Corrections to the hydrodynamics of liquids

A. F. Andreev

Institute of Physics Problems, USSR Academy of Sciences
Submitted 23 April 1978

The principal correction terms of the linear equations of the hydrodynamics of liquids are obtained. The principal mechanism stems from long-wave thermal fluctuations. The low-frequency dispersion of sound is calculated.

PACS numbers: 47.35. + i

The hydrodynamics equations are obtained by expanding the equations of motion in the gradients of the velocity and of the thermodynamic quantities up to terms of second order in the spatial derivatives. In this approximation, the form of the equations, as is well known, follows uniquely from the general conservation laws alone, and is therefore the same for all gases and liquids. Distinctions appear only in the thermodynamic functions and in the values of the kinetic coefficients.

The situation is different, as will be shown below, as going to the next-order approximation. There exist, generally speaking two entirely different types of corrections to hydrodynamics. On the one hand we have the usual "gas-kinetic" corrections obtained by Burnett (see also Ref. 3) on the basis of the Boltzmann equation. If we confine ourselves to linearized equations, then the Navier-Stokes equation acquires in the Burnett approximation an additional term proportional to the third spatial derivative of the temperature. The order of magnitude of this term is $k^3 T$, where $k$ is the wave vector or some other reciprocal of a characteristic length, $T$ is the mean free path of the particles, and $kT$ is the characteristic temperature difference. On the other hand, in the present paper are calculated the fluctuation corrections due to the presence of long-wave thermal fluctuations, particularly acoustic fluctuations. Since sound absorption is proportional to the square of the frequency, acoustic fluctuations with sufficiently low frequency have an arbitrarily large mean free path. This is the physical reason why the fluctuation mechanism is always the basic one at sufficiently small gradients. In fact, the fluctuation correction to the Navier-Stokes equation, as will be shown below, is of the order of $k^2 T$ and $k^3 T$, i.e., at sufficiently low $k$ it greatly exceeds the gas-kinetic correction. It is important, however, that with increasing $k$ the gas-kinetic corrections become the basic one when the condition $k^3 T = (\nu^2 T)^{1/3}$ is satisfied, where $\nu$ is the thermal energy per unit mass.

The presence of thermal fluctuations gives rise to the appearance of small corrections of $\rho_1$, $\omega_1$, . . ., to the hydrodynamic quantities; these corrections oscillate in space and in time. In what follows it is essential to ascertain the relation between the fluctuation wave vectors $\mathbf{q}$, which play the principal role, and the wave vector $k$ of the hydrodynamic motion. Let, for the sake of argument, the hydrodynamic motion of interest to us be a sound wave. From the formulas that follow it will be seen that the main contribution to the correction terms are made by fluctuations whose damping time is of the order of the reciprocal of the frequency of the hydrodynamic motion. Since the damping time of any fluctuation in a liquid is inversely proportional to the square of the wave vector $q$, and the sound frequency is proportional to the first power of $k$, it can be assumed that $q \approx k$. We therefore average all the quantities over volumes whose linear dimensions are much less than $1/k$ but much larger than $1/q$. All the quantities that are linear in the fluctuations vanish after such an averaging:

$$\langle q \rangle = (v/\omega) = 0,$$

and the effect of interest to us appears only in second
order in the fluctuation amplitude.

An arbitrary small perturbation in a liquid is a superposition of acoustic, entropy, and vortical waves. If we choose as the independent thermodynamic variables the pressure $p$ and the entropy per unit mass $s$, then the acoustic fluctuations correspond to oscillations of the pressure and of the longitudinal part $\mathbf{v}_l$ of the velocity ($\nabla s_\mathbf{v}_l = 0$), the entropy fluctuations correspond to oscillations of $s$, and the vortical fluctuations correspond to oscillations of the transverse part $\mathbf{v}_t$ of the velocity ($\text{div} \mathbf{v}_t = 0$), with the remaining variables constant. Since the different types of fluctuation can be regarded as statistically independent, the mean values of some quadratic combinations vanish. For example,

$$\langle \alpha \beta \rangle = \langle \alpha \psi \rangle = \langle \alpha \mathbf{v} \rangle \langle \psi \rangle = \langle \mathbf{v} \mathbf{v} \rangle = 0.$$  

Expanding the equations in (1) accurate to terms quadratic in the fluctuation amplitudes, and carrying out the indicated averaging, we get

$$\frac{\partial}{\partial t} \left( \rho (s, s) + \frac{1}{2} \left( \frac{\partial }{\partial s} s \right) \rho (s, s) + \frac{1}{2} \left( \frac{\partial }{\partial s} s \right) \rho (s, s) \right) + \text{div} \left( \frac{\partial }{\partial s} s \right) \rho (s, s) \rho (s, s) = 0,$$

$$\frac{\partial}{\partial t} \left( \rho (s, s) + \frac{1}{2} \left( \frac{\partial }{\partial s} s \right) \rho (s, s) + \frac{1}{2} \left( \frac{\partial }{\partial s} s \right) \rho (s, s) \right) + \text{div} \left( \frac{\partial }{\partial s} s \right) \rho (s, s) \rho (s, s) = 0,$$

$$\frac{\partial}{\partial t} \left( \rho (s, s) + \frac{1}{2} \left( \frac{\partial }{\partial s} s \right) \rho (s, s) + \frac{1}{2} \left( \frac{\partial }{\partial s} s \right) \rho (s, s) \right) + \text{div} \left( \frac{\partial }{\partial s} s \right) \rho (s, s) \rho (s, s) = 0.$$  

where we have neglected the terms that make no contribution to the linearized equations of interest to us.

The quantity

$$\rho (s, s) = \frac{1}{2} \left( \frac{\partial }{\partial s} s \right) \rho (s, s) + \frac{1}{2} \left( \frac{\partial }{\partial s} s \right) \rho (s, s)$$

in the first equation of (2) is the average renormalized density of the liquid. It is easy to determine analogously the average entropy $S$ per unit volume:

$$S = S(s, s) + \frac{1}{2} \left( \frac{\partial }{\partial s} s \right) S(s, s) + \frac{1}{2} \left( \frac{\partial }{\partial s} s \right) S(s, s)$$

and the average velocity

$$\mathbf{v} = \mathbf{v}(s, s) + \frac{1}{2} \left( \frac{\partial }{\partial s} s \right) \mathbf{v}(s, s) + \frac{1}{2} \left( \frac{\partial }{\partial s} s \right) \mathbf{v}(s, s).$$

If we choose the renormalized quantities $\rho^r, S^r, \mathbf{v}^r$ as the new independent variables, then we can rewrite (3) in the form

$$\rho^r \frac{\partial}{\partial t} \rho^r + \frac{\partial }{\partial s} s \rho^r \rho^r \rho^r = 0,$$

$$\frac{\partial }{\partial s} s = 0,$$

$$\frac{\partial }{\partial s} s = 0,$$

$$\frac{\partial }{\partial s} s = 0,$$

where $c = \text{the speed of sound, } c_s = \text{the heat capacity per unit mass at constant pressure}$. Here and below we shall omit the bar over the letters $\rho, S, \mathbf{v}$, which will henceforth designate the renormalized quantities.

We represent the fluctuations in the form of the expansions

$$\rho^e = \rho^e - \rho^e, \rho^e = \rho^e - \rho^e, \rho^e = \rho^e - \rho^e,$$

$$\text{where } V \text{ is the normalization volume, } l_4 (a = 1, 2) \text{ are mutually perpendicular unit vectors and lie in a plane perpendicular to the wave vector } q \text{ and satisfy the condition } l_4 l_4 = s_4 - q (q, q),$$

and introduce the distribution functions of the acoustic fluctuations

$$\xi (0) = \frac{(q, q)^2}{c_s^2},$$

of the entropy fluctuations

$$\xi (0) = \frac{(q, q)^2}{c_s^2},$$

and the vortical fluctuations

$$\xi (0) = \frac{(q, q)^2}{c_s^2}.$$

The mean values in (5) can be expressed in terms of the distribution functions as follows:

$$\langle \rho^e \rangle = \int d^3 q \xi (q), \langle s^e \rangle = \int d^3 q \xi (q),$$

$$\langle \rho^e \rangle = \int d^3 q \xi (q), \langle s^e \rangle = \int d^3 q \xi (q),$$

$$\langle \mathbf{v}^e \rangle = \int d^3 q \xi (q), \langle \mathbf{v}^e \rangle = \int d^3 q \xi (q),$$

$$\langle \mathbf{v}^e \rangle = \int d^3 q \xi (q), \langle \mathbf{v}^e \rangle = \int d^3 q \xi (q),$$

where $d^3 q = dq^2 (2\pi)^3$. Substituting these formulas in (5), we obtain after simple transformations

$$\rho^e \frac{\partial}{\partial t} \rho^e = 0,$$

$$\rho^e \frac{\partial}{\partial t} \rho^e = 0,$$

$$\rho^e \frac{\partial}{\partial t} \rho^e = 0,$$

$$\rho^e \frac{\partial}{\partial t} \rho^e = 0,$$

where we have introduced the renormalized entropy $s = S/\rho$ per unit mass. Equations (6), with only acoustic fluctuations taken into account, i.e., at $f_{ac} = f_{ac} = 0$, were obtained by the author earlier by another method, by starting from the conservation laws.

The last equation of (6) contains, under the sign of the derivative with respect to time, besides the entropy also a combination of distribution functions; this combination constitutes the "combinatorial" (see Ref. 6) entropy of the fluctuations. In what follows it will be convenient to carry out one more renormalization of the entropy, by including in it the combinatorial entropy. In addition, it is possible to replace in all the equations the distribution functions by their deviations from the equilibrium values, since the equilibrium fluctuations can be incorporated in the definitions of the thermodynamic functions. As a result, the equations take on the form

Sov. Phys. JETP 48(3), Sept. 1978
The acoustic-fluctuation distribution function satisfies the usual Boltzmann equation
\[ \frac{\partial f}{\partial t} + v \cdot \nabla f = -\frac{\partial}{\partial q} \left( q f \right) \]
where
\[ H = c q + q e v, \]
\[ \gamma = q_0 + \xi + \lambda \left( \frac{1}{c} - 1 / \rho \right), \]
\( \gamma \) is the specific heat per unit mass at constant volume, and \( \xi \) and \( \lambda \) are the first and second viscosity coefficients, and \( \lambda \) is the thermal conductivity coefficient. Putting \( \phi = n_0 = n_0 \),

where \( n_0 = T / c q \) is the equilibrium distribution function, and linearizing the kinetic equation, we get
\[ \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f = \frac{1}{e} \left( \mathbf{u} + \frac{\partial f}{\partial q} \right) \]
where \( n = q / c q \).

The nonequilibrium part of the acoustic distribution function is thus equal to
\[ n \phi (q) = \frac{1}{e} \left( \mathbf{u} + \frac{\partial f}{\partial q} \right) \]

where \( \phi = (q / c q) \mathbf{u} + \nabla \cdot \mathbf{u} \) is the acoustic distribution function.

The nonequilibrium part of the acoustic distribution function is thus equal to
\[ n \phi (q) = \frac{1}{e} \left( \mathbf{u} + \frac{\partial f}{\partial q} \right) \]

where \( \phi = (q / c q) \mathbf{u} + \nabla \cdot \mathbf{u} \) is the acoustic distribution function.

A kinetic equation for the entropy distribution function \( \phi (s) \) was derived in the Appendix of the paper by Meierovich and the author(9) from the general theory of hydrodynamic fluctuations. If we are interested in the linearized equations this kinetic equation can be written in the form
\[ \frac{\partial f}{\partial t} + \nabla \cdot \mathbf{u} = \frac{1}{e} \left( \mathbf{u} + \frac{\partial f}{\partial q} \right) \]

which are written in a form that makes clear the contributions of the fluctuations to the momentum and heat fluxes.

The tensors \( \sigma_{ij}, \lambda_{ijk}, \alpha_{ijk}, \) and \( \beta_{ijk} \) are defined by the formulas
\[ \sigma_{ij} = \frac{1}{2} \left( \frac{\partial f}{\partial q_i} \frac{\partial f}{\partial q_j} - \frac{\partial}{\partial q_i} n \frac{\partial}{\partial q_j} n \right) \]

The tensor parts of the entropy distribution function:
\[ \phi_1 = \frac{1}{2} \left( \frac{\partial f}{\partial q_i} \frac{\partial f}{\partial q_j} - \frac{\partial}{\partial q_i} n \frac{\partial}{\partial q_j} n \right) \]

which are written in a form that makes clear the contributions of the fluctuations to the momentum and heat fluxes.

As a result we obtain the following final equations:
\[ \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f = -\frac{1}{e} \left( \mathbf{u} + \frac{\partial f}{\partial q} \right) \]

The distribution function of the vortical fluctuations satisfy the equation
\[ f_\phi = \frac{1}{e} \left( \mathbf{u} + \frac{\partial f}{\partial q} \right) \]

from which we get the nonequilibrium part \( \delta f_\phi \):
Critical phenomena in cholesteric liquid crystals

S. A. Brazovskii and V. M. Filev

L. D. Landau Theoretical Physics Institute, USSR Academy of Sciences

Zh. Eksp. Teor. Fiz. 78, 1140-1150 (September 1978)

Phase transitions in cholesteric liquid crystals are considered. A phase diagram is derived which makes it possible to explain the existence of intermediate phases in a narrow region between the uniform isotropic (UI) phase and the spiral phase. The critical phenomena are investigated in the light of experiments on the supercooling of the UI phase.

PACS numbers: 64.70.Ew

1. INTRODUCTION

The critical properties of cholesteric liquid crystals (CLC) in phase transitions from the uniform isotropic (UI) phase to the spiral phase have a number of important differences from the critical properties of other systems. A number of experimental[1-4] and theoretical[5,6] papers have been devoted to the study of the phase transitions in CLC, but some pertinent problems are still far from being completely solved. In particular, the natural supercooling of the UI phase observed in Ref. 4 and the anomalies in the temperature dependence of the pre-critical scattering of light require deeper investigation.

The relative corrections to the speed of sound and to the damping are thus proportional respectively to $\omega^{\frac{1}{3}}$ and $\omega^{\frac{1}{2}}$.


Critical phenomena in cholesteric liquid crystals

S. A. Brazovskii and V. M. Filev

L. D. Landau Theoretical Physics Institute, USSR Academy of Sciences

Zh. Eksp. Teor. Fiz. 78, 1140-1150 (September 1978)

Phase transitions in cholesteric liquid crystals are considered. A phase diagram is derived which makes it possible to explain the existence of intermediate phases in a narrow region between the uniform isotropic (UI) phase and the spiral phase. The critical phenomena are investigated in the light of experiments on the supercooling of the UI phase.

PACS numbers: 64.70.Ew

1. INTRODUCTION

The critical properties of cholesteric liquid crystals (CLC) in phase transitions from the uniform isotropic (UI) phase to the spiral phase have a number of important differences from the critical properties of other systems. A number of experimental[1-4] and theoretical[5,6] papers have been devoted to the study of the phase transitions in CLC, but some pertinent problems are still far from being completely solved. In particular, the natural supercooling of the UI phase observed in Ref. 4 and the anomalies in the temperature dependence of the pre-critical scattering of light require deeper investigation.

The relative corrections to the speed of sound and to the damping are thus proportional respectively to $\omega^{\frac{1}{3}}$ and $\omega^{\frac{1}{2}}$.


Critical phenomena in cholesteric liquid crystals

S. A. Brazovskii and V. M. Filev

L. D. Landau Theoretical Physics Institute, USSR Academy of Sciences

Zh. Eksp. Teor. Fiz. 78, 1140-1150 (September 1978)

Phase transitions in cholesteric liquid crystals are considered. A phase diagram is derived which makes it possible to explain the existence of intermediate phases in a narrow region between the uniform isotropic (UI) phase and the spiral phase. The critical phenomena are investigated in the light of experiments on the supercooling of the UI phase.

PACS numbers: 64.70.Ew

1. INTRODUCTION

The critical properties of cholesteric liquid crystals (CLC) in phase transitions from the uniform isotropic (UI) phase to the spiral phase have a number of important differences from the critical properties of other systems. A number of experimental[1-4] and theoretical[5,6] papers have been devoted to the study of the phase transitions in CLC, but some pertinent problems are still far from being completely solved. In particular, the natural supercooling of the UI phase observed in Ref. 4 and the anomalies in the temperature dependence of the pre-critical scattering of light require deeper investigation.

The relative corrections to the speed of sound and to the damping are thus proportional respectively to $\omega^{\frac{1}{3}}$ and $\omega^{\frac{1}{2}}$.