H$.

Near the self-intersection point, the amplitudes $|\mathbf{K}_0 \pm \mathbf{K}_s\rangle$ and $|\mathbf{K}_0 + \mathbf{H} \pm \mathbf{K}_s\rangle$ of the plane waves increase to values of the order of unity. The interference of the waves $|\mathbf{K}_0\rangle$, $|\mathbf{K}_0 \pm \mathbf{K}_s\rangle$ and $|\mathbf{K}_0 + \mathbf{H}\rangle$, $|\mathbf{K}_0 + \mathbf{H} \pm \mathbf{K}_s\rangle$ causes intensity oscillations with a fundamental period λ_s in the transmitted and diffracted beams. The oscillations of the intensity of the diffracted beams, under conditions of the Borrmann effect at $\mathbf{K}_s \| \mathbf{H}$, were observed in Refs. 4 and 5. The interpretation of the experimental results was reduced in Refs. 4 and 5 to the fact that the Borrmann effect is suppressed in the most deformed crystal sections. It is easily seen that in this case the period of the oscillations should be equal to $\lambda_s/2$ (the regions of maximum deformation are encountered twice in each wavelength). The period of the oscillations on the topogram[4] is actually close to $\lambda_s/2$, and in Ref. 5 it is close to λ_s. This can be explained in the following manner. At $|\mathbf{H} \cdot \mathbf{U}_0| \ll 1$ the amplitude of the satellites are not small in a narrow region of values of θ, and a noticeable contrast arises when the waves $|\mathbf{K}_0 + \mathbf{H} \pm \mathbf{K}_s\rangle$ and $|\mathbf{K}_0 + \mathbf{H}\rangle$ interfere. With increasing E_0, the contrast of the oscillations with the period $\lambda_s/2$, which are due to interference of the waves $|\mathbf{K}_0 + \mathbf{H} + \mathbf{K}_s\rangle$ and $|\mathbf{K}_0 + \mathbf{H} - \mathbf{K}_s\rangle$ increases. When the branches intersect, a trajectory of the wave $\mathbf{v}'$ appears and is the mirror image of the initial trajectory (Fig. 2). Diffraction in a non-absorbing crystal at $\mathbf{K}_s \| \mathbf{H}$ and $\lambda_s \ll \tau$ was investigated experimentally and theoretically in Refs. 3, 13, and 19. Satisfaction of the condition $\lambda_s \ll \tau$ makes it possible to observe separately satellites of different orders ($|\mathbf{H} \cdot \mathbf{U}_0| \gg 1$) on the rocking curves. In Ref. 20, for the same geometry of the problem and for a thin crystal, the ray approximation was used. Focusing of the wave field on the periodic displacement field leads to oscillations of intensity. In Ref. 20, however, the principal mechanism of contrast formation in the case of a thin crystal, namely the direct image, was not considered.

Figure 3 shows the section of the dispersion surface in the case when the wave vector

$$\mathbf{K}_s = K_n\mathbf{n} - K_H\mathbf{H}/H$$

couples a Bloch state with definite θ^* with a state corresponding to $\theta = \theta_B$,

$$K_n = \mathrm{Re}\{(A^2 + r^{*2}/4)^{1/2} - A\},$$

$$K_H = K\cos\theta_B(\theta^* - \theta_B).$$

The intermixed states are

$$\mathbf{D}^0_{\mathbf{K}_0,1} = \begin{pmatrix} \sin(\beta^*/2) \\ -\cos(\beta^*/2) \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{D}^0_{\mathbf{K}_0+\mathbf{K}_s,1} = \frac{1}{2^{1/2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$$

with eigenvalues

$$\delta^0_{\mathbf{K}_0,1} = -\frac{CK\chi_H}{2\cos\theta_B}\,\mathrm{ctg}\,\frac{\beta^*}{2},$$

$$\delta^0_{\mathbf{K}_0+\mathbf{K}_s,1} = K_n + K_H\,\mathrm{tg}\,\theta_B - \frac{CK\chi_H}{2\cos\theta_B}.$$

At

$$|V^{11}_{\mathbf{K}_s}| \ll \mathrm{Im}\,(\delta^0_{\mathbf{K}_0+\mathbf{K}_s,1} - \delta^0_{\mathbf{K}_0,1})$$

we obtain with the aid of (4)

$$\delta^+ \approx \delta^0_{\mathbf{K}_0,1} + \frac{(V^{11}_{\mathbf{K}_s})^2}{\delta^0_{\mathbf{K}_0,1} - \delta^0_{\mathbf{K}_0+\mathbf{K}_s,1}},$$

$$\delta^- \approx \delta^0_{\mathbf{K}_0+\mathbf{K}_s,1} - \frac{(V^{11}_{\mathbf{K}_s})^2}{\delta^0_{\mathbf{K}_0,1} - \delta^0_{\mathbf{K}_0+\mathbf{K}_s,1}}$$

where

$$V^{11}_{\mathbf{K}_s} \approx \frac{a}{2^{1/2}}\left(\sin\frac{\beta^*}{2} - \cos\frac{\beta^*}{2}\right).$$

The absorption coefficient of the Bloch wave $\mathbf{D}_{\mathbf{K}_0+\mathbf{K}_s,i}$ described by the imaginary part δ^- is smaller than the absorption coefficient of the wave $\mathbf{D}^0_{\mathbf{K}_0,1}$, and tends, when the displacement amplitude is decreased to the minimum value possible for a perfect crystal, possessed by a Borrmann wave field at $\theta = \theta_B$. Consequently in this case the acoustic field enhances the Borrmann effect (scattering into a state corresponding to $\theta = \theta_B$ takes place). The amplitude of a weakly absorbed field is

$$\psi_{\mathbf{K}_0+\mathbf{K}_s} \approx \sin\frac{\beta^*}{2}\,\frac{V^{11}_{\mathbf{K}_s}}{\delta^0_{\mathbf{K}_0,1} - \delta^0_{\mathbf{K}_0+\mathbf{K}_s,1}}.$$

Thus, in analysis of the diffraction of x rays by a crystal with a superlattice reduces to consideration of different cases of self-intersection of a dispersion surface which is periodic in reciprocal space with a period $\mathbf{K}_s$. Three effects are produced in this case: change of the absorption (suppression or enhancement of the anomalous passage), appearance of new trajectories of the wave field in the crystal, and resonant amplification of the oscillations of the intensity of the diffracted and transmitted beams with the fundamental period λ_s.

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[1)] At $|a| > \frac{1}{2}\mathrm{Im}(\Delta K_H)$ the degeneracy of the real parts of the eigenvalues is lifted and the branches do not intersect.

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Finite-dimension ring interferometer in an external magnetic field

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A theory is developed for a single ring interferometer (SQUID) having a finite contact with L and placed in an external static magnetic field H_e. Numerical methods are used to study the nonlinear-equation solutions that yield the distributions of the field in the ring and inside the contact. The obtained solutions are investigated for stability, and the stable and unstable configurations are determined. The free energies of the different states are obtained. The points of equilibrium transition from one state to another are found, as are the boundaries of the hysteresis region. The dependence of the field H_i inside the ring on the external field H_e is plotted at different values of the width L of the contact and of the area σ of the internal opening of the ring. In the limiting case of small widths ($L << 1$), and also in the case of strong fields ($H_e >> 1$), analytic formulas are obtained. The general expression for the self-induction coefficient of the ring interferometer is found. A comparison is made with the results of other studies of this subject.

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1. INTRODUCTION

We consider in this paper the problem of penetration of an external static magnetic field into a superconducting ring that is closed by a Josephson junction of finite width L (see Fig. 1). The external magnetic field H_e is directed along the z axis perpendicular to the plane of the figure. It is assumed that the superconductor has an infinite length along the z axis (cylinder with cuts along L). It is obvious that all the quantities in the plane of the barrier depend in this case only on the angle coordinate $x (0 \leq x \leq L)$.

The distribution of the field and of the current in a Josephson barrier of finite width is described by the nonlinear equation[1-3]

$$d^2\varphi/dx^2 = \sin\varphi, \tag{1}$$

where the density of the current through the barrier is $j(x) = \sin\varphi(x)$ and the magnetic field is $H(x) = d\varphi/dx$. (The quantity $\varphi(x)$ is called the phase difference of the superconductor order parameter.)

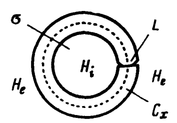

FIG. 1. Schematic view of a ring SQUID in an external field.

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