$$f_2(\langle I\rangle)=(2/\pi)^{1/2}(b/a_{\rm Fr})^{-(2-\alpha)}\langle I\rangle^{1/2(2-\alpha)} \times \exp\{-{}^1/_2(b/a_{\rm Fr})^{-2(2-\alpha)}\langle I\rangle^{4-\alpha}\}, \tag{19}$$

here and below we shall omit the subscript of $\langle I\rangle_{foc}$.

We calculate the integral (4), with (3) and (19) included, by using the saddle-point method; as a result we get

$$f(I)\approx\frac{2}{(5-\alpha)^{1/2}}\left(\frac{2}{4-\alpha}\right)^{(3-\alpha)/2(5-\alpha)}\left\{\left(\frac{b}{a_{\Phi p}}\right)^{2(2-\alpha)}I\right\}^{-1/(5-\alpha)} \times\exp\left\{-\left(\frac{2}{4-\alpha}\right)^{-1/(5-\alpha)}\left(\frac{5-\alpha}{4-\alpha}\right)\left[\left(\frac{b}{a_{\Phi p}}\right)^{2(2-\alpha)}I\right]^{-1/(5-\alpha)}I\right\} \tag{20}$$

This result can be written in a different form

$$f(I)\approx\frac{2}{(5-\alpha)^{1/2}}\left(\frac{2}{4-\alpha}\right)^{(3-\alpha)/2(5-\alpha)}m_0^{4(2-\alpha)/\alpha(5-\alpha)}I^{-1/(5-\alpha)} \times\exp\left\{-\left(\frac{2}{4-\alpha}\right)^{-1/(5-\alpha)}\left(\frac{5-\alpha}{4-\alpha}\right)m_0^{4(2-\alpha)/\alpha(5-\alpha)}I^{(4-\alpha)/(5-\alpha)}\right\}. \tag{21}$$

Relations (20) and (21) are valid at $I>m_0^{(4/\alpha)(2-\alpha)}$.

3. CONCLUSIONS

We have calculated above the distribution functions of the intensity fluctuations in the region of saturated flicker, for the case when the initial type of radiation is a plane wave (i.e., in the Fresnel zone of a laser emitter). Analogous calculations can also be carried out for other boundary conditions. It thus becomes possible to interpret also in the saturation region the extensive experimental material on flicker observation, which until recently was analyzed for the intensity logarithm.

However, the results are also of independent interest. They are necessary for the calculation of the parameters of the random intensity-field spikes,[1] for the determination of the error probability in two-way communication systems and in laser radars,[12] and for other applications.

In conclusion, we are deeply grateful to A. M. Prokhorov for interest in the work.

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Advance and delay effects in photon echo signals

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An experimental investigation was made of the advance and delay of photon echo signals. The experiments were carried out on a ruby single crystal kept at a temperature of 2.2°K and the wavelength was 6935 Å. A theoretical description of these effects is given. Dephasing of the electric dipoles during the action of the exciting pulses and in the course of the first echo is allowed for the first time in the case of stimulated and multiple echo signals. The results of a theoretical calculation of the shifts of the optical coherent response are in agreement with the experimental data.

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INTRODUCTION

Optical (photon) echo is a coherent optical response of a resonant system to the action of two (or more) laser pulses separated by a time interval. The ratio of the power of an optical echo signal to the power of the exciting pulses is frequently $10^{-4}-10^{-5}$, i.e., for pulses of powers of hundreds of kilowatts a coherent response is a relatively strong signal. Therefore, optical echo may find technical applications, particularly in dynamic holography, memory cells of optical computers, and quantum counters of low-intensity signals.[1] A very important task is the control of the process of generation of the optical echo. The present paper is concerned with changes in the shape of an optical echo signal and in the time of its appearance, which occur when the parameters of the exciting pulses are varied under conditions such that we cannot ignore the dephasing of electric dipoles during the action of these pulses. Such investigations are also of interest in connection with possible applications of the echo method in high-resolution optical spectroscopy.

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1. INTENSITY AND SHAPE OF OPTICAL COHERENT RESPONSES

The coherent response of a resonant system to two pulses is known as the primary optical echo. This response is formed at a time $2\tau_1$ and it is emitted along the direction of the wave vector $\mathbf{k}=2\mathbf{k}_2-\mathbf{k}_1$ (τ_1 is the time interval between the centers of two rectangular exciting pulses; $\mathbf{k}_1$ and $\mathbf{k}_2$ are the wave vectors of these pulses). In the case of three pulses acting on a resonant system, a stimulated optical echo signal is formed at a time $2\tau_1+\tau_2$ (τ_2 is the time between the centers of the second and third pulses). The phase-matching condition for this coherent response is $\mathbf{k}=-\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3$, where $\mathbf{k}_3$ is the wave vector of the third exciting pulse. It is worth noting that the echo pulses may themselves act as the exciting pulses. For example, the primary optical echo, together with the exciting pulses can generate new echo signals known as multiple optical echos. These are formed at moments $3\tau_1-\Delta t_p$ and $4\tau_1-\Delta t_p$ (Δt_p is the duration of the primary optical echo) and are emitted along the directions $\mathbf{k}_{m1}=3\mathbf{k}_2-2\mathbf{k}_1$ and $\mathbf{k}_{m2}=4\mathbf{k}_2-3\mathbf{k}_1$, respectively. The moments of appearance of the response pulses given above are measured from the leading edge of the first rectangular pulse. However, exact measurements can be carried out only at the pulse centers. Therefore, we shall take the origin at the center of the first pulse and make all the measurements from center to center. Since the actual pulses in the experiments are not rectangular but more likely bell-shaped, the pulse duration will be understood to be their width at midamplitude. The moments of the appearance of the exciting pulses and of generation of the optical echo pulses are shown schematically in Fig. 1. We recall that the formation of echo signals requires times shorter than the irreversible relaxation times.

The moments of generation of the optical echo signals shown in Fig. 1 correspond to the conditions when the duration of the exciting pulses Δt is much less than the times τ_1 and τ_2. However, in experimental studies of the optical echo the pulse duration is usually comparable with the time intervals between the pulses and the signals are formed at the moments other than those given above. Following Kopvillem *et al.*,[2] we shall call these the anomalous coherent responses. The shape of the primary echo has already been studied theoretically in the rf range[3-5] and in the optical range.[6-9] The shape of the anomalous stimulated and multiple echoes has not yet been considered theoretically. In this section we shall describe a calculation method which gives relatively simply the functions describing the echo signal shape and we shall then calculate these functions numerically.

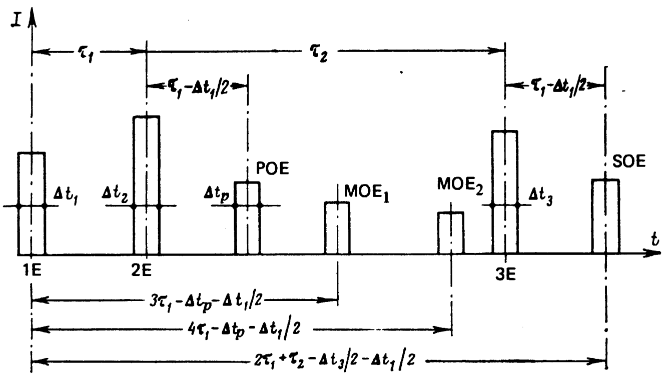

FIG. 1. Schematic representation of the order of action of laser excitation pulses (1E, 2E, and 3E) on a resonant system and the coherent optical response generated by these pulses.

In the interaction representation the equation for the density matrix ρ of a resonant system of interest to us is

$$d\rho/dt=i\hbar^{-1}[\mathcal{H},\rho],\tag{1}$$

where $\mathcal{H}$ is the Hamiltonian of a two-level system and includes allowance for the interaction of this system with the exciting fields in the interaction representation. It follows from Eq. (1) that at a moment t the density matrix of the system may be calculated from

$$\rho(t)=L(t)\rho_0L(t)^{-1},\quad \rho_0=2^{-N}\prod_{j=1}^{N}\left[1-2\,\mathrm{th}\left(\frac{\hbar\omega_0^j}{2k_BT}\right)R_z^j\right],\tag{2}$$

where R_z^j is the z component of the energy spin $R=\frac{1}{2}$, N is the number of active particles with the levels split by $\approx\hbar\omega_0$, k_B is the Boltzmann constant, T is the absolute temperature of the sample, and $L(t)$ is the evolution operator of the system acted upon by external and internal fields, whose form is

$$L(t)=\exp\left[i\sum_j\Delta\omega_j\left(t-\sum_{\xi=1}^{\eta-1}\tau_\xi-\frac{\Delta t_\eta}{2}\right)\right]$$
$$\times\exp\left[i\left(\sum_j a_\eta^jR_z^j+b_\eta^jR_+^j+b_\eta^{j*}R_-^j\right)\right]\prod_{\xi=1}^{\eta-1}\left\{\exp\left[i\sum_j\Delta\omega_j\left(\tau_\xi-\frac{\Delta t_\xi}{2}-\frac{\Delta t_{\xi+1}}{2}\right)\right]\exp\left[i\sum_j a_\xi^jR_z^j+b_\xi^jR_+^j+b_\xi^{j*}R_-\right]\right\}.\tag{3}$$

In this expression we have $R_\pm=R_x\pm iR_y$; R_x and R_y are the transverse components of the energy spin, τ_ξ is the time interval between the centers of the $(\xi-1)$-st and ξ-th pulses, Δt_ξ is the duration of the ξ-th pulse, $\Delta\omega_j=\omega-\omega_0^j$, ω is the central frequency of a laser pulse ($\hbar\omega\approx\hbar\omega_0$),

$$a_\xi^j=\Delta\omega_j\Delta t_\xi,\quad b_\xi^j={}^1/_2\theta_\xi\exp(i\mathbf{k}_\xi\mathbf{r}_j),$$

$\mathbf{r}_j$ is the radius vector of the position of the j-th particle, $\mathbf{k}_\xi$ is the wave vector of the ξ-th pulse, and θ_ξ is a dimensionless parameter known as the pulse "area" (for a rectangular pulse we have $\theta_\xi=\hbar^{-1}p\mathcal{E}_{0\xi}\Delta t_\xi$, where p is the modulus of the electric dipole moment of the resonant transition and $\mathcal{E}_0$ is the amplitude of the electric field of the laser pulse). The exponential functions under the product sign in Eq. (3) are expanded in increasing order from the right to left. The following expression for the Hermitian operators $\hat{x}$ and $\hat{y}$[10]

$$e^{\hat{x}}\hat{y}e^{-\hat{x}}=\hat{y}+[\hat{x},\hat{y}]+\frac{1}{2!}[\hat{x},[\hat{x},\hat{y}]]+\frac{1}{3!}[\hat{x},[\hat{x},[\hat{x},\hat{y}]]]+\cdots\tag{4}$$

yields the formulas

$$L_jR_zL_j^{-1}=A_kR_z+B_k^*R_++B_kR_-,\quad L_jR_+L_j^{-1}=2B_k^*R_z+D_kR_++M_k^*R_-,$$
$$L_jR_-L_j^{-1}=2B_kR_z+M_kR_++D_k^*R_-,\quad L_mR_\pm L_m^{-1}=R_\pm\exp\{\pm i\Delta\omega t\},\tag{5}$$

where

$$A_k=\frac{1}{\gamma_k^2}(a_k^2+4|b_k|^2\cos\gamma_k),\quad B_k=\frac{1}{\gamma_k^2}(1-\cos\gamma_k)a_kb_k^*+i\frac{1}{\gamma_k}b_k^*\sin\gamma_k,$$

$$\gamma_k=[a_k^2+4b_k^2]^{1/2},\qquad M_k=2(1-\cos\gamma_k)b_k^2/\gamma_k^2,$$

$$D_k=\frac{1}{\gamma_k^2}[a_k^2\cos\gamma_k+2|b_k|^2(1-\cos\gamma_k)]+i\frac{a_k}{\gamma}\sin\gamma_k,$$

$$L_i=\exp\{a_kR_z+b_kR_++b_k^*R_-\},\quad L_m=\exp\{i\Delta\omega tR_z\}.$$

The application of the formulas in Eq. (5) gives the expression for the density matrix of the system at the moment of response generation

$$\rho(t)=2^{-N}\prod_l^N\left[1-\mathrm{th}\left(\frac{\hbar\omega_0^j}{2k_BT}\right)(Q_l+Q_l')\right],\tag{6}$$

where the function Q has the following form:

a) for the primary optical echo,

$$Q_p\sim B_1M_2\exp\{-i\Delta\omega(t-2\tau_1-\Delta t_1/2)\}R_+;$$

b) for the stimulated optical echo,

$$Q_s\sim 2B_1B_2B_3^*\exp\left\{i\Delta\omega\left(t-2\tau_1-\tau_2+\frac{\Delta t_3}{2}+\frac{\Delta t_1}{2}+\frac{\Delta t_2}{2}\right)\right\}R_+.$$

The intensity of the coherent spontaneous radiation emitted by a system of particles can be calculated from[11]

$$I(\mathbf{k})=I_0(\mathbf{k})\,\mathrm{Sp}\{\rho(t)R_{\mathbf{k}+}R_{\mathbf{k}-}\},\quad R_{\mathbf{k}\pm}=\sum_j R_\pm^j\exp\{\pm i\mathbf{k}\mathbf{r}_j\},\tag{7}$$

where $I_0(\mathbf{k})$ is the intensity of the spontaneous radiation of an isolated particle in the direction of the wave vector $\mathbf{k}$ per unit solid angle. The summation over $\Delta\omega_l$ in Eq. (7) can be replaced by integration of the distribution function $g(\Delta\omega)$. The function Q is a complex function of $\Delta\omega$ ($Q=Q'+iQ''$) and, as was done by Bloom,[3] we can easily show that

$$\int Q'(\Delta\omega)g(\Delta\omega)d(\Delta\omega)=0,\quad i\int Q''(\Delta\omega)g(\Delta\omega)d(\Delta\omega)\neq 0.$$

The double summation over $\mathbf{r}_l$ may be carried out using a method described by Abella *et al.*[12] We consequently find that the total intensity of the optical echo is

$$I=I_0N^2\frac{\lambda^2}{S\varepsilon}\Phi(t')^2,\tag{8}$$

where ϵ is the permittivity, S is the working part of the cross section of the sample, $\Phi(t')$ is the function describing the shape of the optical echo, $t'=t_\eta-\tau_1+\frac{1}{2}\Delta t_1+\frac{1}{2}\Delta t_2$, and t_η is the aftereffect time of the η-th pulse. For the primary optical echo the function $\Phi(t')^2$ for $\Delta t_1=\Delta t_2=\Delta t$ has the form

$$\Phi(t')^2=\left\{\theta_1^2\frac{\theta_2^2}{2}\int_{-\infty}^{\infty}d(\Delta\omega)g(\Delta\omega)\left[\cos\Delta\omega t'\frac{\sin\theta_\mathrm{I}}{\theta_\mathrm{I}}+\Delta\omega\,\Delta t\sin\Delta\omega t'\frac{(1-\cos\theta_\mathrm{I})}{\theta_\mathrm{I}^2}\right]\frac{(1-\cos\theta_\mathrm{II})}{\theta_\mathrm{II}^2}\right\}^2,\tag{9}$$

where $\theta_{\mathrm{I,II}}=[\theta_{1,2}^2+\Delta\omega^2\Delta t^2]^{1/2}$. The expression (8), subject to Eq. (9), reduces, for $\Delta\omega\Delta t\to 0$, to the well-known expression for the intensity of the primary optical echo obtained without allowance for the dipole dephasing[12]:

$$I=I_0N^2\frac{\lambda^2}{4\varepsilon S}\sin^2\theta_1\sin^4\frac{\theta_2}{2}.$$

An approximate analytic integration of Eq. (9) was carried out by Samartsev and Shagidullin.[7]

In the case of the stimulated optical echo the function $\Phi(t')$ has the form ($\Delta t_1=\Delta t_2=\Delta t_3=\Delta t$)

$$\begin{aligned}\Phi(t')^2=\Bigg[&\frac{1}{4}\theta_1\theta_2\theta_3\Bigg\{\int_{-\infty}^{\infty}g(\Delta\omega)d(\Delta\omega)\Bigg[\frac{(1-\cos\theta_\mathrm{I})}{\theta_\mathrm{I}^2}\frac{(1-\cos\theta_\mathrm{II})}{\theta_\mathrm{II}^2}\\
&\times\frac{(1-\cos\theta_\mathrm{III})}{\theta_\mathrm{III}^2}\Delta t^3\Delta\omega^3\sin\Delta\omega t'+\frac{\sin\theta_\mathrm{I}}{\theta_\mathrm{I}}\frac{\sin\theta_\mathrm{II}}{\theta_\mathrm{II}}\frac{\sin\theta_\mathrm{III}}{\theta_\mathrm{III}}\cos\Delta\omega t'\\
&+\frac{(1-\cos\theta_\mathrm{I})}{\theta_\mathrm{I}^2}\frac{\sin\theta_\mathrm{II}}{\theta_\mathrm{II}}\frac{(1-\cos\theta_\mathrm{III})}{\theta_\mathrm{III}^2}\Delta\omega^2\Delta t^2\cos\Delta\omega t'+\frac{(1-\cos\theta_\mathrm{I})}{\theta_\mathrm{I}^2}\\
&\times\frac{\sin\theta_\mathrm{II}}{\theta_\mathrm{II}}\frac{\sin\theta_\mathrm{III}}{\theta_\mathrm{III}}\Delta\omega\,\Delta t\sin\Delta\omega t'+\frac{\sin\theta_\mathrm{I}}{\theta_\mathrm{I}}\frac{(1-\cos\theta_\mathrm{II})}{\theta_\mathrm{II}^2}\\
&\times\frac{(1-\cos\theta_\mathrm{III})}{\theta_\mathrm{III}^2}\Delta\omega^2\Delta t^2\cos\Delta\omega t'+\frac{\sin\theta_\mathrm{I}}{\theta_\mathrm{I}}\frac{(1-\cos\theta_\mathrm{II})}{\theta_\mathrm{II}^2}\frac{\sin\theta_\mathrm{III}}{\theta_\mathrm{III}}\\
&\times\Delta\omega\,\Delta t\sin\Delta\omega t'-\frac{(1-\cos\theta_\mathrm{I})}{\theta_\mathrm{I}^2}\frac{(1-\cos\theta_\mathrm{II})}{\theta_\mathrm{II}^2}\frac{\sin\theta_\mathrm{III}}{\theta_\mathrm{III}}\Delta\omega^2\Delta t^2\cos\Delta\omega t'\\
&-\frac{\sin\theta_\mathrm{I}}{\theta_\mathrm{I}}\frac{\sin\theta_\mathrm{II}}{\theta_\mathrm{II}}\frac{(1-\cos\theta_\mathrm{III})}{\theta_\mathrm{III}^2}\Delta\omega\,\Delta t\sin\Delta\omega t'\Bigg]\Bigg\}\Bigg]^2\end{aligned}\tag{10}$$

where θ_{III} is defined in the same way as $\theta_{\mathrm{I,II}}$.

Exact integration of Eqs. (9) and (10) with respect to $\Delta\omega$ can be carried out numerically on a computer. The results of such an integration for a number of cases which we studied experimentally are given in Fig. 2 and, in the case of the primary optical echo, they can be summarized as follows: 1) the value of θ_2 (i.e., the power of the second pulse) does not affect the posi-

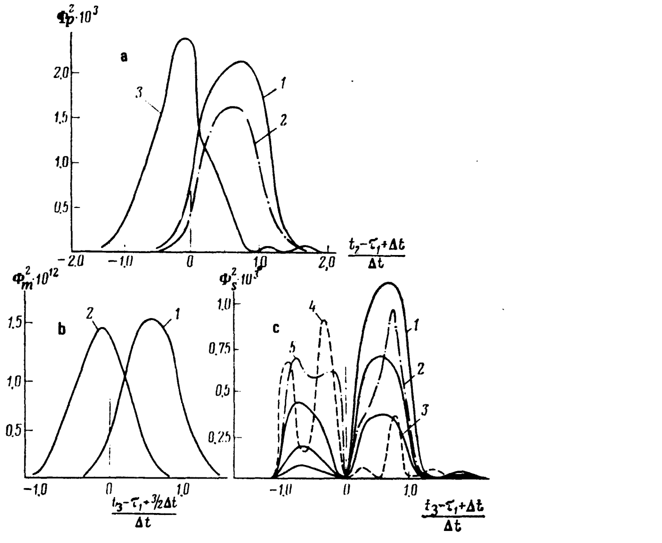

FIG. 2. Results of a numerical calculation of the shape of the optical coherent response pulses for $T_2^*/\Delta t=10^{-2}$ and a Gaussian distribution function $g(\Delta\omega)$. a) Shape of the primary optical echo: 1) $\theta_1=\pi/2$, $\theta_2=\pi$; 2) $\theta_1=0.4\pi$, $\theta_2=0.8\pi$; 3) $\theta_1=1.35\pi$, $\theta_2=\pi$. b) Shape of the multiple optical echo: 1) $\theta_2=\pi/2$, $\theta_e=10^{-2}$; 2) $\theta_2=1.35\pi$, $\theta_e=10^{-2}$. c) Shape of the stimulated optical echo: 1) $\theta_1=2\pi/3$, $\theta_2=\theta_3=\pi/2$; 2) $\theta_1=\pi/2$, $\theta_2=\theta_3=\pi/2$; 3) $\theta_1=\pi/3$, $\theta_2=\theta_3=\pi/2$; 4) $\theta_1=1.5\pi$, $\theta_2=\theta_3=\pi/2$; 5) $\theta_1=\pi$, $\theta_2=\theta_3=\pi/2$.

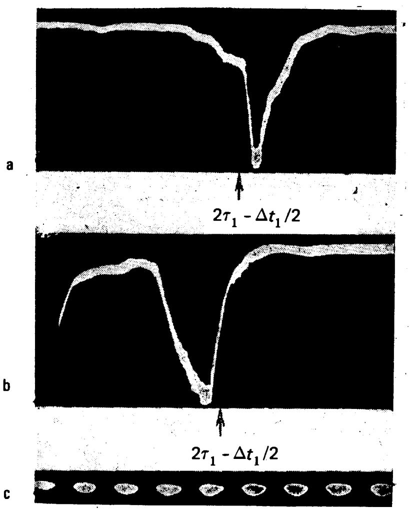

FIG. 3. Oscillograms illustrating the delay (a) and advance (b) of the primary echo signals in ruby. The bottom part of the figure (c) shows timing marks at intervals of 10 nsec.

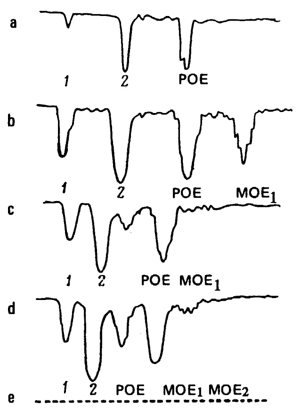

FIG. 5. Oscillograms of the coherent response pulses observed for ruby subjected to two laser exciting pulses: a) primary optical echo (first signal on the right) for $\tau_1 = 75$ nsec, $\Delta t = 10$ nsec; b), c) primary and multiple echo signals for $\tau_1 = 75$ nsec and $\tau_1 = 38$ nsec, respectively, and $\Delta t = 12$ nsec; d) primary and multiple (MOE_1 and MOE_2) echo signals for $\tau_1 = 38$ nsec, $\Delta t = 15$ nsec; e) timing marks at intervals of 10 nsec.

tion of the "center of gravity" of the echo signal; 2) for $\theta_1 \leq \pi$, $\theta_2 = \pi$, and $T_2^*/\Delta t = 10^{-2}$ (ruby; T_2^* is the transverse reversible relaxation time), the primary optical echo signal should be "delayed" in the resonant medium, i.e., the center of gravity of this signal should be later than the moment $2\tau_1 - \Delta t_1/2$; 3) for $\pi < \theta_1 \leq 1.5\pi$, $\theta_2 = \pi$, and $T_2^*/\Delta t = 10^{-2}$, the primary optical echo signal should be advanced. The absence of influence of the parameters of the second exciting pulse on the time of generation of the primary optical echo easily follows from the determination of the extremum of Eq. (9) for a single "spin packet" showing that the positions of the extrema t_{ext} satisfy

$$t_{ext} = \frac{1}{\Delta\omega} \operatorname{arctg}\left\{\Delta\omega\,\Delta t \frac{(1-\cos\theta_I)}{\theta_I}\right\}$$

which does not contain θ_{II}. A similar situation occurs also for the multiple optical echo signals, in which the second exciting pulse is the primary echo of area θ_e. Calculations for typical values $\theta_e = 10^{-2}$ and $T_2^*/\Delta t = 10^{-2}$ (ruby) then show that both delay ($\theta_e \leq \pi$) and advance ($1.5\pi > \theta_2 > \pi$) of the moment of generation of the first multiple optical echo signal (Fig. 2b) is possible, compared with the position of the multiple echo signals in Fig. 1. We can see that in the case of the second multiple echo signal the results are identical if the ordinate of Fig. 2b is replaced with $(t_e - 2\tau_1 + \frac{3}{2}\Delta t)/\Delta t$. In the case of the stimulated optical echo the calculations show (Fig. 2c) that again there are delay ($\theta_1 \leq \pi$, $\theta_2 = \theta_3 = \pi/2$, $T_2^*/\Delta t = 10^{-2}$) and advance [$\theta_1 = (1.35-1.5)\pi$, $\theta_2 = \theta_3 = \pi/2$, $T_2^*/\Delta t = 10^{-2}$) effects. Curves 1–3 in Fig. 2c represent the dynamics of the formation of the shape of the stimulated optical echo with the change in θ_1 and reduction of θ_1 from π to $\pi/4$ alters the stimulated optical echo from two humps to one.

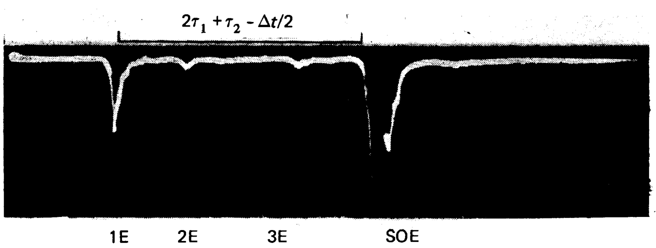

FIG. 4. Oscillogram of the stimulated optical echo signal in ruby. The first signal on the right is the stimulated echo (SOE) and the other signals are the exciting pulses (1E, 2E, 3E); $\Delta t = 10$ nsec, $\tau_1 = 38$ nsec, $\tau_2 = 58$ nsec.

2. EXPERIMENTAL INVESTIGATION OF ADVANCE AND DELAY OF ECHO SIGNALS

We investigated a ruby single crystal with the Cr^{3+} concentration ≈0.05 at.%. This crystal was a plate of 1×1 cm area and 0.07 cm thick. It was placed in an optical helium cryostat. The excitation was provided by a ruby laser (whose active element was also kept at liquid nitrogen temperature) emitting at the wavelength λ ≈ 6935 Å. Under these conditions the emission frequency of the ruby laser coincided with the frequency of the $^4A_2 \to {}^2E(\bar{E})$ transition. The pulse duration was 10 nsec and the interval between the pulses was varied in the range 40–120 nsec. The optical signals were detected by an ÉLU-FT fast-response photomultiplier with a resolution of at least 2.7 nsec. The background of scattered light in the optical elements and helium cryostat was reduced by including a Kerr cell acting as a shutter. A stop was placed in front of the Kerr cell and this passed only the echo signal but blocked the exciting pulses. The coherent response was time-selected by the application of a high-voltage (from a modulator) to the electrode of the Kerr shutter. The signals were studied using an I2-7 nanosecond time-interval meter. The ruby laser power was 800 kW. The system of optical filters was used to vary the pulse "areas" θ_n. The arrangement made it possible to apply a static magnetic field H_0, created in Helmholtz coils ($|H_0| \leq 200$ G). In a static field H_0 directed along the optic axis c the intensity of the echo signals was considerably higher than in the absence of this field, which was due to the lifting of the degeneracy of the resonating levels and also due to the suppression of the irreversible relaxation resulting from the interaction between the paramagnetic centers and nuclei. Oscillograms of the primary optical echo signals corresponding to the conditions for the observation of the advance and delay effects are shown in Fig. 3. A comparison of the time shifts of the centers of gravity of the primary optical echo signals with the theoretical values (Fig. 2a) demonstrates that the theory and experiment are in agreement.

Figure 4 shows an oscillogram of the signals observed in a study of the stimulated optical echo. The first three signals on the left are the exciting pulses and the last one is the stimulated optical echo, whose shape corresponds to the $\theta_1 \leq \pi/4$, $\theta_2 = \theta_3 = \pi/2$, $T_2^*/\Delta t = 10^{-2}$ case. The delay of the stimulated optical echo is about 6 nsec, which is in agreement with the results of calculations (Fig. 2c).

The advance and delay effects appear much more clearly in multiple optical echo because the shift due to dephasing during the primary echo has to be supplemented to the doubled shift of the primary echo signal. The oscillograms of the primary and multiple optical echo signals are shown in Fig. 5. The oscillogram in Fig. 5b corresponds to the case when the position of the center of gravity of the first multiple optical echo signal coincides with the position identified as MOE_1 in Fig. 1. The signals MOE_1 and MOE_2 in the oscillograms of Figs. 5c and 5d are delayed by 9–11 nsec for pulse durations ≈15 nsec. This shift is greater than the theoretical value (Fig. 2b) and this is clearly associated with the incorrectness of the approximation that the primary echo and exciting pulses are rectangular, and also with the inaccuracy of the model of formation of the multiple echo signals. Nevertheless, the directions of the shifts of the MOE_1 and MOE_2 signals are in qualitative agreement with the results of the numerical calculation.

CONCLUSIONS

The first experimental investigation of anomalies in the moments of generation of optical echo signals is reported and explained theoretically. It should be noted that in the spin echo method[3,4] only the delay of the primary echo by $\Delta t/2$ has been pointed out and the shape of the stimulated and multiple echo signals has not been investigated at all. It is shown above that the primary, stimulated, and multiple echo signals can form earlier or later than the predicted moments (Fig. 1) and that the time shift may differ from $|\Delta t/2|$. The formulas obtained above describe the experimental data quite accurately. Thus, for specific values of θ_η and Δt_η, the application of two or more laser pulses to a sample under the coherent interaction conditions may produce at certain moments and along certain directions the primary, stimulated, and multiple optical echo signals.

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