the number of particles in a critical nucleus is fairly large. However, it is important to note that the adopted method can be applied effectively to those cases when the size of a critical nucleus is finite and the macroscopic description is difficult. This becomes particularly clear when we turn to the results of Sec. 4, which demonstrate proximity of the subbarrier transition amplitudes for the evolution of a compact nucleus and purely random formation of the interior of a critical nucleus. In particular, the results make it possible to analyze an intrinsic class of problems corresponding to decay of a metastable phase by formation of clusters with very much modified atomic configurations. It should be noted that in the latter case the discrete nature of the energy levels is very pronounced and the inelastic interaction with the phonon subsystem, considered in Sec. 5, may be of fundamental importance.

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Motion of domain walls in an external magnetic field

V. M. Eleonskiĭ, N. N. Kirova, and N. E. Kulagin

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It is shown that in a range of magnetic fields exceeding the known value of the Walker limiting field, there can exist stationary-profile waves corresponding to moving domain walls with a definite internal structure.

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1. Investigations of steady-state motions of domain walls[1,2] have shown that with allowance for uniaxial anisotropy of the ferromagnet, dissipation, and an external magnetic field directed along the anisotropy axis, the velocity of a stationary-profile wave is bounded from above by the value

$$U_-=2|\gamma|\frac{(AK)^{1/2}}{M_s}u_-(\varepsilon). \tag{1.1}$$

Here the following notation is used:

$$u_-(\varepsilon)\equiv(1+\varepsilon)^{1/2}-1,\quad \varepsilon=2\pi M_s^2/K \tag{1.2}$$

γ is the gyromagnetic ratio, A and K are the exchange and the uniaxial-anisotropy energy constants, and M_s is the saturation magnetization. Furthermore, as was first mentioned by Walker,[1] the range of existence of stationary motions of domain walls of the Bloch-Landau or Néel type, characterized by a constant orientation of the plane of rotation of the magnetic moment, is bounded from above by a value of the magnetic field equal to

$$H_1=\frac{2K}{M_s}h_1,\quad h_1=\frac{1}{2}\alpha\varepsilon, \tag{1.3}$$

α is the damping parameter.

A solution of the Landau-Lifshitz equations that corresponds to a stationary-profile wave with a constant orientation of the plane of rotation of the magnetic moment cannot be continued into the external magnetic field range $H_z>H_1$.

The present paper discusses the possibility of existence of stationary-profile waves in the external-field range $H_z>H_1$. Waves of this type correspond to stationary motions of domain walls that are characterized by a definite internal structure. Specifically, turning of the plane of rotation of the magnetic moment leads to the appearance of a definite number of "internal" domain walls, because of the fact that the projection of the magnetic moment on the direction of the external field changes sign several times during passage from the region of uniform magnetization along the external field to the region of uniform magnetization opposite to the external field.

In the case considered, the system of Landau-Lifshitz equations has the form

$$\begin{gathered}\theta''-(1+\omega^2+\varepsilon\cos^2\varphi)\sin\theta\cos\theta-h_z\sin\theta=u\omega\sin\theta-\alpha u\theta',\\ (\omega\sin^2\theta)'+\varepsilon\sin^2\theta\cos\varphi\sin\varphi=-u\theta'\sin\theta-\alpha u\omega\sin^2\theta.\\ \omega=\varphi'.\end{gathered} \tag{1.4}$$

Here u is the velocity of the stationary-profile wave divided by the characteristic velocity $2|\gamma|(AK)^{1/2}/M_s$; h_z is the external magnetic field divided by the anisotropy field $2K/M_s$; θ and φ are the polar and azimuthal angles of the vector magnetic moment; the differentiation is with respect to the variable $\xi\equiv x-ut$ (the spatial variable x has been divided by the characteristic thickness $(A/K)^{1/2}$ of a Bloch wall).

For a stationary-profile wave, corresponding to steady-state motion of a domain wall, the asymptotic boundary conditions are

$$\lim_{\theta\to 0}\varphi=\varphi_-,\quad \lim_{\theta\to\pi}\varphi=\varphi_+,\quad \lim_{\theta\to 0,\pi}\omega=0. \tag{1.5}$$

An exact consequence of the Landau-Lifshitz equations (1.4), with allowance for the boundary conditions (1.5), is the integral relation

$$u=2h_z/\alpha\int_{-\infty}^{+\infty}d\xi\,(\theta'^2+\omega^2\sin^2\theta). \tag{1.6}$$

Analysis of the asymptotic behavior of solutions of the system (1.4) in a region where uniform magnetization is established leads to the relation[3]

$$u^2=\frac{\varepsilon^2\cos^2\varphi_\pm\sin^2\varphi_\pm}{1+\varepsilon\cos^2\varphi_\pm\mp(h_z+{}^1/_2\alpha\varepsilon\sin 2\varphi_\pm)}, \tag{1.7}$$

which shows that the velocity of motion of all types of domain walls is bounded from above by the limiting value (1.1). But here there is no necessity for a condition that the external magnetic fields be bounded by the values (1.2). The condition $h_z<h_1$ means that in this case solutions of the stationary-profile wave type are possible with the same orientation of the plane of rotation of the magnetic moment in the regions where uniform magnetization is established ($\varphi_-=\varphi_+$). It is to this type that the well-known exact solution belongs, in which the orientation of the plane of rotation of the magnetic moment is constant throughout space[1,2]:

$$\varphi_-=\varphi(x-ut)=\varphi_+. \tag{1.8}$$

When $h_z>h_1$, the only possible stationary-profile waves are ones in which the plane of rotation of the magnetic moment turns ($\varphi_-\neq\varphi_+$). Then according to (1.7), the upper bound to the velocity is much less than the limiting velocity (1.1) and decreases with increase of the external field.

Numerical analysis of the problem of steady-state motion of domain walls, with allowance for two angular degrees of freedom of the magnetic moment, indicates the existence of a series of stationary-profile waves that can be classified according to the number of internal domain walls (that is, according to the number of zeros of $\cos\theta$). Such formations can be regarded qualitatively as a "coupled" state (or cluster) of an odd number of moving domain walls. The integral relation (1.6) leads to the following simple estimate of the velocity of a stationary-profile wave corresponding to a coupled state of $2n+1$ domain walls:

$$u_{2n+1}(h_z)\sim\text{const}\cdot 2\frac{h_z\alpha^{-1}}{2n+1},\quad n=0,1,2,\ldots. \tag{1.9}$$

The relation (1.9) indicates a decrease of the mobility of a cluster with increase of the number of internal domain walls. Since all values u_{2n+1} of the velocity must satisfy the relation

$$u_{2n+1}(h_z)\leqslant u_-(\varepsilon), \tag{1.10}$$

we arrive at the conclusion that every stationary-profile wave corresponding to steady-state motion of a domain wall with internal structure is characterized by a limiting value of the external magnetic field

$$h_{2n+1}\sim\text{const}\cdot{}^1/_2(2n+1)\alpha u_-(\varepsilon). \tag{1.11}$$

Figure 1 shows the variation of the velocity of propagation of a stationary-profile wave with external magnetic field h_z, as it depends on the internal structure of the wave, according to the estimates (1.9) and (1.11) and also according to the results of numerical analysis. On increase of the number of internal domain walls, the mobility of the cluster decreases, but the limiting value of the external field increases.

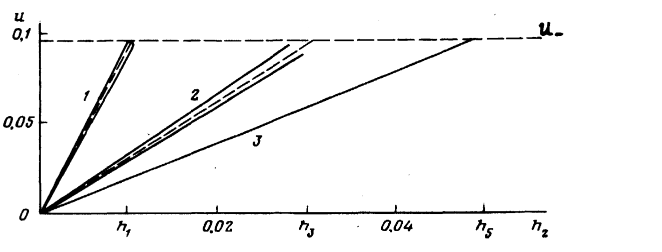

FIG. 1. Variation of velocity of motion of domain walls with external field ($\varepsilon=0.1$; $\alpha=0.1$). Curves 1 correspond to motion of a domain wall with constant orientation of the plane of rotation of the magnetic moment; curves 2 and 3 correspond to moving clusters of three and of five domain walls. The relations obtained from the estimates (1.9) are represented by the dotted curves.

A stationary-profile wave with a constant orientation of the plane of rotation of the magnetic moment, (1.8), is represented in the (u, h_z) plane by two curves, close in proportion to the smallness of the parameter ε of the magnetic medium, corresponding to moving domain walls of Bloch and of Néel types. At $h_z\to h_1$ these curves run together. As a result of numerical analysis, a similar splitting was observed in an investigation of the $u_3(h_z)$ relation for a cluster formed by three domain walls. The corresponding distributions of the polar and azimuthal angles are shown in Figs. 2 and 3. The

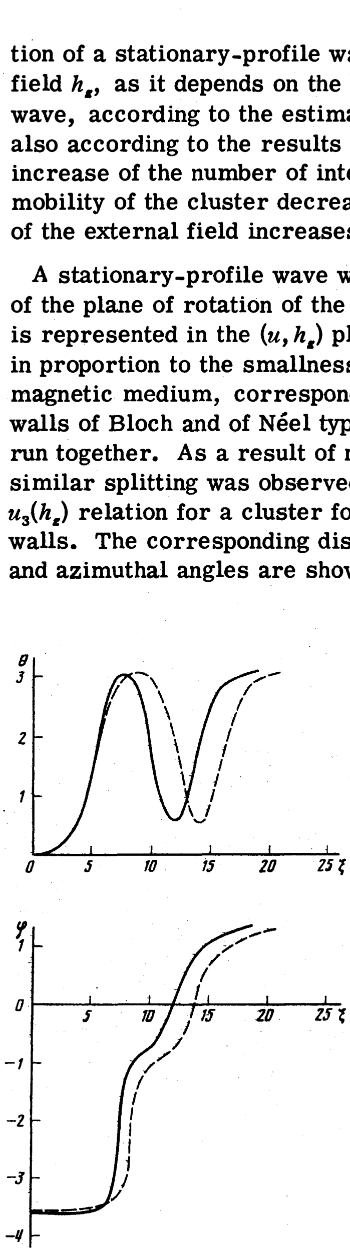

FIG. 2. Domain-wall structure corresponding to a cluster consisting of three domain walls, with monotonic variation of the azimuthal angle φ. The solid curve corresponds to $\varepsilon=0.2$, $\alpha=0.1$, $h_z=0.02$, $u=0.06$; the dotted curve corresponds to $\varepsilon=0.2$, $\alpha=0.01$, $h_z=0.002$, $u=0.06$.

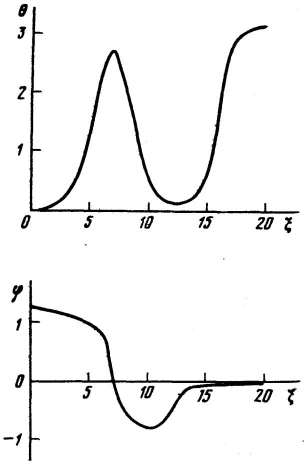

FIG. 3. Domain wall structure corresponding to a cluster of three domain walls with a change of sign of φ; $\varepsilon=0.2$, $\alpha=0.1$, $h_z=0.005$, $u=0.016$.

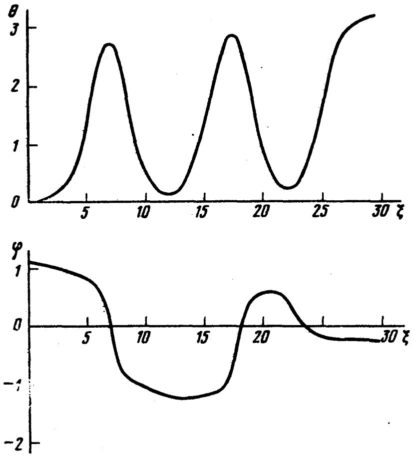

FIG. 4. Domain wall structure corresponding to a cluster of five domain walls with a change of sign of φ; $\varepsilon=0.2$, $\alpha=0.1$, $h_z=0.0125$, $u=0.024$.

behavior of the split states at $h_z\to h_3$ was not investigated because of computational difficulties. For the same reason, the splitting of the $u_5(h_z)$ curves for a cluster formed by five domain walls was not investigated. The solutions found were of a single type (see Fig. 4).

2. We shall give expressions that determine the asymptotic behavior of the solutions of the Landau-Lifshitz equations (1.4) in a region where uniform magnetization is established. For $\theta\to 0$, the system (1.4) can be written in the form

$$\omega\frac{d\Gamma}{d\varphi}+\Gamma^2+\alpha u\Gamma-\omega^2-u\omega=1+h_z+\frac{1}{2}\varepsilon+\frac{1}{2}\varepsilon\cos 2\varphi,$$
$$\omega\frac{d\omega}{d\varphi}+2\omega\Gamma+u\Gamma+\alpha u\omega=-\frac{1}{2}\varepsilon\sin 2\varphi,\quad \Gamma=\frac{\theta'}{\theta},\quad \omega=\varphi'. \qquad (2.1)$$

Here the azimuthal angle φ has been introduced as the independent variable. The case

$$\alpha=h_z=0$$

was treated earlier.[4] Omitting some simple calculations, we shall give the final result. The solution of the system (2.1) has the form

$$\Gamma(\varphi)=\Gamma_0+\frac{1}{2}\frac{\varepsilon}{[(2\Gamma_0+\alpha u)^2+u^2]^{1/2}}\cos 2(\varphi+\delta),$$
$$\omega(\varphi)=-\frac{u\Gamma_0}{2\Gamma_0+\alpha u}-\frac{1}{2}\frac{\varepsilon}{[(2\Gamma_0+\alpha u)^2+u^2]^{1/2}}\sin 2(\varphi+\delta), \qquad (2.2)$$
$$\operatorname{tg} 2\delta=\frac{u}{2\Gamma_0+\alpha u}.$$

The following condition for solvability of the system must be satisfied:

$$\frac{1}{4}\left[1-\left(\frac{\alpha u}{2\Gamma_0+\alpha u}\right)^2\right][(2\Gamma_0+\alpha u)^2+u^2]+\frac{1}{4}\frac{\varepsilon^2}{(2\Gamma_0+\alpha u)^2+u^2}=1+h_z+\frac{1}{2}\varepsilon. \qquad (2.3)$$

Similar relations arise for $\theta\to\pi$. The asymptotic boundary conditions (1.5) correspond to zeros of the precession frequency, which is determined by the second of the relations (2.2). The condition

$$\omega(\varphi)=0$$

leads to the relation (1.7).

In the numerical analysis of the problem of stationary-profile waves corresponding to steady-state moving domain walls with a turning of the plane of rotation of the magnetic moment, the following algorithm was used.

We consider the function

$$\mathscr{H}=\left(\frac{d\theta}{d\xi}\right)^2+\left[\left(\frac{d\varphi}{d\xi}\right)^2-1-\varepsilon\cos^2\varphi\right]\sin^2\theta+2h_z\cos\theta, \qquad (2.4)$$

which by virtue of

$$\frac{d\mathscr{H}}{d\xi}=-\alpha u\left[\left(\frac{d\theta}{d\xi}\right)^2+\left(\frac{d\varphi}{d\xi}\right)^2\sin^2\theta\right] \qquad (2.5)$$

decreases monotonically along solutions of the system (1.4) and takes values from $2h_z$ to $-2h_z$ at the self-localized solutions. We consider two one-parameter families of trajectories, "exiting" from the region of uniform magnetization along the external field ($\theta\to 0$ at $\xi\to-\infty$) and "entering" the region of uniform magnetization opposite to the external field ($\theta\to\pi$ at $\xi\to\infty$); and let C_- and C_+ be the respective parameters of these families. These two familes form, on intersection with the three-dimensional surface $\mathscr{H}=0$, two curves

$$\{\theta_-(C_-),\ \theta_-'(C_-),\ \varphi_-(C_-),\ \varphi_-'(C_-)\},$$
$$\{\theta_+(C_+),\ \theta_+'(C_+),\ \varphi_+(C_+),\ \varphi_+'(C_+)\}. \qquad (2.6)$$

Thus in order to find a self-localized solution of the system (1.4), it is necessary to solve the system of three equations

$$\theta_-(C_-)=\theta_+(C_+),\quad \varphi_-(C_-)=\varphi_+(C_+),\quad \varphi_-'(C_-)=\varphi_+'(C_+) \qquad (2.7)$$

with the two unknowns C_- and C_+. For arbitrary values of the parameters α, ε, h_z, and u of the system (1.4), this is in general impossible. Therefore the system of three equations (2.7) was solved for the three unknowns $C_\pm$ and u at fixed values of the parameters α, ε, and h_z.

3. Thus analysis of the problem of steady-state motion of domain walls with internal structure indicates the possibility of existence of stationary-profile waves in magnetic fields exceeding the Walker limiting field (1.3).

We remark that the impossibility of continuing the particular Walker solution (1.8) into the range of external magnetic fields $h_z > h_1$ led to the model of oscillating motion of domain walls.[2,5] The basis of the model is the assumption that in external fields $h_z > h_1$, a spatially uniform precession of the magnetic moment is excited, with preservation of the simple and well-known functional structure of the domain wall with respect to the polar angle θ. Application of a variational principle leads to "contraction" of the system of differential equations determining the time dependence of the variational parameters. In fields larger than the limiting field h_1, there occurs an oscillating motion of the domain wall, leading to the occurrence of a characteristic N-shaped variation of the mean velocity of motion of the domain wall with external field.

The existence of two essentially complementary models of the motion of domain walls in external magnetic fields indicates two possibilities for evolution of the system on attainment of the limiting values of external magnetic fields. Namely, on passage through critical field values either definite types of stationary-profile waves may be excited, or self-neutralization of the precessional motion leads to the result that the rotation of the magnetic moment is independent of the precession. Both models lead to a decrease of the mobility of domain walls on passage through a limiting field.

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Population and lifetime of excited states of shallow impurities in Ge

E. M. Gershenzon, G. N. Gol'tsman, and N. G. Ptitsina

V. I. Lenin State Pedagogical Institute, Moscow

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An investigation was made of the dependences of the intensities of photothermal ionization lines of excited states of shallow impurities in Ge on the intensity of impurity-absorbed background radiation and on temperature. The results obtained were used to find the density and lifetime of carriers of lower excited states of the impurity centers. The lifetimes of the excited states of donors in Ge were 10^{-9}–10^{-11} sec and the lifetime of the lower excited state of acceptors was $\sim 10^{-7}$ sec. In the presence of background radiation the population of the excited states was very different from the equilibrium value and, in particular, a population inversion of the $2p_{\pm 1}$ state relative to the $3p_0$ and $3s$ states was observed.

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1. INTRODUCTION

Information on the lifetimes of excited states of impurities in semiconductors and on the distribution of nonequilibrium carriers between such excited states under various conditions is essential for the understanding of the recombination of free carriers at impurity centers, establishment of an equilibrium between impurity states and a vacant band in the case of impact ionization of impurities, optical heating of free carriers, and other experiments.

The generally accepted classical cascade recombination model of Lax,[1] greatly refined and developed by Abakumov *et al.*,[2,3] does not allow for the discrete nature of the energy spectrum of the impurity electrons. Quantum calculations[4] show that lower excited states may play a fairly important role in the process of electron capture but not all of them are equivalent from the point of view of capture: for example, the states with a finite projection of the orbital momentum can be ignored. The lifetimes of excited donor states τ calculated in these treatments amount to 10^{-10}– 10^{-8} sec at low temperatures. Recent investigations of oscillations of the photoconductivity and photo-emf of p-type Ge subjected to a magnetic field[5] can be explained assuming anomalously long carrier lifetimes of the first excited acceptor state (10^{-6}–10^{-7} sec). Even longer lifetimes (exceeding seconds) of the $2s$ and split $1s$ states of donors in Si are suggested by Lehto and Proctor[6] to explain the impurity breakdown kinetics. Such a very great difference between the values of τ obtained using the approximate theory and indirect experimental data makes it highly desirable to determine directly the excited-state lifetimes.

The energies of transitions between excited impurity states correspond to the submillimeter wavelength range; only recently it has become possible to carry out

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