New method in the theory of a weakly ideal one-dimensional Fermi gas. Correlation functions

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We consider the problem of a weakly ideal one-dimensional Fermi gas. A regular method is proposed for calculating the corrections to the energy and to the correlation functions of a zero-spin Fermi gas and of a Fermi gas with spin. The method is based on reducing the Fermi Hamiltonian to an equivalent Bose Hamiltonian whose wave functions are sought in exponential form. The distribution of the ground state of the Fermi gas in momentum is obtained in first order in the particle interaction constant. In the zero-spin case and for a δ-function interaction potential, this distribution has a regular behavior in contrast to the expressions found in the literature. For a Fermi gas with spin, correlators of the superconducting and density-density type have been calculated. The latter is functionally close to the corresponding correlator of an ideal Fermi gas.

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1. INTRODUCTION

Despite the many exact results obtained in the problem of a nonideal one-dimensional Fermi gas,[1-3] this problem is by far not completely solved. In particular, if the interaction potential, while small, is not a δ function, there is no regular method for calculating the dependences of the energy, of the spectrum, and of the correlation functions on the interaction constant. The methods of summation of "parquet diagrams" and of the renormalization group yield only the leading terms of the corresponding quantities.[4,5] Although an approach based on the Bethe representation[1,2] for the wave functions of the one-dimensional problem does yield the exact ground-state energy and an accurate spectrum of the lowest excitations (for a δ-function potential), it is too complicated for the calculation of the correlation functions. It is known, for example, that a one-dimensional Fermi gas with attraction has a gap in the spectrum of the single-particle excitations,[3] and the expression for the gap coincides basically with the BCS formula. Yet it is still not clear whether this system is a superconductor.

A similar problem (we have in mind antiferromagnetism) arises in the one-dimensional Hubbard model with repulsion.[6] On the other hand, there is a group of papers, starting with Tomonaga's well known work,[7-10] in which a real quadratic spectrum reduces to a linear one near the Fermi "surface" and two sorts of Fermi particles corresponding to two points on the Fermi surface. This leads immediately to a lower bound on the spectrum. The introduction of the "Dirac sea" in this situation is by no means a well substantiated operation. However, perhaps the most serious shortcoming of this approach is that the total wave function of the system has no symmetry. There are no arguments whatever favoring the opinion that neglect of the regular symmetry is justified in the sense of some expansion in powers of some small interaction constant. A consequence of this shortcoming is, for example, the fact that a zero-spin Fermi gas with a δ-function potential has in the linear approximation[8,9] correlation functions that do not correspond to the formulas of an ideal gas.

The purpose of the present paper is to develop a regular method of obtaining the corrections for the energy and for the correlation functions of a weakly non-ideal Fermi gas. We shall consider separately a Fermi gas without and with spin. A brief exposition of the gist of the paper is given in Ref. 11.

2. THE BOSON REPRESENTATION

We consider first the question of a zero-spin Fermi gas. We seek the wave function $\psi(x_1,\ldots,x_N)$ of a system of zero-spin particles with Hamiltonian

$$\hat{H}=-\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2}+\sum_{i>j}V(x_i-x_j),\quad \hbar=2m=1, \tag{1}$$

in the form

$$\psi(x_1,\ldots,x_N)=\psi_0(x_1,\ldots,x_N)\,\Phi(x_1,\ldots,x_N), \tag{2}$$

where $\Phi(x_1,\ldots,x_N)$ is a symmetrical function, $\psi_0(x_1,\ldots,x_N)$ is the wave function of the ground state (1) at $V(x)=0$, and is given, apart from normalization (N is odd), by

$$\psi_0(x_1,\ldots,x_N)=\prod_{i>k}\sin\frac{\pi}{L}(x_i-x_k), \tag{3}$$

L is the length of the system.

Substituting (2) in the equation $\hat{H}\psi=E\psi$ we arrive at an equation for Φ:

$$\hat{\bar{H}}\Phi=(E-E_0)\Phi, \tag{4}$$

where E_0 is the energy of the ground state (1) at $V(x)=0$, and H is of the form

$$\hat{\bar{H}}=-\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2}-2\frac{\pi}{L}\sum_{i>j}\operatorname{ctg}\frac{\pi}{L}(x_i-x_j)\left(\frac{\partial}{\partial x_i}-\frac{\partial}{\partial x_j}\right)+\sum_{i>j}V(x_i-x_j). \tag{5}$$

Since $\Phi(x_1,\ldots,x_N)$ is a symmetric function, the problem of finding the wave function of zero-spin Fermi particles reduces according to (4) to the corresponding problem for a system of bosons with Hamiltonian (5).

In similar fashion, the transition from the fermion to the boson Hamiltonian can be made also in the presence of spin.

In fact, the coordinate part of the wave function

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$\psi(x_1, \ldots, x_N; y_1, \ldots, y_N)$ of a system of electrons in a state with $S=0$ is a solution of the equation

$$H_{sp}\psi(x_1,\ldots,x_N;y_1,\ldots,y_N)=E\psi(x_1,\ldots,x_N;y_1,\ldots,y_N),$$

$$\hat{H}_{sp}=-\sum_{i=1}^{N}\left(\frac{\partial^2}{\partial x_i^2}+\frac{\partial^2}{\partial y_i^2}\right)+\sum_{i,j}V(x_i-y_j), \tag{6}$$

x_i and y_i are the coordinates of the particles with spin $\uparrow(\alpha)$ and $\downarrow(\beta)$, respectively.

Although the Hamiltonian (6) takes into account only interaction of electrons with antiparallel spins, the analysis of the spin case can be easily extended to include also a more realistic Hamiltonian containing two types of interaction potentials, $V_{\sigma\sigma}(x)$ and $V_{\sigma,-\sigma}(x)$. In particular, this generalization will be made in Sec. 5 when the momentum distribution function is considered.

We seek $\psi(x_1, \ldots, x_N; y_1, \ldots, y_N)$ in the form

$$\psi(x_1,\ldots,x_N;y_1,\ldots,y_N)=\psi_0(x_1,\ldots,x_N)\psi_0(y_1,\ldots,y_N)\Phi(x_1,\ldots,x_N;y_1,\ldots,y_N), \tag{7}$$

where $\psi(x_1, \ldots, x_N)$ and $\psi(y_1, \ldots, y_N)$ are defined in accord with (3), and $\Phi(x_i, \ldots, x_N; y_i, \ldots, y_N)$ are functions that are symmetric in the coordinates x_i and y_i taken separately.

The equation for $\Phi(x_i, \ldots, x_N; y_i, \ldots, y_N)$ is

$$\hat{H}_{sp}\Phi(x_1,\ldots,x_N;y_1,\ldots,y_N)=(E-E_0)\Phi(x_1,\ldots,x_N;y_1,\ldots,y_N), \tag{8}$$

where E_0 is the energy of the ground state of the electrons, and

$$\hat{H}_{sp}=-\sum_{i=1}^{N}\left(\frac{\partial^2}{\partial x_i^2}+\frac{\partial^2}{\partial y_i^2}\right)-2\frac{\pi}{L}\sum_{i>j}\left\{\operatorname{ctg}\frac{\pi}{L}(x_i-x_j)\left(\frac{\partial}{\partial x_i}-\frac{\partial}{\partial x_j}\right)+\operatorname{ctg}\frac{\pi}{L}(y_i-y_j)\left(\frac{\partial}{\partial y_i}-\frac{\partial}{\partial y_j}\right)\right\}+\sum_{i,j}V(x_i-y_j). \tag{9}$$

Formula (8) effects the transition to the boson representation for an electron system.

The Hamiltonian (9) can be expressed in an equivalent second-quantization form:

$$\hat{H}_{sp}=\sum_{k,\sigma}k^2b_{k\sigma}^+b_{k\sigma}-\frac{\pi}{L}\sum_{\substack{q\neq0\\k_1,k_2,\sigma}}\operatorname{sign}q(k_1-k_2)\,b^+_{k_1+q\sigma}b^+_{k_2-q\sigma}b_{k_2\sigma}b_{k_1\sigma}+\frac{1}{2L}\sum_{k_1,k_2,q,\sigma}\nu(q)\,b^+_{k_1+q,\sigma}b^+_{k_2-q,-\sigma}b_{k_2,-\sigma}b_{k_1,\sigma} \tag{10}$$

$\nu(q)$ is the Fourier transform of $V(x)$.

3. PERTURBATION THEORY FOR THE WAVE FUNCTION

We consider now the task of solving Eqs. (4) and (8). We start with the zero-spin case. If the potential $V(x_i - x_j)$ in (4) is equal to zero, then the function Φ from (4) is equal to a constant. As shown by Bijl,[12] the solution of (4) at $V(x_i - x_j)\neq 0$ should be sought in the form

$$\Phi(x_1,\ldots,x_N)=C\exp\{S(x_1,\ldots,x_N)\}. \tag{11}$$

If $V(x_i - x_j)$ is small, then $S(x_1, \ldots, x_N)$ is also small. Perturbation theory yields then for S a regular behavior over the "volume" of the system L. Bijl's method[12] was subsequently refined by Bogolyubov and Zubarev,[13] who used it to determine the spectrum of a Bose gas. The results were the same as in Bogolyubov's well known paper on a weakly ideal Bose gas.[14] The use of Bijl's method, however, does not presuppose the presence a condensate, a most important factor in the one-dimensional case, which has been shown in Ref. 15 to have no condensate. It is natural to seek $S(x_1, \ldots, x_N)$ in the form of an expansion in two-, three-, etc. particle functions

$$S(x_1,\ldots,x_N)=\sum_{ij}S_2(x_i-x_j)+\sum_{ijl}S_3(x_i-x_j,x_j-x_l)+\ldots. \tag{12}$$

Going over to the momentum representation, we rewrite (12) in the form

$$S(x_1,\ldots,x_N)=\frac{1}{L}\sum_{k\neq0}\sum_{ij}\sigma_2(k)\exp(ik(x_i-x_j))+\frac{1}{L^2}\sum_{\substack{k_1+k_2+k_3=0\\k_i\neq0}}\sum_{i,j,l}\sigma_3(k_1,k_2,k_3)\exp(ik_1x_i+ik_2x_j+ik_3x_l)+\ldots. \tag{13}$$

For the two-, three-, etc. particle functions to be linearly independent it is necessary that the summations in (12) and (13) be carried out with the restriction $i\neq j$ in the first term, $i\neq j\neq l$ in the second, etc. In this case the n-th term of (13) can be obtained by integrating the expression

$$S(x_1,\ldots,x_N)\sum_{1\neq2\neq3\neq\ldots}\exp(-iq_1x_1-iq_2x_2-\ldots-iq_nx_n)$$

with respect to $x_1, \ldots, x_N$. In other words, "orthogonality" obtains for the two-, three-, etc. particle functions.

A consequence of this orthogonality is that the vanishing of $S(x_1, \ldots, x_N)$ means vanishing of each term of the expansion. Such an expansion of $S(x_1, \ldots, x_N)$ (with the restrictions $i\neq j$ etc.) will be called irreducible. Obviously, we could use also another expansion, such that the summation in (12) and (13) over i, j, etc. is effected without restriction. We shall call this expansion reducible. The reducible expansion was used in fact in the already cited paper.[13] It turns out to be appropriate for the Bose gas because the expansion in S_2, S_3, etc. coincides with the expansion in powers of the coupling constant. In the case of Eq. (4), however, to which the Fermi-gas problem reduces, the irreducible representation is of principal significance.

As can be shown below, the principal role in the determination of the correlation properties of the system is played by the two-particle function $S_2(x_i - x_j)$.

We proceed now to the problem of finding $S(x_1, \ldots, x_N)$ is first order in $V(x)$. Substituting (11) in (12), we arrive at the equation

$$-\sum_{i=1}^{N}\frac{\partial^2S}{\partial x_i^2}-\sum_{i=1}^{N}\left(\frac{\partial S}{\partial x_i}\right)^2-2\frac{\pi}{L}\sum_{i>j}\operatorname{ctg}\frac{\pi}{L}(x_i-x_j)\left(\frac{\partial S}{\partial x_i}-\frac{\partial S}{\partial x_j}\right)+\sum_{i<j}V(x_i-x_j)=E-E_0. \tag{14}$$

Neglecting in (14) the nonlinear term [this is justified in first-order perturbation theory in $V(x)$] and changing over to the function $\tilde{S}=\psi_0 S$, we obtain for S the equation

$$(H_0-E_0)\tilde{S}=-\sum_{i<j}V(x_i-x_j)\psi_0+(E-E_0)\psi_0, \tag{15}$$

$$H_0=-\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2}.$$

Since (15) is linear, S can be written as

$$S=\frac{1}{L}\sum_{q\neq0}\nu(q)\tilde{S}(q), \quad (16)$$

and $S(q)$ takes the form

$$\tilde{S}(q)=-i\int_0^\infty dt\exp\{-\varepsilon t+i(\hat{H}_0-E_0)t\}\sum_{i<j}\exp(iq(x_i-x_j))\psi_0, \quad \varepsilon\to0. \quad (17)$$

Formula (17) can be transformed into

$$\tilde{S}(q)=-i\int_0^\infty dt\exp(-\varepsilon t)\sum_{i<j}\exp(2iq^2t)\exp(iq(x_i-x_j))$$
$$\times\exp\left(2qt\frac{\partial}{\partial x_i}\right)\exp\left(-2qt\frac{\partial}{\partial x_j}\right)\psi_0, \quad (18)$$

where $\exp(2qt\partial/\partial x_i)$ is the $x_i\to x_i+2qt$ shift operator. Since we are interested in final analysis in the function $\tilde{S}(q)/\psi_0$, and ψ_0^{-1} has poles that do not coincide with the zeros of each of the terms of the sum over $i<j$ in (18) (although, of course, $\tilde{S}(q)/\psi_0$ has no singularities), it is convenient to employ the device

$$\psi_0^{-1}=\prod_{i>j}\sin^{-1}\frac{\pi}{L}(x_i-x_j)\to\prod_{i>j}\sin^{-1}\frac{\pi}{L}(x_i-x_j+i\delta), \quad \delta>0, \quad (19)$$

and let $\delta\to0$ at the end of the calculations.

The quantity $\sigma_2(k)$ in (13) is determined from the formula

$$\sigma_2(k)=\frac{L^{-N+1}}{2}\int S(x_1,\ldots,x_N)\exp(-ik(x_1-x_2))dx_1\ldots dx_N. \quad (20)$$

The integral in (20) is evaluated by changing to integration along the unit circle in the z plane, where $z_i=\exp(2\pi x_i/L)$; $i=1,\ldots,N$. We calculate the integral in (20) [for the i,j term of the sum (18)] by residues in succession: first with respect to all variables except z_i, z_j, z_1, and z_2, and then with respect to the remainder. In the limit as $L\to\infty$ and $N\to\infty$ (with $N/L=\rho$) we get

$$\sigma_2(k)=-\frac{1}{4}\nu(k)(k^2+2p_F|k|)^{-1}$$
$$+\frac{1}{2}\int_{|k|}^\infty\nu(q)[(q-2p_F)^2+8p_F|k|]^{-1/2}dq, \quad (21)$$
$$p_F=\pi\rho.$$

As $k\to0$ the function $\sigma_2(k)$ takes the form

$$\sigma_2(k)=-\frac{\nu(0)-\nu(2p_F)}{8p_F|k|}+\frac{\nu''(2p_F)}{4}\ln\frac{2p_F}{|k|}+O(1). \quad (22)$$

For a δ-function interaction potential, i.e., for constant $\nu(q)$, as seen from (21), $\sigma_2(k)=0$ because in this case we have an ideal Fermi gas.

In analogy with the preceding, we can calculate the energy $E-E_0$, the terms of the type $\sigma_3(k_1,k_2,k_3)$, etc. In particular, we have

$$E-E_0=L^{-N}\int S(x_1,\ldots,x_N)dx_1\ldots dx_N.$$

By calculating this integral we obtain the correction to the energy in the Hartree-Fock approximation.

In analogy with the case of the system of zero-spin Fermi particles, we seek a solution of Eq. (8) in the form

$$\Phi(x_1,\ldots,x_N;y_1,\ldots,y_N)=C\exp\{S(x_1,\ldots,x_N;y_1,\ldots,y_N)\}. \quad (23)$$

The expansions $S(x_1,\ldots,x_N;y_1,\ldots,y_N)$ of the type (12) and (13) are given by

$$S=\sum_{i,j}S_{\alpha\beta}(x_i-y_j)+\sum_{i,j}\{S_{\alpha\alpha}(x_i-x_j)+S_{\beta\beta}(y_i-y_j)\}$$
$$+\sum_{i,j,l}S_{\alpha\beta\beta}(x_i-y_j,y_j-y_l)+\ldots, \quad (24)$$

$$S=L^{-1}\sum_{k\neq0}\sum_{i,j}\sigma_{\alpha\beta}(k)\exp(ik(x_i-y_j))+L^{-1}\sum_{k\neq0}\sum_{i,j}\sigma_{\alpha\alpha}(k)\exp(ik(x_i-x_j))$$
$$+L^{-2}\sum_{\substack{k_1+k_2+k_3=0\\k_i\neq0}}\sum_{i,j,l}\sigma_{\alpha\beta\beta}(k_1,k_2,k_3)\exp(ik_1x_i+ik_2y_j+ik_3y_l)+\ldots \quad (25)$$

It is easily seen that considerations connected with the orthogonality of $S_{\alpha\beta}(x_i-y_j)$, $S_{\alpha\alpha}(x_i-x_j)$, etc. permit the summation over i and j in the first term of (24) or (25) without any restrictions, in the second term $i\neq j$, in the third $j\neq l$, etc.

The problem of finding the function $S(x_1,\ldots,x_N;y_1,\ldots,y_N)$ in first order in $V(x)$ is fully analogous to the zero-spin case. Introducing the function

$$\tilde{S}=\psi_0(x_1,\ldots,x_N)\psi_0(y_1,\ldots,y_N)S,$$

we obtain for S the equation

$$(\hat{H}_0-E_0)\tilde{S}=-\sum_{i,j}V(x_i-y_j)\psi_0(x_1,\ldots,x_N)\psi_0(y_1,\ldots,y_N)$$
$$+(E-E_0)\psi_0(x_1,\ldots,x_N)\psi_0(y_1,\ldots,y_N), \quad (26)$$

where

$$\hat{H}_0=-\sum_{i=1}^N\left(\frac{\partial^2}{\partial x_i^2}+\frac{\partial^2}{\partial y_i^2}\right).$$

The function $S(q)$ [see (16)] is expressed in the form

$$\tilde{S}(q)=-i\int_0^\infty dt\exp(-\varepsilon t)\sum_{i,j}\exp(2iq^2t)\exp(iq(x_i-y_j))\exp\left(2qt\frac{\partial}{\partial x_i}\right)$$
$$\times\exp\left(-2qt\frac{\partial}{\partial y_j}\right)\psi_0(x_1,\ldots,x_N)\psi_0(y_1,\ldots,y_N). \quad (27)$$

In (27), $\exp(2qt\partial/\partial x_i)$ and $\exp(-2qt\partial/\partial y_j)$ are the shift operators of the arguments x_i and y_i of the functions $\psi_0(x_1,\ldots,x_N)$ and $\psi_0(y_1,\ldots,y_N)$, respectively. In the calculation of the function

$$\tilde{S}(q)/\psi_0(x_1,\ldots,x_N)\psi_0(y_1,\ldots,y_N)$$

of interest, we use again a device involving the introduction of an infinitesimally small quantity δ in accord with (19). The quantity $\sigma_{\alpha\beta}(k)$ of (25) is determined from the formula

$$\sigma_{\alpha\beta}(k)=L^{-2N+1}\int\ldots\int S(x_1,\ldots,x_N;y_1,\ldots,y_N)$$
$$\times\exp(-ik(x_1-y_1))dx_1,\ldots,dx_N\,dy_1\ldots dy_N. \quad (28)$$

Changing in (28) to integration over the unit circle in the (z,u) plane, where

$$z_i=\exp\left(\frac{2\pi}{L}x_i\right),\quad u_j=\exp\left(\frac{2\pi}{L}y_j\right),$$

and calculating (28) by residues, we obtain the limit as $L\to\infty$, $N\to\infty$, $N/L=\rho/2$

$$\sigma_{\alpha\beta}(k)=-{}^1/_2\nu(k)/(k^2+2p_F|k|), \quad p_F=\pi\rho/2. \quad (29)$$

Evaluating integrals of the form

$$\int S(x_1,\ldots,x_N;y_1,\ldots,y_N)\exp(-ik_1x_1-ik_2x_2$$
$$\ldots-ik_Nx_N)dx_1\ldots dx_N\,dy_1\ldots dy_N, \quad (30)$$

it is easy to verify that

$$\sigma_{\alpha\alpha}(k)=\sigma_{\beta\beta}(k)=\sigma_{\alpha\alpha\alpha}(k_1, k_2, k_3)=\ldots=0.$$

We calculate also the function $\sigma_{\alpha\beta\beta}(k_1, k_2, k_3)$ of (25). It is obtained from the formula

$$\sigma_{\alpha\beta\beta}(k_1,k_2,k_3)=\frac{L^{-2N+2}}{2}\int S(x_1,\ldots,x_N;y_1,\ldots,y_N)$$
$$\times\exp[-i(k_1x_1+k_2y_1+k_3y_2)]\,dx_1\ldots dx_N\,dy_1\ldots dy_N. \qquad (31)$$

Calculating the integral (31) in analogy with (28), we get

$$\sigma_{\alpha\beta\beta}(k_1, k_2, k_3)=2^{-1}\pi L^{-1}v(k_1)\,|k_1|\,(k_1^2+2p_F|k_1|)^{-2}$$
$$\times\{\theta(k_2)\theta(k_3)(1-\theta(k_1))+(1-\theta(k_2))(1-\theta(k_3))\theta(k_1)\}. \qquad (32)$$

4. MOMENTUM DISTRIBUTION IN THE GROUND STATE OF A FERMI GAS

Inasmuch as the momentum distribution function in the ground state $n_p\langle\psi|a_p^+a_p|\psi\rangle$ is the Fourier transform of the correlation function

$$g(x, x')=\langle\psi|a^+(x)a(x')|\psi\rangle$$

[$a^+(x)$ and $a(x)$ are the operators for the creation and annihilation of the zero-spin particles], we shall consider the correlation function. It is given by

$$g(x, x')=\langle\exp(T_0/2)a^+(x)a(x')\exp(T_0/2)\rangle_0/\langle\exp T_0\rangle_0, \qquad (33)$$

where $\langle\ldots\rangle_0$ denotes averaging with the wave function ψ_0, and

$$\hat{T}_0=2\int_0^L dz\int_0^L dz'S_2(z-z')n(z)n(z')=\frac{2}{L}\sum_{p,p',q}\sigma_2(q)a_{p+q}^+a_pa_{p'-q}^+a_{p'},$$
$$n(z)=a^+(z)a(z).$$

It is convenient to transform (33) into

$$g(x, x')=\exp\{2S_2(0)-2S_2(\xi)\}\langle\exp(T_0+T_1)a^+(x)a(x')\rangle_0/\langle\exp T_0\rangle_0, \qquad (34)$$

where

$$\hat{T}_1=2\int_0^L[S_2(z-x')-S_2(z-x)]n(z)\,dz$$
$$=2\frac{1}{L}\sum_{p,q}\sigma_2(q)[\exp(iqx')-\exp(iqx)]a_{p+q}^+a_p, \qquad \xi=x'-x,$$

and when changing from (33) to (34) we use the fact that $\hat{T}_0$ and $\hat{T}_1$ commute.

We calculate (34) by a diagram technique. Since the mean values in (34) are expressed in terms of connected diagrams, we can represent $g(x, x')$ in the form

$$g(x, x')=b(x, x')\exp[2S_2(0)-2S_2(\xi)]\langle\exp(T_0+T_1)\rangle_0/\langle\exp T_0\rangle_0. \qquad (35)$$

We consider first the factor $b(x, x')$ in (35):

$$b(x,x')=g_0(\xi)+L^{-1}\sum_{p,q}\sum_{n=1}\exp(-ip\xi+iqx)\frac{1}{n!}\langle(T_0+T_1)^na_{p+q}^+a_p\rangle_{0\,\mathrm{conn}}, \qquad (36)$$

where $g_0(\xi)=(\pi\xi)^{-1}\sin p_F\xi$ is the correlation function of the system without interaction. Since T_0 has the usual four-fermion form, the construction of the connected diagrams (36) can be carried out in the standard manner. The corresponding diagrams consist of closed loops connected by the interaction lines $2\sigma_2(q)$. In each loop it is convenient to sum over all the particle and hole lines. We illustrate this with one of the diagrams $\langle T_0^3\rangle_{0\,\mathrm{conn}}$ as an example (see Fig. 1). The $b(x, x')$ diagrams contain two types of loops, shown graphically in Fig. 2. The contribution of a loop of n-th order (to which n interaction lines are connected) is

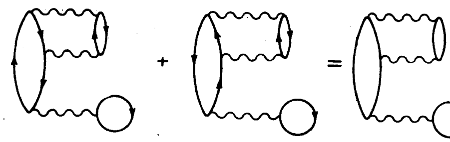

FIG. 1.

$$\sum_{k=0}Q_{n+k,\alpha}(q_1,\ldots,q_n,p_1,\ldots,p_k)\bar{\sigma}_2(p_1)\ldots\bar{\sigma}_2(p_k), \qquad (37)$$

where $\alpha=1, 2$ corresponds to Figs. 2a and 2b,

$$\bar{\sigma}_2(q)=\sigma_2(q)(\exp(iq\xi)-1),$$
$$Q_{n,1}(q_1,\ldots,q_n)=L^{-1}\langle\rho(q_1)\ldots\rho(q_n)\rangle_{0\,\mathrm{c}}\,\delta_{q_1+\ldots+q_n,0}, \qquad (38)$$

$$Q_{n,2}(q_1,\ldots,q_n)=\frac{1}{L}\sum_{p,q}\exp(-ip\xi)\langle\rho(q_1)\ldots\rho(q_n)a_{p+q}^+a_p\rangle_{0\,\mathrm{c}}\,\delta_{q_1+\ldots+q_n,-q}; \qquad (39)$$

$$\rho(q)=\sum_p a_{p+q}^+a_p.$$

The following diagram equations hold in this case (see Fig. 3): circles on the diagrams denote "intrusion" of operators T_1 into the loops, i.e., they correspond inclusion of operators of T_1 in the mean values (38) and (39). We note also that each diagram (36) contains only one loop of the type of Fig. 2b.

In our earlier paper[11] we established a number of proproperties of the functions $Q_{n,1}(q_1,\ldots,q_n)$ and $Q_{n,2}(q_1,\ldots,q_n)$ (see Ref. 11, pp. 647 of original and 611 of the translation), and have shown on their basis that the main contribution to $g(x, x')$ is made by diagrams of type of Fig. 4. At $\xi\gg p_F^{-1}$ the diagrams of this type, which contain n interaction lines, are of the order of $\sim\ln^n(p_F\xi)$. Their summation leads to the result:

$$g(x, x')=g_0(\xi)\xi^{-\varkappa}, \quad \varkappa\sim v^2.$$

It will be shown below that all the remaining diagrams are "extraneous" at $p_F\xi\gg1$ and their summation does not alter the character of the singularity obtained in Ref. 11.

We turn now to the problem of calculating $b(x, x')$. All the possible types of diagrams for this quantity are shown in Fig. 5. We denote the factor corresponding to the "generalized" loops of Fig. 5 (i.e., the shaded squares to which n interaction lines are connected) by $\Pi_n(q_1,\ldots,q_n)$. We consider first diagrams of type 5a. Let such a diagram contain n_1 generalized loops of first-order, n_2 of second order, etc., with $n_1+2n_2+\ldots+tn_t=m$ (m is the number of interaction lines on the diagram). The total contribution of this diagram is

$$\sum_{n=0}\frac{1}{n!}N_m\sum_{\{p,q\}}Q_{n+m,2}(\{p,q\})\Pi_1(q_1^1)\ldots\Pi_1(q_{n_1}^1)\Pi_2(q_1^2,q_2^2)$$
$$\times\Pi_2(q_3^2,q_4^2)\ldots\Pi_2(q_{2n_2-1}^2,q_{2n_2}^2)\Pi_3(q_1^3,q_2^3,q_3^3)\ldots\bar{\sigma}_2(p_1)\ldots\bar{\sigma}_2(p_n)$$
$$\times\sigma_2(q_1^1)\ldots\sigma_2(q_{n_1}^1)\sigma_2(q_1^2)\sigma_2(q_2^2)\sigma_2(q_3^2)\sigma_2(q_4^2)$$
$$\ldots\sigma_2(q_{2n_2-1}^2)\sigma_2(q_{2n_2}^2)\sigma_2(q_1^3)\sigma_2(q_2^3)\sigma_2(q_3^3)\ldots, \qquad (40)$$

FIG. 2.

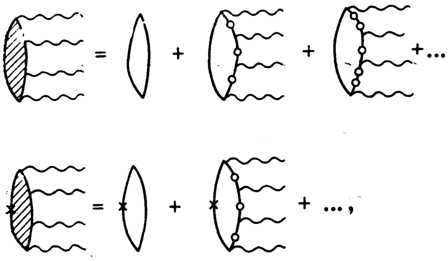

FIG. 3.

where $\{p, q\} = (p_1, \ldots, p_n; -q_1^1, \ldots, -q_{n_1}^1;\ -q_2^2, \ldots, -q_{2n_1-1}^2, -q_{2n_1}{}^2; -q_1^3, -q_2^3, -q_3^3 \ldots$, the superscript of the momentum q is the order of the loop to which it pertains; N_m is the number of diagrams and equals

$2^m m!/n_1!(2!)^{n_2}n_2!\ldots(t!)^{n_t}n_t!$.

It can be shown that the quantity $Q_{n,2}(q_1, \ldots, q_n)$ can be represented in the form

$$Q_{n,2}(q_1,\ldots,q_n)=g_0(\xi)+\sum_{i=1}^{n} f_1(q_i)+\sum_{i\neq j}^{n} f_2(q_i,q_j)$$
$$+\sum_{i\neq j\neq l}^{n} f_3(q_i,q_j,q_l)+\ldots+f_n(q_1,q_2,\ldots,q_n). \tag{41}$$

We include in (41) initially only the first term. Substituting for $Q_{n+m,2}(\{p,q\})$ in (40) the function $g_0(\xi)$, which does not depend on $\{p, q\}$ and taking the sum $m, n_1, n_2, \ldots, n_t$ (so that $n_1+2n_2+\ldots=m$), we find that the corresponding contribution to $b(x, x')$ is equal to $g_0(\xi)a(\xi)$, where

$$a(\xi)=\exp\Big\{2S_2(\xi)-2S_2(0)+\frac{2}{1!}\sum_{q_1}\Pi_1(q_1)\sigma_2(q_1)+\frac{2^2}{2!}\sum_{q_1q_2}\Pi_2(q_1,q_2)\sigma_2(q_1)$$
$$\times\sigma_2(q_2)+\frac{2^3}{3!}\sum_{q_1q_2q_3}\Pi_3(q_1,q_2,q_3)\sigma_2(q_1)\sigma_2(q_2)\sigma_2(q_3)+\ldots\Big\}. \tag{42}$$

In the expansion (41) we consider now, for example, the term $f(\{\tilde{p}, \tilde{q}\})$, where

$$\{\tilde{p},\tilde{q}\}=(p_1,\ldots,p_{s_0};-q_1{}^1,\ldots,-q_{s_1}{}^1;-q_1{}^2,-q_2{}^2,\ldots,-q_{2s_2-1}^2;\ldots),$$
$$s_0\leqslant n,\quad s_1\leqslant n_1,\ldots,\quad s_t\leqslant n_t.$$

We substitute $f(\{\tilde{p}, \tilde{q}\})$ in (40) in place of $Q_{n+m,2}(\{p, q\})$ and introduce new summation indices:

$$s_0,\ \tilde{s}_0=n-s_0;\quad s_1,\ \tilde{s}_1=n_1-s_1;\quad s_2,\ \tilde{s}_2=n_2-s_2;\ldots;\quad s_t,\ \tilde{s}_t=n_t-s_t.$$

We can sum in (40) over $\tilde{s}_0, \tilde{s}_1, \ldots, \tilde{s}_t$ independently, and this yields $a(\xi)$. Taking also the sum over $s_0, s_1, \ldots, s_t$, we obtain as the result

$$a(\xi)\sum_{\{\tilde{p},\tilde{q}\}}\sum_{s_0,s_1,\ldots,s_t} f(\{\tilde{p},\tilde{q}\})\frac{\tilde{\sigma}_2(p_1)\ldots\tilde{\sigma}_2(p_{s_0})}{s_0!}\frac{2^{s_1}}{s_1!}\Pi_1(q_1{}^1)\ldots\Pi_1(q_{s_1}{}^1)$$
$$\times\frac{2^{2s_2}}{s_2!}\Pi_2(q_1{}^2,q_2{}^2)\Pi_2(q_3{}^2,q_4{}^2)\ldots\Pi_2(q_{2s_2-1}^2,q_{2s_2}^2)\ldots\sigma_2(q_1{}^1)\ldots\sigma_2(q_{s_1}{}^1)$$
$$\times\sigma_2(q_1{}^2)\sigma_2(q_2{}^2)\ldots\sigma_2(q_{2s_2-1}^2)\sigma_2(q_{2s_2}^2)\ldots G, \tag{43}$$

where G is the topological factor of the diagram and depends on $\{\tilde{p}, \tilde{q}\}$.

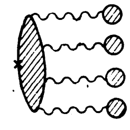

FIG. 4.

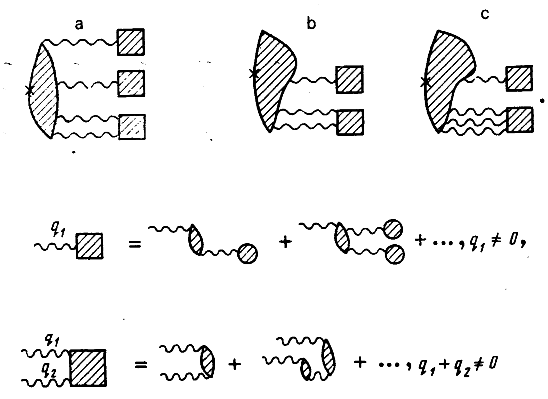

FIG. 5.

The total contribution made to $b(x, x')$ by diagrams of type 5a is thus

$$a(\xi)\Big[g_0(\xi)+\sum_{n=2}\nu^n\varphi_{n,1}(\xi)\Big], \tag{44}$$

where the functions $\varphi_{n,1}(\xi)$ are sums of quantities of the type of the coefficient $a(\xi)$ in (43). We can similarly calculate contributions from the diagrams of type 5b, 5c, etc. It turns out that they take the form

$$a(\xi)\sum_{n=2}\nu^n\varphi_{n,2}(\xi),\quad a(\xi)\sum_{n=2}\nu^n\varphi_{n,3}(\xi),\ldots$$

etc. We get ultimately for $b(x, x')$

$$b(x,x')=a(\xi)\Big[g_0(\xi)+\sum_{n=2}\nu^n\varphi_n(\xi)\Big],\quad \varphi_n(\xi)=\sum_i\varphi_{n,i}(\xi). \tag{45}$$

We now clarify the character of the behavior of $b(x, x')$ at $p_F\xi \gg 1$. It must be noted here that since $\sigma_2(q) \sim |q|^{-1}$ as $q \to 0$, it is obvious that the most substantial contributions to the quantities in (45) come from integration in the region of small q. Analysis of the expression for $Q_{n,1}(q_1, \ldots, q_n)$ shows that it differs from zero if all $|q_i| \gtrsim 2p_F$, $i = 1, \ldots, n$ (then $Q_{n,1}(q_1, \ldots, q_n)$), or if it is small ($|q_i| \ll 2p_F$) then one of the momenta $q_1, \ldots, q_n$ (in this case $Q_{n,1} = |q_i|/2\pi$). As a result, the expressions

$$\Pi_n(q_1,\ldots,q_n)\sigma_2(q_1)\ldots\sigma_2(q_n)$$

for $n \geqslant 2$ will be finite as $q \to 0$ and upon integration with respect to $q_1, \ldots, q_n$ they yield functions that are not singular as $\xi \to \infty$. As to the quantity $\Pi_1(q_1)$, it includes, in particular, the diagrams of Fig. 6 and therefore

$$\sum_{q_1}\Pi_1(q_1)\sigma_2(q_1)\sim\ln(p_F\xi).$$

This is precisely the class of diagrams that was summed in Ref. 11. It is also easy to see that the diagrams of other types in $\Pi_1(q)$ do not lead to singularities as $\xi \to \infty$. Summing the diagrams of Fig. 6 we obtain for $a(\xi)$

$$a(\xi)=\exp\Big\{2S_2(\xi)-2S_2(0)-4\pi^{-2}\int_0^\infty(1-\cos q\xi)q\sigma_2{}^2(q)\,dq$$
$$-8\pi^{-3}\int_0^\infty q^2\sigma_2{}^3(q)(1-\cos q\xi)(1-2q\pi^{-1}\sigma_2(q))^{-1}\,dq\Big\}. \tag{46}$$

We now estimate the quantities $\varphi_n(\xi)$, which enter in (45), as $\xi \to \infty$. We consider, in particular the coefficient of $a(\xi)$ in (43). The most "dangerous" from the

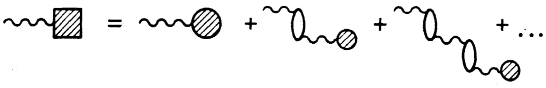

FIG. 6.

point of view of the appearance of singularities in ξ is the integration with respect to $p_1, \ldots, p_{s_0}$ and $q_1^1, \ldots, q_{s_1}^1$ at small values of these momenta. [Integration over all other momenta does not lead to singularities by virtue of the aforementioned properties of the function $Q_{n,1}(q_1, \ldots, q_n)$]. We note now the following fact. At small $q_1, \ldots, q_n$, or more accurately speaking at

$$|q_i|<2p_F; \quad |q_i+q_j|<2p_F; \quad \ldots; \quad |q_1+\ldots+q_n|<2p_F,$$

we have

$$f_n(q_1,\ldots,q_n)=i(-1)^n(2\pi\xi)^{-1}\prod_{l=1}^{n}\exp\left(-\frac{iq_l\xi}{2}\right)$$
$$\times\left\{\exp(-ip_F\xi)\prod_{l=1}^{n}\left[\theta(-q_l)\exp\left(-\frac{iq_l\xi}{2}\right)+\theta(q_l)\exp\left(\frac{iq_l\xi}{2}\right)\right]\right.$$
$$\left.-\exp(ip_F\xi)\prod_{l=1}^{n}\left[\theta(-q_l)\exp\left(\frac{iq_l\xi}{2}\right)+\theta(q_l)\exp\left(-\frac{iq_l\xi}{2}\right)\right]\right\}. \quad (47)$$

Substitution of (47) in (43) leads to the appearance of factors

$$\prod_{l=1}^{n}\sin\frac{p_l\xi}{2}$$

and integration with respect to p_i yields zero.

For this reason (the presence of "odd" functions), the integrals with respect to $q_1^1, \ldots, q_s^1$ are equal to zero in the region of small q. Integration over large q, on the other hand, can be readily seen to lead to $\varphi_n \sim \xi^{-1}$.

We consider next the quantity

$$\langle\exp(T_0+T_1)\rangle_0/\langle\exp T_0\rangle_0$$

of (35). We express it in terms of the connected diagrams:

$$\langle\exp(T_0+T_1)\rangle_0/\langle\exp T_0\rangle=\exp\left\{\sum_{n_1=0}\sum_{n_2=1}\langle T_0^{n_1}T_1^{n_2}\rangle_{0\,\mathrm{conn}}\right\}. \quad (48)$$

Examination of the corresponding diagrams with account taken of the properties of the functions $Q_{n,1}(a_1, \ldots, q_n)$ shows that the largest ($\sim\ln p_F\xi$) contribution to the argument of the exponential (48) is made by the term $\langle T_1^2\rangle_{0\,\mathrm{conn}}$, and also by the diagrams $\langle T_0^{n_1}T_1^{n_2}\rangle_{0\,\mathrm{conn}}$ of the type shown in Fig. 7. The remaining diagrams in (48) are nonsingular as $\xi\to\infty$. On the other hand, calculating $\langle T_1^2\rangle_{0\,\mathrm{conn}}$ and summing the diagrams of Fig. 7 we get

$$\langle\exp(T_0+T_1)\rangle_0/\langle\exp T_0\rangle_0=\exp\left\{2\pi^{-2}\int_0^\infty(1-\cos q\xi)\,q\sigma_2^2(q)\,dq\right.$$
$$\left.+4\pi^{-3}\int_0^\infty q^2\sigma_2^3(q)\,(1-\cos q\xi)\,(1-2q\pi^{-1}\sigma_2(q))^{-1}\,dq\right\}. \quad (49)$$

Taking (35), (42), (45), (46), and (49) into account we get ultimately for $g(\xi)$:

$$g(\xi)=\exp\{-F_0(\xi)+F_1(\nu,\xi)\}\left\{g_0(\xi)+\xi^{-1}\sum_{n=2}\nu^n\Phi_n(\xi)\right\}, \quad (50)$$

where

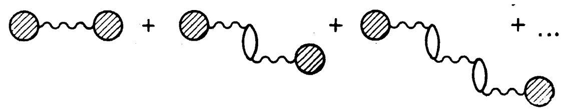

FIG. 7.

$$F_0(\xi)=2\pi^{-2}\int_0^\infty(1-\cos q\xi)\,[1-2q\pi^{-1}\sigma_2(q)]^{-1}q\sigma_2^2(q)\,dq,$$

and $F_1(\nu, \xi)$ and $\Phi_n(\xi)$ are functions that are not singular as $\xi\to\infty$. The distribution function

$$n_p=\langle\psi|a_p^+a_p|\psi\rangle$$

takes at $|p|\approx p_F$, according to (5), the form

$$n_p=\frac{1}{2}\{1+\mathrm{sign}(p_F-p)\,|p-p_F|^\beta\},\qquad \beta=\frac{[\nu(0)-\nu(2p_F)]^2}{32\pi^2p_F^2}. \quad (51)$$

5. CORRELATION FUNCTIONS OF THE GROUND STATE OF A FERMI GAS WITH SPIN

We consider now the question of the calculation of the correlation functions for a system of particles with spin. Our primary interest here is whether the singularity obtained in the zero-spin case in the distribution function n_p (51) of the particle momenta in the ground state is preserved also for a system of particles with spin. We consider in this connection the correlation function

$$g_{\mathrm{sp}}(x,\ x')=\langle\psi|a_\alpha^+(x)a_\alpha(x')+a_\beta^+(x)a_\beta(x')|\psi\rangle.$$

It is easily seen that $g_{\mathrm{sp}}(x, x')$ can be expressed in the form

$$g_{\mathrm{sp}}(x,\ x')=2\langle\exp(W+T_{1\beta})a_\alpha^+(x)a_\alpha(x')\rangle_0/\langle\exp W\rangle_0, \quad (52)$$

where

$$\hat{W}=2\int_0^L dz\int_0^L dz'S_{\alpha\beta}(z-z')n_\alpha(z)n_\beta(z'),$$

$$\hat{T}_{1\sigma}=\int_0^L dz[S_{\alpha\beta}(x'-z)-S_{\alpha\beta}(x-z)]n_\sigma(z).$$

Expressing the mean values in (52) in terms of contributions of only connected diagrams, we represent $g_{\mathrm{sp}}(x, x')$ in the form

$$g_{\mathrm{sp}}(x,x')=\frac{\langle\exp(W+T_{1\beta})\rangle_0}{\langle\exp W\rangle_0}\left\{g_0(\xi)+\frac{1}{L}\sum_{p,q}\sum_{n=1}\exp(-ip\xi+iqx)\right.$$
$$\left.\times\frac{1}{n!}\langle(W+T_{1\beta})^n a_{p+q\alpha}^+a_{p\alpha}\rangle_{0\,\mathrm{conn}}\right\}. \quad (53)$$

The connected diagrams (53) are constructed in accordance with the same rules as in the zero-spin case, subject however to the simplifying circumstance that the contributions of the closed loops, due to pairings of Fermi operators with different spin indices, are equal to zero. In particular, loops of the type of Fig. 2b contain no intrusions of the operators $T_{1\beta}$ i.e., Eq. (37) has no terms with $k\neq 0$ and $\alpha=2$. It is also easily seen that all the considerations concerning the estimates of the behavior of the diagram contributions as $\xi\to\infty$ and separation of the class of singular diagrams, which were made for the zero-spin case, are valid here, too. In particular, the diagrams that make the principal contribution to the curly-bracket term of (53) take the form shown in Fig. 8. We recall that the unshaded loops in Fig. 8 correspond to absence of terms with $k\neq 0$ from (37). Summing the diagrams of Fig. 8 and taking into account, just as in the

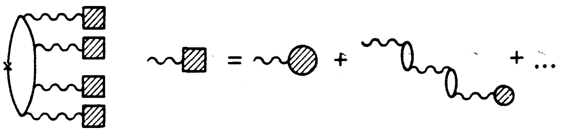

FIG. 8.

zero-spin case, the principal contributions to

$$\langle \exp(W+T_{1\beta})\rangle_0/\langle \exp W\rangle_0,$$

we obtain for $g_{sp}(x, x')=g_{sp}(\xi)$

$$g_{sp}(\xi)=2g_0(\xi)(p_F\xi)^{-\bar\beta}, \quad \bar\beta=\nu^2(0)/32\pi^2 p_F^2. \tag{54}$$

In accord with (54), the distribution function n_p takes the form (51) with $\beta=\bar\beta$.

The same form is possessed also by the distribution function n_p for a spin system with a more realistic Hamiltonian containing two types of interaction potentials (see Sec. 2). In this case $\nu^2(0)$ and (54) is replaced by

$$\nu^2_{\sigma,-\sigma}(0)+[\nu_{\sigma\sigma}(0)-\nu_{\sigma\sigma}(2p_F)]^2.$$

For a system of particles with spin, it is also of interest to calculate the correlator

$$G(x, x')=\langle\psi|a_\alpha^+(x)a_\beta^+(x)a_\beta(x')a_\alpha(x')|\psi\rangle.$$

It is easy to show that it takes the form

$$G(x, x')=\exp\{2S_{\alpha\beta}(0)-2S_{\alpha\beta}(\xi)\}$$
$$\times\langle\exp(W+T_{1\alpha}+T_{1\beta})a_\alpha^+(x)a_\alpha(x')a_\beta^+(x)a_\beta(x')\rangle_0\langle\exp W\rangle_0^{-1}. \tag{55}$$

where $\hat W$ and $\hat T_{1\sigma}$ are defined in accordance with (52). The separation of the singular diagram is easily done in analogy with the preceding cases. In this case $G(x, x')=G(\xi)$ is given by

$$G(\xi)=g_0^2(\xi)(p_F\xi)^{-2\bar\beta}. \tag{56}$$

We consider, finally, the density-density correlator

$$R(\xi)=\langle\psi|n(x)n(x')-\rho^2|\psi\rangle, \quad n(x)=\sum_\sigma n_\sigma(x).$$

It takes the form

$$R(\xi)=\langle\exp(W)[n(x)n(x')-\rho^2]\rangle_0/\langle\exp W\rangle_0. \tag{57}$$

Analysis of the expression shows that the diagrams corresponding to (57) are nonsingular and their contributions $\sim\xi^{-2}$ as $\xi\to\infty$. Accurate to terms of first order in the interaction constant, we have

$$R(\xi)=\rho\delta(\xi)-2g_0^2(\xi)+\pi^{-3}\left\{\int_0^{2p_F}\sigma_{\alpha\beta}(q)q^2\cos q\xi\,dq\right.$$
$$\left.+(2p_F)^2\int_{2p_F}^{\infty}\sigma_{\alpha\beta}(q)\cos q\xi\,dq\right\}. \tag{58}$$

The first two terms in (58) correspond to $R(\xi)$ for an ideal one-dimensional Fermi gas, while the last two terms likewise behave like ξ^{-2} (as $\xi\to\infty$), but contain the small factor ν. We note, to conclude this section, that in the calculation of $g_{sp}(\xi)$ we have neglected in $S(x_1,\dots,x_N;y_1,\dots,y_N)$ all but the two-particle terms. The basis for this is that $\sigma_{\alpha\beta\beta}(q)$, $\sigma_{\alpha\beta\beta\beta}(q)$, etc. are nonsingular functions that are integrable at small q (see, e.g., (32)), and consequently contribute as $\xi\to\infty$ only the higher-order terms of the expansion in ξ^{-1} to the correlation functions.

6. DISCUSSION OF RESULTS

It is appropriate to compare in conclusion the results with the available published expressions for the correlation functions. We note that expression (51) for n_p is of the same form as the analogous expressions of Refs. 8–10, i.e., the step in the Fermi distribution in the momenta becomes smeared out, but, on the contrary, β has in (51) a different value and vanishes, as it should, for a δ-function interaction potential. We note that the smearing of the step in the Fermi distribution for the ground state takes place at arbitrarily small interaction and that this agrees with the conclusions obtained in the linear model.[10]

According to (58), as $\xi\to\infty$ the density-density correlator $R(\xi)\sim\xi^{-2}$ just as for an ideal gas. This fact agrees with Feynman's formula for collective excitations

$$\varepsilon(k)=k^2/R(k). \tag{59}$$

Here $R(k)$ is the Fourier transform of $R(\xi)\sim\xi^{-2}$ as $\xi\to\infty$ we have $R(k)\sim|k|$ and according to (59) we get $\varepsilon(k)\sim|k|$ in agreement with the existing exact solutions for a δ-function interaction potential.[6,16]

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