Transition radiation and transition scattering produced in a vacuum in the presence of a strong electromagnetic field

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Zh. Eksp. Teor. Fiz. 74, 1621-1635 (May 1978)

In the presence of a strong electromagnetic field (in particular, a constant magnetic field), the vacuum behaves, as is well known, like a medium with permittivity and permeability that depend on the strong-field intensities. Transition radiation and transition scattering can therefore take place in vacuum. The article considers the transition radiation produced when a charge crosses the boundary between a strong magnetic field and a field-free region. The problems solved are those of transition scattering of sufficiently long strong electromagnetic waves by an immobile charge with frequency doubling, and of scattering without a change of frequency in the presence of a strong magnetic field. The same problems are considered also for a moving charge (in all considered cases the scattering takes place also for a charge with mass \( M \rightarrow \infty \)).

PACS numbers: 41.70.+t

Transition radiation is a rather common phenomenon which occurs when a charge or some other source having no natural (frequency) moves with constant velocity in or near an inhomogeneous medium. If the properties of the medium (the refractive index etc.) vary periodically in space and (or) in time, then the transition radiation acquires distinct features and can be called resonant transition radiation or transition scattering. The use of the last term is quite natural when one deals with a charge that is immobile relative to the medium and scatters a permittivity wave.\(^{111}\) An effect analogous to transition scattering takes place in vacuum when a gravitational wave is incident on an immobile electric charge or dipole (electric or magnetic).\(^{111}\)

We consider in this article transition radiation and transition scattering produced likewise in vacuum, but in the presence of a strong electromagnetic field. The gist of the matter that in a strong field electrodynamics becomes, as is well known, nonlinear even in vacuum, since the field gives rise to a vacuum polarization that is analogous to some extent to polarization of a medium. Transition radiation should therefore take place in an inhomogeneous strong field, and when a sufficiently strong wave is incident on a charge, transition scattering should take place. Of course, in a consistent quantum-electrodynamic calculation the transition effects are taken into account in the corresponding problems, but this calls for cumbersome computations. An exam-
The polarization and the magnetization $P$ and $M$ of the vacuum are given by

$$P = \frac{\partial L'}{\partial E}, \quad M = \frac{\partial L'}{\partial B}.$$ (3)

Since $P$ and $M$ depend on $E$ and $B$ in nonlinear fashion, we cannot introduce different material tensors. We confine ourselves to the particular case when

$$B = B_0, \quad E = E_0, \quad B, \leq B_0, \quad E, \leq E_0.$$ (4)

i.e., we have a strong magnetic field $B_0$ and a weak field, say a wave field $(E_0, B_0)$. Then

$$P = P_0 = \{-2(E_0'B_0 + E_0(B_0'B_0))\}, \quad M = M_0 = \{-2(E_0'B_0 + E_0(B_0'B_0))\},$$ (5)

and it is convenient to introduce the material tensors for the weak field

$$P_\nu = \frac{\delta P_{\nu}}{\delta E_\nu}, \quad M_\nu = \frac{\delta M_{\nu}}{\delta B_\nu}.$$ (6)

In connection with the last expression it should be noted that, by definition, $M_{\nu} = -j\omega\epsilon_{\nu}, \frac{\partial E_\nu}{\partial E_\nu}$. We, however, are interested only in the case when

$$|\beta_{\mu}| < 1, \quad |\beta_\nu| < 1.$$ (7)

Under these conditions we can replace $H_0$ in (6) by $E_0$. Recognizing that the field $B_0$ is assumed directed along the $z$ axis, we have

$$\delta_{\mu,0} = \frac{2\omega M_{\mu}}{cE_0}, \quad \delta_{\nu,0} = \frac{2\omega P_{\nu}}{cE_0}.$$ (8)

(All the components $\delta_{\mu,0}$ and $\delta_{\nu,0}$, except those written out, are equal to zero.) The weak field of the wave propagating in the "medium" (vacuum) with constant permittivity and permeability $(8)$ is written in the form

$$E = E_0 e^{i(-\omega t - \frac{x}{c} + \nu)}, \quad B = B_0 e^{i(-\omega t + \mu)}, \quad \nu = \nu_0, \nu = \nu_0.$$ (9)

Obviously, this field satisfies the ordinary Maxwell's equations, by virtue of which

$$\nabla \times E = \mu_0 \frac{\partial B}{\partial t}, \quad \nabla \times B = -\epsilon_0 \frac{\partial E}{\partial t}. \quad B_0 = \mu_0 B_0, \quad E_0 = \epsilon_0 E_0.$$ (10)

(where $\nu$ and $\mu$ are the vector indices; of course, Eqs. (10) are equally valid for the fields $E$ and the amplitudes $E_0)$. Substituting in (10) the tensors (8), we obtain the dispersion equation (the dependences of $\epsilon_{\mu}$ and $\mu_{\nu}$ on the angles are disregarded here—the field will be "joined together" on the boundary $\nu < 0$). From the dispersion equation we obtain the refractive index $n$, defined by the relation $|k| = \omega n/c$.

In the case (8) the indices $\nu$ depend only on the angle

$$n, = \frac{\epsilon_0}{\mu_0} = \frac{1}{\beta}.$$ (11)

The polarization and the magnetization $P$ and $M$ of the vacuum are given by

$$P = \frac{\partial L'}{\partial E}, \quad M = \frac{\partial L'}{\partial B}.$$ (12)

Then, and at sufficiently low frequencies $\omega$ of the propagating waves, the solution of the electrodynamical problems can be based on the Lagrangian (see, e.g., (13) Sec. 126, where the induction $B$ is designated $H$):

$$L = \Delta L', \quad L = \Delta L'/h, \quad L = \Delta L'/h(E, -B + 1(E, E, B)), \quad x = \frac{1}{\Delta L'/h}, \quad \Delta = \frac{1}{15} x^2.$$ (13)

The polarization and magnetization $P$ and $M$ of the vacuum are given by

$$P = \frac{\partial L'}{\partial E}, \quad M = \frac{\partial L'}{\partial B}.$$ (14)

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\( \theta \) between \( \mathbf{B}_0 \) and \( \mathbf{E} \). The normal waves are polarized (we are referring to the directions of the vectors \( \mathbf{E}_n, \mathbf{n} \)) perpendicular to \( \mathbf{B}_0 \) and in the \( (\mathbf{k}, \mathbf{B}_0) \) plane. The direction of \( \mathbf{B}_0 \) (the \( z \) axis) is the optical axis. For this direction, the refractive index is

\[
\eta(0) = (\eta_0)^2 \frac{1 + \sin^2 \theta}{1 + \sin^2 \theta}.
\]

For waves propagating across the field \( \mathbf{B}_0 \) (angle \( \theta = \pi/2 \)),

\[
\eta = (\eta_0)^2 \frac{1 + \sin^2 \theta}{1 + \sin^2 \theta} + \frac{1}{2} \frac{e_{\mathbf{k}} + e_{\mathbf{k}}^*}{\sin \theta}, \quad \eta = (\eta_0)^2 \frac{1 + \sin^2 \theta}{1 + \sin^2 \theta} + \frac{1}{2} \frac{e_{\mathbf{k}} + e_{\mathbf{k}}^*}{\sin \theta},
\]

(11)

where the symbol \( \perp \) pertains to a wave polarized along \( \mathbf{B}_0 \), and \( \perp \) to a wave polarized in the perpendicular direction \( \mathbf{E} \). Expressions (11) agree (to the degree of accuracy with which they were derived) with those given in [14].

If the particle (charge) moves along \( \mathbf{B}_0 \), then in the ultrarelativistic limit it radiates in practice by virtue of some acceleration (which we assume to take place) only in the same direction. This should pertain also to Cerenkov radiation, because of exceedingly small deviation of the refractive indices \( \eta(\theta) \) from unity. In this approximation, however (and in fact in a much more general approximation), we have \( \eta(0) = 1 \). Therefore when the source moves along the field \( \mathbf{B}_0 \), the polarization of the vacuum exerts no influence on its radiation (this pertains, more accurately, to the changes due to the influence of the refractive index \( \eta \), which are the only significant ones in the relativistic case at \( \eta = 1 \)). Here, however, the field \( \mathbf{B}_0 \) is assumed to be homogeneous, corresponding to a homogeneous medium. As to the transition radiation produced when a boundary between two media is crossed, its intensity depends not only on \( \eta \) but also on \( \epsilon_\mu \) and \( \mu_\perp \). However, both the transition radiation and other radiative effects are much stronger under the considered conditions at \( \theta = 0 \) than at \( \theta = \pi/2 \).

Both in the calculation of Cerenkov radiation in a strong field, and in the case of transition radiation, it is necessary to integrate over the frequencies. It is therefore necessary to know the frequency region in which expressions of the type (11) are applicable, and how these expressions change at higher frequencies. The initial Lagrangian (2) is suitable only at sufficiently low frequencies and, specifically, formulas (1) and the more general ones at \( \theta = \pi/2 \) are valid under the condition [12]

\[
\lambda = 3 \eta_0 B_0 / 2 e_{\mathbf{k}}^* \sin \theta < 1.
\]

(12)

where in the case of (11) we must put \( \sin \theta = 1 \). At \( \lambda \approx 1 \), and in practice also at \( \lambda < 1 \), the radiation absorption due to production of the electron–positron pairs is negligibly small. To the contrary, at \( \lambda \approx 1 \) the absorption must be taken into account, and accordingly the constants \( \epsilon \) and \( \mu \) contain imaginary parts. To be sure, the product \( \epsilon_\mu = 1 \) and at the angle \( \theta = 0 \) we have \( \eta(0) = (\Re \epsilon_\mu)^{1/2} = 1 \). Using (4)-(6), we have at \( \lambda > 1 \)

\[
\text{where } \phi \text{ is the angle between } \mathbf{k} \text{ and the particle velocity}
\]

\[
W = \int \int \int W(\phi) d\phi = \int \int \int W(\phi) d\phi d\psi d^2 \mathbf{q}
\]

\[
\lambda = 3 \eta_0 B_0 / 2 e_{\mathbf{k}}^* \sin \theta < 1.
\]

(12)

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v (i.e., the y axis; \( \phi \) is the azimuthal angle that determines the projections on the axes x and z near \( k \)); in the integration with respect to \( \phi \) only small angles are significant, so that the upper limit of the integral is set only arbitrarily equal to \( \pi \).

For waves of type \( A \) we have

\[
W \propto \int W(\omega) d\omega = \int \left( \frac{\omega}{2\pi} \right)^{\frac{3}{2}} \left( 1 + \frac{1}{\omega} \right)^{-\frac{1}{2}} d\omega.
\]

We use for \( \omega \) and \( \phi \) first the expressions (11). Then in the particle-energy region

\[
W(\omega) \propto \left( \frac{\omega}{2\pi} \right)^{\frac{3}{2}} \left( 1 + \frac{1}{\omega} \right)^{-\frac{1}{2}}
\]

we can assume in (16) and (17) that \( (mc^2)^2 \rho \) is much larger than \( 1 - \epsilon_0 \) or \( 1 - \epsilon_1 \). In this case

\[
W(\omega) \propto \left( \frac{\omega}{2\pi} \right)^{\frac{3}{2}} \left( 1 + \frac{1}{\omega} \right)^{-\frac{1}{2}} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}},
\]

where, of course, account is taken of condition (1); the pole that can appear in principle in the integrand of (16) and (17) corresponds to the Cerenkov condition (see below). The flat frequency spectrum (19) extends all the way to frequencies \( \omega = \omega_0 \) [see (14)], after which \( W(\omega) \) begins to decrease with frequency. This yields an estimate of the total radiative energy

\[
W = \int W(\omega) d\omega = \frac{3}{15} \frac{\alpha^2 \epsilon_0}{2\pi} \left( \frac{m_e c^2}{B_0} \right)^2 \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}} \frac{\rho}{\rho_0} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}}.
\]

where the numerical factor \( s \), generally speaking, is of the order of unity (its exact value can be obtained with the aid of very cumbersome formulas that are given in [12]; if expression (20) is "joined" with the value (22) pertaining to the case \( \rho > \rho_0 \), then \( s = 7.4 \).

At \( \rho = \rho_0 \), the term \( \epsilon_0 - 1 \) or \( \epsilon_1 - 1 \) predominates in the denominators of (16) and (17), until these terms drop as a result of the dispersion that is taken into account by formulas (13). Assuming \( \epsilon_0 = \epsilon_1 = 1 - 1/1 + \rho_0/\rho_0 \) we obtain from (7) an estimate for the highest radiated frequencies:

\[
\omega = \sqrt{\frac{m_e c}{\rho_0} \frac{B_0}{\rho_0}} \frac{\epsilon_0}{\rho_0} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}}.
\]

As is clear from (16) and (17), at frequencies \( \omega = \omega_{\text{max}} \) (and in practice also at \( \omega = \omega_{\text{max}} \)) the spectrum \( W(\omega) \) is independent of frequency, and then at \( \omega > \omega_{\text{max}} \) it begins to decrease like \( \omega^{3/2} \). The contribution to the integrated radial energy from the frequency region \( \omega = \omega_0 \) turns out to be negligible. Consequently, we can use expressions (13) when integrating by parts in (16) and (17). As a result of integration over the angles and frequencies, we obtain

\[
W = \frac{2\pi a m_e c}{\sqrt{2\pi}} \frac{m_e c^2}{B_0} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}}.
\]

Of course, at \( \rho > \rho_0 \) formulas (20) in (22) give approximately the same results,

\[
W \left( \rho > \rho_0 \right) = \frac{2\pi a m_e c}{\sqrt{2\pi}} \frac{m_e c^2}{B_0} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}}.
\]

The foregoing results are limited in the sense that we took into account only vacuum polarization of electron-positron types. Inasmuch as for muons the field \( B_0 \), is \( (m_\mu/m_e)^2 \) - 4000 times stronger than \( B = B_0 \) (see (1); \( m_\mu \) is the muon mass), this limitation plays no role from the practical point of view. As is clear from (15), at the frequencies \( \omega > \omega_0 \), meaning all the more in the frequency region (21), the role of the plasma is also insignificant.

The boundary of the magnetic field was assumed above to be abrupt, whereas physically this condition cannot be satisfied and the formulas used for \( \epsilon_0 \) and \( \epsilon_1 \) are valid, in any case, if the thickness of the "boundary" is

\[
\delta \gg \lambda/\rho_0.
\]

It is known from the theory of transition radiation (18) that its intensity is determined by the dimension \( L_0 \) of the zone in which the radiation is formed near the boundary, with

\[
L_0 \sim \frac{a m_e c^2}{\rho_0 B_0}.
\]

where no account is taken of the influence of the plasma, and the angle \( \psi \) is set equal to zero; more accurately, it is assumed that

\[
\left( \frac{m_e c}{\rho_0} \right) \sim \frac{\lambda_0}{\omega}.
\]

It is obvious [see (14) and (21)]

\[
L_0(\psi) = \frac{4\pi}{3} \frac{\lambda_0}{\psi} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}}.
\]

Thus, at least at \( \delta \gg \delta_0 \), the length \( L_0 \gg \rho_0/\rho_0 \) and the conditions \( L_0 \gg \delta_0 \gg \rho_0/\rho_0 \) are compatible.

We compare now transition radiation with synchrotron radiation and Cerenkov radiation. The synchrotron radiation power is (the angle between \( v \) and \( B_0 \) is \( \pi/2 \), and \( \delta / Mc^2 = 1 \))

\[
Q = \frac{2\pi a m_e c}{\sqrt{2\pi}} \frac{m_e c^2}{B_0} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}} \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{2}}.
\]

It is reasonable to compare the transition-radiation en-
ergy with the energy radiated via the synchrotron mechanism over the length of the formation zone, i.e., within in the time $L/c$. This energy is

$$W_{\text{sc}} = \frac{Q_{\text{sc}}}{c} = \frac{4\pi n(0)\langle \beta \rangle}{\omega} \left( \frac{m_e c^2}{\omega} \right) \left( \frac{B_e}{B_0} \right)^2 \frac{e^2}{2\sigma_e} \frac{\ln B_e}{\ln B_0}$$

and

$$W_{\text{syn}} = \frac{Q_{\text{syn}}}{c} = \frac{4\pi n(0)\langle \beta \rangle}{\omega} \left( \frac{m_e c^2}{\omega} \right) \left( \frac{B_e}{B_0} \right)^2 \frac{e^2}{2\sigma_e} \frac{\ln B_e}{\ln B_0}.$$

Comparing (30) and (28), we see that the transition radiation can exceed the synchrotron radiation only for particles with very large mass $M \gg m(45\pi^2/2a)$ (here $a = [c/\omega]^{1/2}$). If furthermore we compare (29) with (32), then we arrive at an analogous conclusion. In fact the situation is more complicated, since it is necessary to distinguish between the formation zones in both media (in this case—in the field and in the region outside the field). Therefore the ratio of the synchrotron radiation to the transition radiation changes under certain conditions in favor of the latter. On the whole, there is no doubt that in the considered example (motion of a charge across a magnetic field) the synchrotron radiation is generally predominant.

The Cerenkov-radiation power under the same conditions (for more details see (23)) is

$$Q_{\text{Cerenkov}} = \frac{E_{\text{total}}}{\lambda} = \frac{\pi e^2}{2\sigma_e} \left( \frac{m_e c}{\omega} \right)^2 \left( \frac{B_e}{B_0} \right)^2 \left( \frac{\ln B_e}{\ln B_0} \right)$$

Substituting (8) we obtain (see also (11))

$$Q_{\text{Cerenkov}} = \frac{E_{\text{total}}}{\lambda} = \frac{\pi e^2}{2\sigma_e} \left( \frac{m_e c}{\omega} \right)^2 \left( \frac{B_e}{B_0} \right)^2 \left( \frac{\ln B_e}{\ln B_0} \right)$$

Obviously, radiation is possible only under the condition

$$\frac{E_{\text{total}}}{\lambda} > \left( \frac{11}{15} \right) e^2 \left( \frac{m_e c}{\omega} \right)^2 \left( \frac{B_e}{B_0} \right)^2 \left( \frac{\ln B_e}{\ln B_0} \right).$$

When this condition is satisfied, we can roughly estimate the power by integrating in (31) up to the frequency $\omega_i = (mc^2/k)(\beta \gamma_i B_e)$ above which the refractive index of the vacuum begins to decrease with frequency. Such an estimate yields

$$Q_{\text{Cerenkov}} = \frac{11\pi e^2}{180} \left( \frac{B_e}{B_0} \right)^2 \left( \frac{\ln B_e}{\ln B_0} \right) \omega_i^{-3} \ln \omega_i \gamma_i^{-3} \left( \frac{m_e c}{\omega_i} \right)^{-3},$$

The energy radiated over the formation zone is [see (25) and (26)]

$$W_{\text{Cerenkov}} = Q_{\text{Cerenkov}} = \frac{E_{\text{total}}}{\lambda} = \frac{\pi e^2}{2\sigma_e} \left( \frac{m_e c}{\omega} \right)^2 \left( \frac{B_e}{B_0} \right)^2 \left( \frac{\ln B_e}{\ln B_0} \right).$$

Comparing this expression with (22), we see that the transition radiation exceeds the Cerenkov radiation if

$$\frac{E_{\text{total}}}{\lambda} > 3\pi e^2 \left( \frac{m_e c}{\omega} \right)^2 \left( \frac{B_e}{B_0} \right)^2 \left( \frac{\ln B_e}{\ln B_0} \right).$$

By virtue of (32) the condition (35) is always satisfied.

4. If an ultrarelativistic particle moves along the field $B_0$, then, as already noted, the refractive index is $\approx (0.1$ and there is neither synchrotron nor Cerenkov radiation. On the other hand, transition radiation is produced if, of course, there is a boundary between the region with the field and without the field. By virtue of the equation $\omega \gamma > 0$ it is not easy to produce a boundary along the field, although within certain limits this is possible if external currents are available. It must furthermore be recognized that the boundary need not necessarily be perpendicular to the particle velocity, i.e., in this case it need not be located in the plane $(x, y)$. On the other hand, for a boundary perpendicular to the field, the transition radiation (in the case of motion along the field) is much weaker than in the case considered above. The point is that in an approximation such as (10) and (17) the radiated energy depends only on $(a)$ (according to (11), Eqs. (16) and (17) contain only the quantities $a_0$ and $a_1$). We therefore need to use for the calculations more exact formulas, which are given in (14). We confine ourselves here to the result for the case $\ln B_e/\ln B_0 = 1$:

$$W_{\text{trans}} = \frac{a_0^2}{a_0} = \frac{a_1^2}{a_0^2} \ln \frac{B_e}{B_0} $$

The total radiated energy can be estimated by integrating up to the frequency $\omega_i$ [see (14)]:

$$W_{\text{trans}} = \frac{a_0^2}{a_0} \ln \frac{B_e}{B_0} $$

At $\omega < c$ we obtain

$$W = 3 \times 10^{-8} \left( \frac{B_e}{B_0} \right)^2 \left( \frac{\ln B_e}{\ln B_0} \right).$$

At pulsar surfaces the field $B_0$ is usually estimated at $10^12 G$, but it is possible that in some cases it is stronger by one order of magnitude. Thus, the parameter $B_e/B_0$ for pulsars can quite readily reach a value 0.1. As to the particle concentration near pulsars, the estimates here are less reliable. We confine ourselves to the remark that the condition $B_e/B_0 \gg N\bar{\sigma}$ ($N$ is the concentration of particles with energy $\delta$) at which the pressure of the field predominates, is satisfied up to concentrations $N = 10^6$ cm$^{-3}$ at $B_e = 5 \times 10^12 G$ and $10^9$ eV $\times 10^9$ erg. Even at $N = 10^9$ the total pulsar radiation power (38) for a particle flux $N\bar{\sigma}$ through an area $\delta \times 10^8$ cm$^2$ amounts to $(B_e/B_0 \approx 0.1$ and $\ln B_e/\ln B_0 = 1$)

$$Q = 0 \times 10^{-8} \approx 3 \times 10^{-8} \text{erg/sec}. $$

At the same parameters we have for motion across the field in accordance with (22) with $\omega < c$.

$$Q = 0 \times 10^{-8} \approx 3 \times 10^{-8} \text{erg/sec}. $$

We do not regard these estimates at all as realistic (as applied to pulsars). They indicate, however, that transition radiation can play a substantial role in pulsar physics. There are also some prospects of producing in the laboratory, conditions in which the parameters $B_e/B_0$ and $\delta/Mc^2$ are large enough for nonlinear phenomena to manifest themselves in vacuum (the same holds for certain atomic nuclei). It must be remembered here that the transition radiation, taken in its broader mean-

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ing, takes place whenever a medium (or the vacuum in a strong field) is inhomogeneous in space and (or) in time. It is advantageous, as already noted in the introduction, to make partial use of classical theory of transition radiation.

We note in conclusion that in semiconductors it is possible to simulate within certain limits the nonlinear phenomena that occur in a vacuum, by using much weaker fields \( B = 10^{-2} - 10^6 \) G. Roughly speaking, what is done here is replacing the "gap" \( 2mc^2 \approx 1 \) eV in vacuum by the width of the forbidden band in the semiconductors \( \approx 1 \) eV. In particular, in a strong constant magnetic field a semiconductor has rather unique electrodynamic properties (see, e.g., [11]). It is possible that interest attaches in this connection to an analysis of certain features of the transition radiation and scattering in semiconductors situated in strong fields. Incidentally, the foregoing pertains not so much specifically to semiconductors as in general to nonlinear media (for more details see [5]).

5. We dwell now on transition radiation, which is the simplest mechanism of conversion of waves of one type into waves of another type by a charge (or by some other source of polarization of the medium), without requiring (sometimes an important factor) a change in the motion of the charge itself.

We assume below for simplicity that the charge \( q \) is immobile (pinned). Its electric field, without allowance for the nonlinearity of the vacuum can be written in the form

\[
E_0 = q\rho / r^3.
\]

Let there be incident on this charge an electromagnetic wave whose field is equal to

\[
E_0^i = E_0^i E_0 \cos (\omega t - k\cdot r + \phi), \quad E_0^i = |E_0^i|, \quad \omega = |\omega|, \quad k = |k|.
\]

If the wave is coherent and monochromatic. If the incident waves have a broad spectrum, are not correlated in phase, and are not polarized, then

\[
|E_i|^2 = |E_0^i|^2 / 8 \pi \Delta \omega.
\]

The magnetic field \( B \) and the time-average energy \( \langle W \rangle \) of the waves are respectively, for the coherent case,

\[
B = E_0^i \cos (\omega t - k\cdot r + \phi), \quad E_0^i = |E_0^i|, \quad \langle W \rangle = \frac{1}{8n} (B^2 / \omega^2).
\]

For an incoherent field, the average energy is

\[
\langle W \rangle = \frac{1}{8n} E_0^i E_0^j \delta_{ij}.
\]

The values corresponding to a random quasimonochromatic field in (43) and (5) are

\[
|E_i|^2 = E_0^i^2 / \delta_{ij}, \quad (W) = E_0^i^2 / 8n.
\]

Consider now a process in which the incident wave of frequency \( \omega_i \) is converted into a scattered wave with frequency \( 2\omega_i \). In quantum language this corresponds to absorption of two quanta \( E_0^i \) with emission of one quantum of energy \( 2E_0^i \). In the case of the spectrum, we are dealing here with doubling of each of the frequencies \( \omega = |\omega| \). Assuming that \( (E^i)^* = (E^i)^2 \) and \( (E^i, B^j) = 0 \), it is necessary for the process in question to retain in formulas (3) for \( P \) and \( \mathcal{M} \) the terms that are linear in the field \( E^i \) of the charge and quadratic in the field \( B^j \) of the wave

\[
P = \langle |E^i|^2 E^j + (B^i)^2 \rangle,
\]

\[
\mathcal{M} = \langle -|E^i|^2 B^j + (B^i)^2 \rangle.
\]

For the wave (42) we obtain hence

\[
P = \langle |E^i|^2 E^j + (B^i)^2 \rangle,
\]

\[
\mathcal{M} = \langle -|E^i|^2 B^j + (B^i)^2 \rangle.
\]

Thus, the polarization and the magnetization have oscillating terms with frequency \( 2\omega_i \) and wave vector \( 2k \).

The alternating polarization and magnetization produce transition scattering by the charge \( q \). The intensity of the scattering can be easily obtained by the method described in [11] being equal to the power radiated by a current of density

\[
\mathcal{J}_q = \mu M \cdot \mathcal{E}.
\]

The radiation power for the scattered wave (set over both polarizations), is

\[
\mathcal{P} = \frac{1}{8\pi} \int_0^{2\pi} \int_0^\pi \int_0^\pi \mathcal{J}_q \cdot \mathcal{E} \sin \theta \sin \varphi \sin \psi \sin \theta \sin \varphi \sin \psi \sin \theta \sin \varphi \sin \psi \sin \theta \sin \varphi \sin \psi \sin \theta \sin \varphi \sin \psi.
\]

The result (50) coincides in fact with that obtained in [11] on the basis of a cumbersome quantum electrodynamic calculation (the calculations in [11] were carried out also for the region of high frequencies \( \omega \)).

For arbitrary nonmonochromatic waves, the Fourier components \( \mathcal{J} \) of the current density are given by

\[
\mathcal{J}_{\mathcal{J}}(\mathcal{F}) = \int \mathcal{J}_q(\mathcal{F}) \cdot \mathcal{E}(\mathcal{F}) \sin \theta \sin \varphi \sin \psi \sin \theta \sin \varphi \sin \psi \sin \theta \sin \varphi \sin \psi \sin \theta \sin \varphi \sin \psi \sin \theta \sin \varphi \sin \psi.
\]

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\]

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The radiation power of the current (51) at the frequency \( \nu = c |k_i| |k_f| \) for random waves, averaged over the phases with the aid of (43), is of the form

\[
Q = \int \left| \Phi(k_i,k_f) \right|^2 d\Omega_i d\Omega_f dE_i dE_f. \tag{53}
\]

\[
\Phi(k_i,k_f) = \int d\Omega_i d\Omega_f dE_i dE_f \Phi(k_i,k_f). \tag{54}
\]

where \( n = k/k_i, h = k_i/k_f, k_i = k_{i1}/h, \) and \( k_f \) are the wave vectors of the scattered wave, \( k \) is the wave vector, and \( d\Omega_i \) is the solid angle of the scattered waves; finally

\[
n = n_{i1} - h = n_{i2} + h.
\]

In the case of a quasimonochromatic random wave it is necessary to substitute (46) in (53) and (54). The result coincides with (50). This means that the result (50) remains in force for a scattered wave packet of sufficiently general form in the case when its spectral width is \( \Delta \omega = \omega_i \) and the total energy is equal to \( E_i^2/2h \). The latter is important for a possible experimental confirmation of the effect of the aid of, for example, scattering of intense laser radiation by massive ions.

In connection with the nonlinearities of the vacuum, the effect most frequently discussed was the so-called Delbrück scattering (see [13], Sec. 125, and [14]). This effect can be considered by using the nonlinear polarizabilities \( P \) and \( M \), if two of the fields in the terms cubic in the field in (3) are taken to be the charge field \( E_a \), and in the remaining term \( P \) and \( M \) leads to integrals that diverge at large \( E \) of the field of the scattered wave, \( k \) is the wave vector, and \( d\Omega \) is the solid angle of the scattered waves; finally

\[
\Phi(k_i,k_f) = \int d\Omega_i d\Omega_f dE_i dE_f \Phi(k_i,k_f). \tag{54}
\]

The maximum frequency (59) is radiated in the direction of motion of the particle \( n = -q/\omega_0 \). As \( \nu = c \) we have

\[
Q = \int \left| \Phi(k_i,k_f) \right|^2 d\Omega_i d\Omega_f dE_i dE_f. \tag{53}
\]

The intensity of the scattering of a coherent or quasimonochromatic random wave is determined by the relation

\[
Q(0) = \frac{2 \pi^2}{c} \frac{E_i^2}{E_f^2} \frac{\sin^2(\theta)}{2} \int \left| \Phi(k_i,k_f) \right|^2 d\Omega_i d\Omega_f dE_i dE_f. \tag{55}
\]

Attention is called to the strong dependence of the scattering intensity on the particle energy.

6. We dwell also on the nonlinear effect in vacuum-scattering of a wave by a charge in the presence of a strong magnetic field \( B_0 \). We assume first that the charge is immobile and consider the scattering process when the frequency of the scattered wave is equal to the frequency of the incident wave. In this case

\[
E = E_i + E_f, \quad B = B_0 + B_f.
\]

Assuming the field of the charge to be weak, we take in-
to account the terms that are linear in $\delta^2$ in the expressions (3) for the polarizations $P$ and the magnetization $M$. In the corresponding expressions that are quadratic in the field $\mathbf{E}$ and $\mathbf{B}$, the terms of order $\delta^2$ and $\delta^2 \mathbf{E}^2$ describe the two types of processes which have already been discussed above, and the process of interest to us here is described by terms of order $\delta \mathbf{E} \cdot \mathbf{B}$ and $\delta \mathbf{B} \cdot \mathbf{E}$. These parts of the polarization and magnetization are of the form

$$
\delta P = -\frac{1}{3} \varepsilon_0 c^2 \mu_0 \mathbf{E} \cdot \mathbf{B} \delta \mathbf{E} 
$$

and

$$
\delta M = -\frac{1}{3} \varepsilon_0 c^2 \mu_0 \mathbf{E} \cdot \mathbf{B} \delta \mathbf{B}.
$$

We present here the final formula for the scattering intensity of a wave propagating in the direction of the magnetic field $\mathbf{B}$ ($\mathbf{k} \cdot \mathbf{B} = 0$) in the case of a charge at rest:

$$
Q = \frac{4\pi \varepsilon_0 c^4}{3 \varepsilon_0} \frac{\mathbf{E}^2 \mathbf{E}^*}{\mathbf{B}^2} \frac{\mathbf{k}^2}{\mathbf{k}} \frac{(\mathbf{k}^2 + \mathbf{E}^2)}{(\mathbf{E}^2)}.
$$

For ultrarelativistic particles moving opposite to the wave in the magnetic field, we obtain

$$
Q = \frac{16}{3} \varepsilon_0 c^4 \frac{\mathbf{E}^2 \mathbf{E}^*}{\mathbf{B}^2} \frac{\mathbf{k}^2}{\mathbf{k}} \frac{(\mathbf{k}^2 + \mathbf{E}^2)}{(\mathbf{E}^2)}.
$$

The maximum frequency is radiated in a direction close to the particle velocity, and is equal to

$$
\omega = -\nu_0 \frac{\mathbf{E}}{\mathbf{B}}.
$$

We emphasize that formulas (55) and (66) do not contain integration with respect to large $k$ and are therefore exact within the framework of the assumptions.

Pulsars possibly accelerate ions to $\mathbf{E} / \mathbf{B} \approx 10^9$. Under these conditions, the discussed scattering with power (66) can be of interest. However, as already indicated, no estimates were made in the present article to the field of the charge $q$ at the distance $r = \mathbf{B}/\mathbf{E}$ from its center. Then $4 \mathbf{B} / \mathbf{E} = \mathbf{E}/\mathbf{B}$, and by taking into account the fact that the dimension of the area responsible for the scattering is small by a factor $\mathbf{E}/\mathbf{B}^2 = \mathbf{E}/\mathbf{B}^2$. This yields qualitatively the two small factors in (66).

