neutron star. This distance increases with increasing power of the GW source. It is interesting to note that at this distance from a neutron star one can also detect GW. In fact, at this distance the GW energy flux density equals

$$S=3.1\cdot10^{-3}\ \mathrm{erg/cm^2 s}.$$

According to Braginskiĭ et al.,[2] one can, with the help of an electromagnetic detector, register GW in the radio spectrum with an energy flux density

$$S=10^{-4}\ \mathrm{erg/cm^2 s}.$$

This makes it possible to select an object for setting up an experimental detection of GW on the basis of the singularities of the EMW (10) resulting from the interaction and coming from a neutron star.

In conclusion we note that, in the case of GW incident from the outside on a rotating magnetic dipole, the expressions for $\mathbf{j}_{\rm int}$ and $q_{\rm int}$ (6) can be expanded in a series with an infinite number of terms of spherical harmonics of the first kind. Consequently the solution of Eq. (5) for the potentials $\mathbf{a}$ and Φ obtained in this case also as an infinite series of spherical harmonics of the first kind, while the coefficients of this series have a form which significantly complicates further analysis of the obtained solution.

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Theory of hadron plasma

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The characteristics are calculated of a new state of matter corresponding to densities n and temperatures T such that $\hbar c n^{1/3}$, $T \gtrsim 1$ GeV. This is the hadron plasma in which quarks that are the components of hadrons under ordinary conditions are collectivized. The calculations are performed within the framework of the so-called quantum chromodynamics, i.e., the theory of strong interactions. The results obtain for cold plasma with high density of baryon charge are applied to the collapse of neutron stars and to the problem of the repulsive core of nucleons. Results on the properties of hot neutral plasmas are applied to cosmology and to hadron collisions at high energies.

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1. INTRODUCTION

The present paper is concerned with the description of the properties of matter at densities much greater than the density of atomic nuclei, or temperatures exceeding the characteristic hadron mass $m\sim1$ GeV. It is based on the theory of strong interactions with non-Abelian gauge fields[1] which is frequently called quantum chromodynamics[2] (see also the review by Politzer[3]). Under the above conditions, the separation between the hadrons becomes smaller than their dimensions, and they cease to behave as individual objects, i.e., the quarks of which they are made up are collectivized.[4,5] By analogy with the similar behavior of atomic electrons, this state of matter can be referred to as hadron plasma, in contrast to the normal state, i.e., the hadron gas. This analogy is considerably enhanced by the similarities between chromodynamics and electrodynamics: in both cases, the interaction is transported by massless vector fields.

To explain the notation used below, we recall that the Lagrangian density in chromodynamics has the form

$$L=-\frac{1}{4}F_{\mu\nu}^aF_{\mu\nu}^a+\sum_{N=1}^{N_K}\bar{\Psi}_N\left(i\hat{\partial}+\frac{g}{2}\lambda^a b_\mu^a\gamma_\mu-m_N\right)\Psi_N,\qquad F_{\mu\nu}^a=\partial_\mu b_\nu^a-\partial_\nu b_\mu^a+gf^{abc}b_\mu^b b_\nu^c, \tag{1}$$

where the fields b_μ^a correspond to the gauge vector fields, the gluons, and Ψ_N represent quarks of flavor N, i.e., ordinary quarks u and d, the strange quark s, the charmed quark c, and, possibly, other quarks up to flavor N_k. Greek subscripts correspond to Lorentz subscripts and run over values between 0 and 3, and Latin superscripts are color indices running over values 1–8; λ^a are the Gell-Mann matrices of the group SU_3, and

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their indices will be omitted. We recall, for the sake of completeness, that the indices of three-dimensional vectors will be indicated by Latin letters.

An important feature of this theory is the so-called asymptotic freedom or antiscreening, i.e., the interaction is turned on at short distances. The asymptotic equation of state of the hadron plasma in the limit of high densities is, therefore, the equation of state of the plasma gas.[5] The aim of this paper is to calculate corrections due to the particle interaction, and to locate more precisely the gas-plasma transition point.

We recall that for momentum transfer k, i.e., over distances $1/k$, the invariant charge $g(k)$ is given by[2,6]

$$\alpha_s(k)=\frac{g^2(k)}{4\pi}=\alpha_s(m_0)\left[1+\left(\frac{11}{2}-\frac{N_k}{3}\right)\frac{\alpha_s(m_0)}{2\pi}\ln\left(\frac{k^2}{m_0^2}\right)\right]^{-1}, \quad (2)$$

where m_0 is an arbitrary normalization point which, to be specific, we define by $\alpha_s(m_0)=1$. The absolute value of m_0 is important because it determines the conditions for the validity of expansions in α_s, which we shall use below. Data on φ and ψ meson decays[7] and on the violation of scaling in eN and μN scattering[8] yield $m_0 \sim 0.2$ GeV, whereas model parametrization of hadron spectra[9] yields $m_0 \sim 0.4$ GeV. These values of m_0 are used below as the upper and lower limits.

Our material is arranged as follows. In Sec. 2, we investigate Green functions for gluons in hadron plasmas and show that antiscreening at short distances is replaced by screening at large distances, and that transverse gluons assume nonzero effective mass, which removes the difficulties in the theory in the infrared. Section 3 gives a calculation of the characteristics of cold plasma, i.e., a medium with high density of baryon charge and zero temperature. It is shown that the transition to the baryon-hyperon gas[10] occurs for baryon densities $\sim 1-2\ \text{fm}^{-3}$. A brief discussion is then given of the correspondence between the results obtained here and the characteristics of the repulsive nucleon core, and of the consequences of these results for neutron-star collapse. The characteristics of hot plasma, i.e., a medium with zero density of all charges and high temperature, are calculated in Sec. 4. Comparison of these results with the characteristics of the hadron gas[11] shows that the "ionization" of hadrons occurs at temperatures of 0.4–0.6 GeV. Apart from cosmologic applications, these results can be used within the framework of the statistical approach to hadron reactions.[12] In particular, this approach can be used to calculate the rate of production of leptons and photons in hadron plasma.

The calculations are performed with the aid of the temperature Green functions.[13–15] A particular technical problem, which is important for calculations and has not been adequately discussed in the literature, namely, the separation of the contributions due to particles in the medium and virtual particles, is elucidated in the Appendix. The presence of the medium determines the coordinate frame, so that the most natural for our purposes is the Coulomb gauge in which quantization has been performed by Khriplovich.[6] The Hamiltonian for the problem is

$$H=\frac{1}{2}p_m^a p_m^a+\frac{1}{2}\partial_n b_m^a\partial_n b_m^a+gf^{abc}\partial_m b_n^a b_m^b b_n^c$$
$$+\frac{g^2}{4}f^{abe}f^{ade}b_m^b b_n^c b_m^d b_n^e-\frac{1}{2}b_0^a\Delta b_0^a \quad (3)$$
$$+\sum_N\bar{\Psi}_N\left[\left(i\partial_m+\frac{g}{2}\lambda^a b_m^a\right)\gamma_m+m_N-\frac{g}{2}\lambda^a b_0^a\gamma_0\right]\Psi_N,$$

where p_m^a is the canonical momentum that is the adjoint of b_m^a and, as in electrodynamics, the zero components of the potential b_0^a are not independent and must be substituted in (3) after the solution of the equations of motion:

$$\Delta b_0^a=gf^{abc}b_m^b(p_m^c+\partial_m b_0^c)+\frac{g}{2}\sum_N\bar{\Psi}_N\gamma_0\lambda^a\Psi_N. \quad (4)$$

However, in contrast to electrodynamics, this can be done only in the form of an infinite series, so that the Hamiltonian (3), in fact, contains an infinite number of vertices.

Finally, we note that, so long as the problem of confinement of quarks remains unresolved, the validity of thermodynamics as a description of strong interactions in general, and the validity of perturbation theory at short distances in particular, cannot be regarded as established. At present, these theories are based on only indirect experimental evidence.[3,7,8]

It is also important to note that the original version of the present paper, published in the form of a preprint, has been substantially extended, and some of the coefficients in the formulas have been corrected.

2. INTERACTIONS IN HADRON PLASMA

Equation (2) gives the interaction between two charges as a function of the distance between them. According to this formula, in vacuum, the coupling constant increases and approaches unity as the hadron size approaches $1/m_0$. The question is: what is the interaction between the charges in plasma? It is clear that, at distances less than the characteristic interparticle distance $r_0 \sim n^{-1/3}$, the presence of the medium should have no effect, and (2) should remain valid. However, it is not immediately clear what happens at distances in excess of r_0, and this is the subject of the present section.

In the Coulomb gauge, the general form of the polarization operator is[15]

$$\Pi_{mn}(\omega,\mathbf{k})=(\delta_{mn}-k_m k_n/\mathbf{k}^2)A(\omega,\mathbf{k})+\frac{\omega^2 k_m k_n}{\mathbf{k}^4}\Pi_{00}(\omega,\mathbf{k}), \quad (5)$$

and the gluon Green function is

$$D_{00}^{ab}(\omega,\mathbf{k})=\frac{-\delta^{ab}}{\mathbf{k}^2-\Pi_{00}(\omega,\mathbf{k})},\quad D_{mn}^{ab}(\omega,\mathbf{k})=\frac{-\delta^{ab}(\delta_{mn}-k_m k_n/\mathbf{k}^2)}{\omega^2-\mathbf{k}^2-A(\omega,\mathbf{k})}. \quad (6)$$

Figure 1 shows the perturbation-theory graphs contributing to $\Pi_{00}(\omega,\mathbf{k})$ and $A(\omega,\mathbf{k})$ in the second order in g. The corresponding analytic expressions can be con-

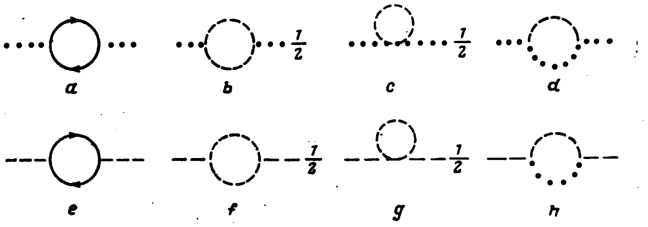

FIG. 1. Graphs giving the second-order contributions to Π_{00} ($a-d$) and Π_{mn} ($e-h$). Here and henceforth, solid lines represent the Green function for quarks; broken lines represent transverse gluon functions; dotted lines represent the Coulomb field. Numerical factors shown against these diagrams represent combinatorial factors by which the expressions obtained from the usual Feynman rules have to be multiplied.

structed in accordance with the standard Feynman rules, but are too unwieldy to be reproduced here. Only the graph of Fig. 1c deserves attention here since the Hamiltonian (3) does not contain this vertex. The point is that the contribution of graph b is

$$\Pi_{00}^{b}(x-y)=\frac{g^2}{2}f^{abc}f^{ade}\langle T[p_m^b(x)\,b_m^c(x)\,p_n^d(y)\,b_n^e(y)]\rangle. \qquad (7)$$

Time derivatives which appear in the canonical momenta $p_m^a=\partial_0 b_m^a$ when they are taken outside the T-product sign give an additional term which is identical with the contribution of diagram c with the vertex taken formally from the Lagrangian (1).

Summation over frequencies is performed by the method described in the Appendix. Since each graph has only one loop, the polarization operator is equal to the sum $\Pi_{\mu\nu}=\Pi_{\mu\nu}^{(\rm vac)}+\Pi_{\mu\nu}^{(\rm med)}$, where $\Pi_{\mu\nu}^{(\rm vac)}$ is the vacuum part calculated by Khriplovich[6] and $\Pi_{\mu}^{(\rm med)}$ is the contribution of the particles in the medium, which takes the form of the same Feynman integral but with one of the Green functions in the loop replaced by the special Green function with the "cross" (see the Appendix), which contains the δ function on the mass surface and the statistical occupation factors. These factors ensure convergence at high momenta such that $\Pi_{\mu\nu}^{(\rm med)}$ need not be normalized. For the same reason, the vacuum contribution $\Pi_{\mu\nu}^{(\rm vac)}\sim g^2k^2\ln(k^2/m_0^2)$ exceeds $\Pi_{\mu\nu}^{(\rm med)}\sim g^2/r_0^2$ at large distances ω, $k\gg 1/r_0$, i.e., the presence of the medium is unimportant at short distances, as indicated above.

Let us now consider the region of large distances, beginning with the case of cold plasma. This case is particularly simple since, for $T=0$, the plasma does not contain real gluons and the only contribution to $\Pi_{\mu\nu}^{(\rm med)}$ is that due to the quark loops (Figs. 1a and e) which are completely analogous to those describing screening in ordinary plasma. For ω, $k\ll\mu$, where μ is the chemical potential, we have

$$\Pi_{00}^{(\rm med)}(\omega,k)=-\frac{g^2N_k\mu^2}{2\pi^2}\left[1-\frac{\omega}{2k}\ln\left|\frac{\omega+k}{\omega-k}\right|+\frac{i\pi\omega}{2k}\theta(k-\omega)\right], \qquad (8)$$

so that, in this region, $\Pi_{00}^{(\rm med)}\gg\Pi_{00}^{(\rm vac)}\sim g^2k^2\ln k^2$ and the interaction is screened at distances of the order of $(g\mu)^{-1}$. If the medium is so dense that $g\mu\gg m_0$, the Coulomb interaction is small at all distances, and the perturbation theory in g is valid.[1)] We note that, in vacuum, the Green function has a nonphysical pole in this approximation for $k^2\sim m^2$, whereas, in plasma, the pole corresponds to plasma oscillations with $\omega_0^2=g^2N_k\mu^2/6\pi^2$. Transverse gluons assume the effective mass $M^2=A(\omega=k)=g^2\mu^2/4\pi^2$, although the static "gluomagnetic" field is not screened, i.e.,

$$A(\omega=0,\ k\to 0)=O(k^2).$$

The case of nonzero temperature is much more complicated because real gluons then appear in the medium. It is important to emphasize that the problem of second-order characteristics of the medium is not exhausted by the eight graphs shown in Fig. 1 because, for the gauge field, the polarization operator depends on the chosen gauge and the simultaneous inclusion of corrections to the vertex functions is essential. However, this can be avoided in certain important special cases.

In the Coulomb gauge, corrections to the vertices with the Coulomb quantum vanish in the limit as $\omega=0$, $k\to 0$ because the total charge of the system is conserved.[6] For gauges using nonphysical degrees of freedom, this argument is no longer valid. The corresponding limit $\Pi_{00}(\omega=0,\ k\to 0)\equiv-\varkappa^2$ therefore assumes an invariant significance where the screening length $\varkappa^{-1}$ is given by

$$\varkappa^2=g^2T^2\left(\frac{N_k}{6}+\frac{1}{2}+\frac{1}{2}+0\right)=g^2T^2\left(1+\frac{N_k}{6}\right). \qquad (9)$$

The individual terms in this expression correspond to the contributions of the diagrams in Figs. 1$a-d$. The polarization operator for the transverse quanta $A^{(\rm med)}$ is gauge-invariant on the mass shell, i.e., for $\omega=k$. It is then expressed in terms of the scattering amplitudes which, since they are physical quantities, are gauge-independent. In fact, by considering the Green-function "crosses," we can readily see that the diagram of Fig. 1e can be expressed in terms of the quark forward-scattering amplitude (see diagrams 5b and c), whereas diagrams f, g, and h can be expressed in terms of the gluon scattering amplitudes (Figs. 5, d, f, and e). The amplitudes summed over the polarizations are, respectively, given by

$$M_{qb}{}^{ab}=M_{\bar qb}{}^{ab}=-2g^2\delta^{ab},\qquad M_{bb}{}^{ab}=-24g^2\delta^{ab}$$

and the effective mass of the transverse quanta is

$$M^2=A^{(\rm med)}(\omega=k)=g^2T^2(1/2+N_k/12). \qquad (10)$$

The Coulomb field in the medium is, therefore, screened and the transverse quanta assume an effective mass. This means that the infrared singularities of chromodynamics are naturally cut off in the medium, so that the characteristics of a sufficiently dense medium can be unambiguously calculated within the framework of perturbation theory.

3. CHARACTERISTICS OF COLD PLASMA

As noted in the Introduction, the zero-order approximation to the cold plasma with high baryon-charge density is the ideal gas of quarks[5] in which the energy density is

$$\varepsilon(\mu)=\frac{3}{8\pi^2}\sum_N\left[p_N\mu(m_N^2+2p_N^2)-m_N^4\ln\left(\frac{p_N+\mu}{m_N}\right)\right], \tag{11}$$

where p_N is the Fermi momentum of quarks of flavor N, $p_N^2=\mu^2-m_N^2$, and $n_N=p_N^3/\pi^2$. The baryon charge density is

$$n_B=\frac{1}{3}\sum_N n_N,$$

and this, together with (11), determines the required equation of state $\epsilon(n_B)$.

The second-order diagrams are shown in Fig. 2. The first of them corresponds to the mean-field approximation and provides no contribution because the medium is "colorless," so that only the contribution of exchange forces remains. According to the "cross rule" (see Appendix), the sum over the frequencies of the two loops in this graph can provide contributions with 0, 1, and 2 "crosses." The first corresponds to the vacuum energy and must be rejected. The second corresponds to the mass operator of the quark on its mass surface and, in the usual renormalization procedure, is zero by definition. However, in thermodynamics, it is nonzero when the Green function is normalized at the point m_0, but is proportional to m_0^2 and small in comparison with the contribution with two crosses, which corresponds to the interaction between real particles of the plasma:

$$\delta\Omega^{(2)}=\frac{g^2}{2}\sum_N\int_{(\varepsilon_1,\varepsilon_2<\mu)}d^3p_1d^3p_2\frac{\mathrm{Sp}[\gamma_\mu(\hat{p}_1+m_N)\gamma_\mu(\hat{p}_2+m_N)]}{\varepsilon_1\varepsilon_2(p_1-p_2)^2(2\pi)^6}. \tag{12}$$

The nonrelativistic limit of this formula is similar to the well-known Wigner-Seitz expression, whereas ultrarelativistic calculations yield

$$\delta\Omega^{(2)}=\frac{\alpha_s N_k\mu^4}{2\pi^3}. \tag{13}$$

Graphs of order g^4 are shown in Fig. 2*c*–*f*. They are very laborious to evaluate and we shall, therefore, confine our attention to the leading logarithmic terms. The emission of soft gluons leads to doubly logarithmic divergences in graphs *d*–*f* separately, but these cancel out in the sum because the charge does not affect direction. We are thus left with the double logarithmic correction to backward scattering in graph *d*. This is analogous to the well-known result[16] in electrodynamics:

$$\delta\Omega^{(d)}=\delta\Omega^{(2)}\left(1+\frac{\alpha_s}{3\pi}\ln^2\alpha_s\right). \tag{14}$$

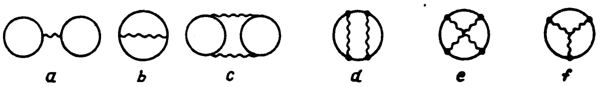

FIG. 2. Graphs corresponding to contributions of order g^2(*a*, *b*) and g^4(*c*–*f*) to the thermodynamic parameters of cold plasma. Wavy lines correspond to gluons, both transverse and Coulomb.

The quantity α_s appears inside the logarithm because the boundaries of the logarithmic region are defined by the momenta $k^2\sim\mu^2$ and $k^2\sim|\Pi_{\mu\nu}|\sim\alpha_s\mu^2$.

Graph *c* provides a logarithmic contribution in the region of small k:

$$\delta\Omega^{(c)}=-\frac{1}{4}\int\frac{d^4k}{(2\pi)^4}\frac{\delta^{aa}N_k{}^2|\Pi_{00}(\mathbf{k})|^2}{\mathbf{k}^4}\approx\frac{0.64}{\pi^5}N_k^2\alpha_s^2\mu^4\ln\alpha_s. \tag{15}$$

As regards logarithms of ultraviolet character, these can all be taken into account by replacing $\alpha_s(m_0)$ by $\alpha_s(\mu)$, which corresponds to the characteristic interparticle separation $1/\mu$. Henceforth, we shall omit the argument of α_s, but it will always be assumed that we are dealing with $\alpha_s(\mu)$.

Thus, collecting all the above terms, we obtain

$$\Omega=-\frac{N_k\mu^4}{4\pi^2}\left[1-\frac{2\alpha_s}{\pi}-\frac{2\alpha_s^2}{3\pi^2}\ln^2\alpha_s-0{,}09\alpha_s^2N_k\ln\alpha_s+\ldots\right], \tag{16}$$

and hence the density of the baryon charge is:

$$n_B=-\frac{1}{3}\frac{\partial\Omega}{\partial\mu}=\frac{N_k\mu^3}{3\pi^2}\left[1+\frac{2\alpha_s}{\pi}+\ldots\right], \tag{17}$$

where we have neglected the relatively small contributions due to $d\alpha_s(\mu)/d\mu$. The final result is

$$\varepsilon=\frac{9}{4}n_B^{4/3}\left(\frac{3\pi^2}{N_k}\right)^{1/3}\left[1+\frac{2\alpha_s}{3\pi}+\frac{2\alpha_s^2}{9\pi^2}\ln^2\alpha_s+0{,}03\alpha_s^2N_k\ln\alpha_s+\ldots\right]. \tag{18}$$

Figure 3 shows a comparison between (18) and the baryon-hyperon gas model[10] (lower part of figure). We note that the exchange correction to the energy is calculated in[17] where α_s is assumed to be 2.2 for all μ.[9]

The next question is the rate at which the validity of the asymptotic expansion (18) is established, and whether it is valid right up to the transition region $n_b=1-2$ fm^{-3}. If we consider this expansion on its own, then since the numerical coefficients are small, the corrections should be small even for $\alpha_s\sim1$. It is clear, however, that this is influenced by effects due to the "nonescape" of quarks, which we have neglected.

Nevertheless, it is possible to argue that these effects are turned on relatively sharply, in the transition region, i.e., the transition from curve *f* in Fig. 3 to curve *g* and h does, in fact, occur in the above transition region. The first of these effects is the sudden onset of scaling in lepton-hadron scattering.[3]

A similar conclusion is reached when we compare our calculation with nuclear-physics data on the nucleon-nucleon interaction. This, it is well known that, when the nucleon separation is 1 fm, the repulsive component is turned on and its height at 0.5 fm is of the order of 0.5-1 GeV. This is the so-called nucleon core which is due to[9,18] the Fermi repulsion between quarks of the same color. In the upper part of Fig. 3, our curves for the plasma (without strange quarks) cut the nucleon-gas curve at $n_B=1-2$ fm^{-3}, i.e., $r_{NN}\sim1$ fm, where the potential turns over and, at $r_{NN}=0.5$ fm, the specific energy ϵ/n_B does, in fact, increase by roughly 1 GeV.

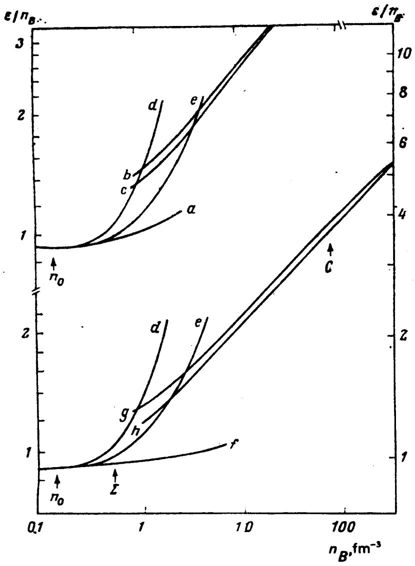

FIG. 3. Specific energy ϵ/n_B (GeV/baryon) as a function of baryon charge density n_B for the isoscalar cold plasma without strange quarks (upper curves, left-hand scale) and with all types of quark u, d, s, c (lower curves, right-hand scale). a—ideal nucleon gas; b, g, and c, h—hadron plasma with $m_0 = 0.2$ and 0.4 GeV; d and e—nuclear-matter calculations; [25]; f—nucleon-hyperon gas. [10] Arrows show n_0—the nuclear density, Σ^- and C—threshold for the appearance of strangeness and charm.

This suggests that the knee on the function $\epsilon(n_B)$ at $n_B \sim 1$-$2\ \mathrm{fm}^{-3}$ does, in fact, correspond to reality.

In accordance with the foregoing, we assume that the equation of state has the form shown in Fig. 3 up to the point of intersection between the curves, and proceed to apply it to the collapse of neutron stars. It follows from the work of Zel'dovich[4] and Saakyan and Vartanyan[19] that, if we use the baryon-hyperon gas model, the central density of the star at the limit of stability is ~2–4 fm^{-3}. This means that collapse and the gas-plasma transition occur practically simultaneously and there are no stable quark stars. This conclusion has also been obtained in the literature[17, 20] on the basis of a somewhat different approach. The stability limit thus corresponds to a stellar mass of 1.6–2 stellar masses, which is in agreement with published data,[19] but the collapse process itself proceeds in a different way, i.e., more rapidly.

4. CHARACTERISTICS OF HOT PLASMA

By hot plasma we understand the medium at a temperature exceeding the chemical potentials of all charges. The zero-order approximation is the ideal gas of quarks, antiquarks, and gluons, for which the energy density is

$$\varepsilon^{(0)}(T)=\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{16p}{\exp(p/T)-1}+\sum\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{12\varepsilon_p}{\exp(\varepsilon_p/T)+1}=\frac{8\pi^2T^4}{15}+\frac{7\pi^2N_kT^4}{20}. \qquad (19)$$

The second equation in this expression neglects the quark masses. General formulas can be found in the

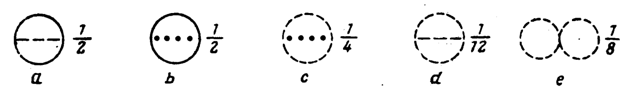

FIG. 4. Second-order graphs for thermodynamic parameters of hot plasma.

paper by Landau and Belen'kiĭ.[21]

At zero chemical potential, the potential Ω, for which the diagram technique is developed, coincides with the free energy $F(t)$. Graphs of second order in g are shown in Fig. 4, and if we use the "cross rule" (see Appendix), we can verify that the graphs include a whole series of physical effects (Fig. 5) such as the exchange interaction between quarks and antiquarks, the direct interaction along the annihilation channel (Fig. 5a), the direct interaction between quarks and gluons (Figs. 5b and c), and the exchange interaction between gluons (Figs. 5d–f). The derivation is relatively laborious but standard, and the final result is

$$\delta F^{(2)}=\pi\alpha_sT^4\left(\frac{N_k}{6}+\frac{2}{3}\right). \qquad (20)$$

Corrections of fourth order in g are of two types and contain two or three Green functions with a "cross" (see Appendix). The first type consists of radiation corrections to the above processes and, as in Sec. 3, they are dominated by the square diagram which provides a contribution of the order of $\delta F \sim \alpha_s^2 T^4 \ln^2\alpha_s$. The second type corresponds to the multiparticle forces and contains diagrams which have a power-type divergence in the infrared. Thus, as $|\mathbf{p}_1-\mathbf{p}_2| \to 0$, the graph of Fig. 6a provides the contribution

$$\delta F=2g^2\int\frac{d^3\mathbf{p}_1d^3\mathbf{p}_2}{p_1p_2(2\pi)^6}\times\frac{p_1p_2+\mathbf{p}_1\mathbf{p}_2}{[\exp(p_1/T)+1][\exp(p_2/T)+1]}\times\frac{\Pi_{00}^{(\mathrm{med})}(\mathbf{p}_1-\mathbf{p}_2)}{(\mathbf{p}_1-\mathbf{p}_2)^4}, \qquad (21)$$

since $\Pi_{00} \to$ const in (8) as $|p_1-p_2| \to 0$. It is clear that, by virtue of (10), this type of divergence does not occur for the transverse gluons. As in ordinary plasma,[22] the sum of a series of divergent diagrams leads to the replacement of one of the Green functions by the complete function, $1/q^2 \to 1/(q^2-\Pi_{00})$, and this removes the divergence. The contribution of the region of small momenta ($k^2 \sim \Pi_{00}$) in graphs a, b is the so-called correlation energy[2)]

$$\delta F=-\frac{g^3T^4}{\pi^3}\left(1+\frac{N_k}{6}\right)^{1/2}\left[(3N_k-24)\zeta(3)+4\pi^2-\frac{\pi^2}{3}N_k\right]. \qquad (22)$$

Collecting together the calculated contributions, and

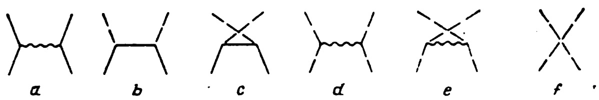

FIG. 5. Physical processes corresponding to the contributions of graphs in Fig. 4 with two "crosses."

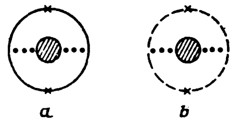

FIG. 6. Graphs containing power-type infrared divergences and leading to correlation energy. Shaded circle corresponds to the sum of graphs in Figs. 1*a–d*.

evaluating the energy densities, we obtain [$\alpha_s = \alpha_s(T)$]:

$$\varepsilon = T^4\left[\frac{8\pi^2}{15} + \frac{7\pi^2 N_k}{20} - \frac{\pi\alpha_s N_k}{2} - 2\pi\alpha_s - 1.3g^2\right]. \tag{23}$$

This expression[3)] is shown in Fig. 7, where $\epsilon_0 = \pi^2 T^4/30$ and can be interpreted as the energy per spinless particle. The ratio ϵ/ϵ_0 is convenient because it is a measure of the effective number of degrees of freedom, for example, for thermal Planck radiation, it is equal to two, i.e., the number of photon polarizations. Curve 1 corresponds to a pion gas, and curves 2 and 3 to the hadron-gas model,[11] including both stable particles and resonances.

Comparison of these curves leads to the conclusion that quark collectivization occurs at $T = 0.4$-0.6 GeV. It is interesting to note that, according to our previous papers,[11] the range of validity of this approach is set by the condition $T \lesssim m_\rho$, where m_ρ is the ρ-meson mass (the exchange of this meson is responsible for most of the forces between the mesons), and this is in agreement with the quark collectivization limit.

The necessity for the fast increase in $\epsilon(T)$ at $T = 0.1$–0.6 GeV was predicted earlier[11] but, from the point of view of chromodynamics, it has a very simple interpretation: this is the region in which we have the transition from the pion gas with three degrees of freedom to the hadron plasma with about forty degrees of freedom. At the same time, it becomes obvious that the approach of Hagedorn and Ranft and of Bugriĭ and Trushevskiĭ[23] is inconsistent, since they tried to extrapolate the Bethe-Uhlenbeck method to arbitrarily high densities and temperatures.

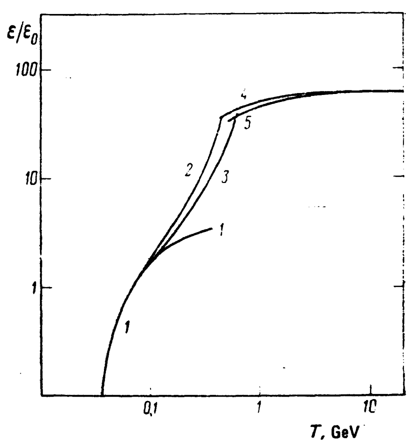

FIG. 7. The ratio ϵ/ϵ_0 as a function of temperature (ϵ is the energy density in hot plasma and $\epsilon_0 = \pi^2 T^4/30$ is the energy density per massless and spinless particle). 1 – gas of pions; 2, 3—gas of hadrons, both stable and resonance[11]; 4,5 —hadron plasma with $m_0 = 0.4$ and 0.2 GeV.

The above results can be used in cosmology to describe the expansion of the Universe at different stages of evolution. The usual equation of state $p = \epsilon/3$, where p is the pressure, is valid in our approach for $T > 0.8$ GeV and for $T < 0.1$ GeV (where photons dominate), whereas, in the intermediate region, $p \approx 0.2\epsilon$ and the expansion of the Universe is somewhat accelerated.

The other area in which these results can be used is much closer to experimental physics: it is the theory of hadron collisions at high energies. Our results show that the results reported by Landau[24] (see also the review by Feĭnberg *et al.*[12]) with the equation of state $p = \epsilon/3$ are valid only for incident particle energies $E_{\text{lab}} \gtrsim 10^4$ GeV, whereas, at most, the initial temperature reaches 0.7 GeV. At lower energies, including those accessible to existing accelerators, the equation of state $p = 0.2\epsilon$ is in better agreement with experiment.[12] We have also given[26] other arguments in favor of low initial temperature. These were based on the "tails" of the spectra of particles with high transverse momenta. The explanation of this again involves the inclusion of a large number of degrees of freedom.

In conclusion, we give one further example of the application of the above theory. Feĭnberg[27] has pointed out that thermal fluctuations in the medium, produced during hadron collisions, may generate lepton pairs e^+e^- and $\mu^+\mu^-$. We can readily use our approach to calculate the rate of this process, i.e., the probability of creation of a lepton pair per unit time per unit volume of plasma consisting of u, d, and s quarks:

$$\frac{dW_{\mu^+\mu^-}}{d^4x} = \frac{\pi\alpha^2}{108}T^4. \tag{24}$$

The rate of production of photons, charmed quarks, and so on, can be calculated similarly.

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APPENDIX

The technique of temperature Green functions[13–15] automatically takes into account the contribution of real particles in the medium as well as the contribution of virtual particles (i.e., radiative corrections). In this Appendix, we shall consider a simple method of separating these two contributions, which simplifies derivations and facilitates interpretation. This problem does not arise in traditional applications of this technique because the nonrelativistic approximation is then adequate and does not involve the radiative corrections.

It is well known that the above technique differs from standard Feynman rules only by the replacement of integrals over frequencies by sums over n, where $\omega_n = i\pi T(2n+1)$ (Fermi statistics) or $\omega_n = 2\pi i Tn$ (Bose statistics). To be specific, we confine our attention to the first case. Standard replacement of a sum by an integral gives[15]

$$T\sum_n \Phi(\omega_n) = \frac{1}{2\pi i}\int_c dz\, f^{\pm}(z)\,\Phi(z),$$

where $f^{\pm}(z) = \pm[1+\exp(\mp z/T)]^{-1}$ and the contour of integration c lies on both sides of the imaginary axis. We now represent $\Phi(z)$ by the sum $\Phi_+(z)+\Phi_-(z)$, where $\Phi_+(z)$ has singularities only in the right half-plane and $\Phi_-(z)$ only in the left half-plane. In the integral including Φ_+, we take f^- and take the right-hand side of the contour c to $+\infty$. The singularities (poles) of $\Phi_+(z)$ give the contributions

$$-\sum_a \operatorname{res} \Phi_+(z_a) f^-(z_a).$$

In the integral of this term over the left-hand part of the contour c, we put $f^- = f^+ - 1$. The term involving f^+ is zero because the contour can be shifted to $-\infty$ without encountering singularities, and there remains the integral

$$\int_{-i\infty}^{i\infty} dz\, \Phi_+(z)$$

over the imaginary axis, which is nonzero if $\Phi_+(z)$ does not fall fast enough in the left-hand half-plane. The result is that the original sum can be written in the form

$$\sum_n \Phi(\omega_n) = -\sum_a \operatorname{res} \Phi_+(z_a) f^-(z_a) + \sum_b \operatorname{res} \Phi_-(z_b) f^+(z_b) + \int_{-i\infty}^{i\infty} \Phi(z) \frac{dz}{2\pi i}.$$

These three terms represent the contributions of particles in the medium, antiparticles (holes), and radiation corrections. We note that the Feynman rules for integrating around poles reappear when the last integral is taken along the real rather than the imaginary axis.

The above result can also be formulated in the form of the following "cross rule": the result of summation over frequencies in each loop in a diagram is the integral over this frequency of the same expression plus the sum of terms in which one of the Green functions in the loop is replaced by singular expressions which we indicate by lines with crosses. In cold plasma, the crosses appear only on quark lines and correspond to

$$G_{(p)}^{(\times)} = \frac{\hat{p}+m}{2\varepsilon_p} 2\pi i \delta(\varepsilon-\varepsilon_p)\theta(\mu-\varepsilon_p).$$

and, in hot plasma, they appear only on the quark and gluon (non-Coulomb) lines, in accordance with

$$G_{(p)}^{(\times)} = \frac{\hat{p}+m}{2\varepsilon_p}\left[\frac{2\pi i\delta(\varepsilon-\varepsilon_p)}{\exp[(\varepsilon_p-\mu)/T]+1} + \frac{2\pi i \delta(\varepsilon+\varepsilon_p)}{\exp[(\varepsilon_p+\mu)/T]+1}\right],$$

$$D_{mn}^{(\times)ab}(\omega, \mathbf{k}) = \frac{2\pi i\delta(\omega^2-\mathbf{k}^2)}{2\omega} \frac{\delta^{ab}(\delta_{mn}-k_m k_n/\mathbf{k}^2)}{\exp(\omega/T)-1}$$

ADDENDUM (October 18, 1977)

In a recent paper, Kislinger and Morley[28] discuss the interaction between gauge fields for $T \neq 0$. They calculate the plasma frequency, i.e., the limit $k=0$, $\omega \to 0$, and not the effective mass, as we have done. The validity of this result is not clear because the initial polarization operator depends explicitly on the chosen gauge (Sec. 2).

The appearance of nonzero effective mass is contrasted by Kislinger and Morley[28] with the conclusions of Kirzhnits *et al.*[29] about the long-range nature of weak interactions in theories with spontaneous symmetry violation at $T > T_c$, where T_c is the point of phase transition with the restitution of symmetry. This important problem requires further analysis.

In the above papers,[29] the long-range interaction is, in fact, the static limit $\omega = 0$, $k \to 0$, whereas the effective mass corresponds to $\omega = k$. It thus turns out that the situation is precisely the same as in electrodynamics, e.g., in superconductivity. For $T > T_c$, the "electric" fields are screened off and the "magnetic" are not. It is clear, however, that the usual plasma screening does not solve the problems encountered in the case of the uncompensated weak charge.[29]

There have also been papers by Chapline *et al.*[30] on cold plasmas and neutron stars. To the extent to which they overlap our work, their conclusions are in agreement with ours.

1) We note that the condition T, $\mu \gg m_0$ is also sufficient to enable us to neglect the quark masses $m = m_d < m_s \sim 0.3$ GeV. Whenever the charmed quark with $m_c \sim 1.5$ GeV is important, it is taken into account but, for simplicity, we reproduce only the ultrarelativistic expressions.

2) In this formula, the charge must be taken to be $g(gT)$, in contrast to the previous formulas. This contribution was calculated incorrectly in the preprint version of this paper.

3) The last term in (23) is not included. From this coefficient onward, the perturbation theory series coefficients are found to increase rapidly and the series diverges.

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Abnormal states of nuclear matter and π condensation

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It is shown within the framework of relativistic field models of the πN interaction that the instability of nuclear matter to π condensation becomes stronger when the Fermi velocity tends to the relativistic limit. The results agree with Migdal's theory and point to the need for taking π condensation into account in the Lee and Wick model for abnormal states of atomic nuclei.

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1. INTRODUCTION

There have been discussions in recent years of the possible existence, at nuclear density, of an energy barrier whose surmounting (e.g., in collisions of heavy nuclei) may be the next step towards a genuine ground state of a system of N nucleons. Thus, for example, calculation of the dependence of the nuclear energy on the effective mass M^* of the nucleon, carried out in the σ model by Lee and Wick[1] (see also [2]) points to the existence of a local energy minimum at $M^* \approx 0$. An increase in the density of nuclear matter may make this minimum absolute[2]—the nucleus may go over via a relativistic phase transition into an abnormal state with $M^* \approx 0$.

On the other hand, Migdal's theory of π condensation[3] (see also later papers by Migdal and co-workers[4]) predicts, at a certain density, the onset of an inhomogeneous classical pion field in the ground state of nuclear matter.

The possibility of π condensation was not considered by Lee and Wick in connection with the problem of abnormal states. The formation of a π condensate in nuclear matter was investigated later, within the framework of the σ model of strong interactions, by Dashen, Campbell, and Manassah.[5] The nonrelativistic approximation used by them does not explain, however, the role of π condensation in the model of abnormal states with $M^* \approx 0$.

We have investigated the stability of nuclear matter to the appearance in it of a classical pion field, using the relativistic quasiclassical approach employed by Lee and Wick. In this approach the solution of the problem is similar to finding the energy $\epsilon = -(1/2)\chi H^2$ of an electron gas in an external magnetic field (χ is the magnetic susceptibility). It is known that the susceptibility of an electron gas is positive, but since the electromagnetic-interaction constant is small, the susceptibility is small compared with unity. Therefore the decrease of the energy of a metal in an external magnetic field is small compared with the self-energy $H^2/8\pi$ of the field.

The situation changes in theories with strong constants. In particular, the gain in the energy of nucleons situated in a classical pion field can exceed the self-energy of the field and may favor the formation of the π condensate.

Starting with Dirac's equation, we find the energy of a relativistic nucleon in an inhomogeneous classical ($\langle\pi\rangle \neq 0$) pion field of small amplitude. We construct next

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