Observation of magnon interaction in an antiferromagnet

A. I. Smirnov

Physical Problems Institute, Academy of Sciences, USSR

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1. INTRODUCTION

The object of the research was to observe the interaction of two magnons with definite values of the frequency and of the wave vectors. The idea for solving this problem experimentally was based on the possibility of parametric excitation of magnons in specimens of antiferromagnetic MnCO₃ and CaMnF₃ by microwave pumping. The phenomenon of parametric excitation of spin waves in antiferromagnets has been studied both experimentally and theoretically and consists of the following.

When the magnetic microwave pumping field h_p with frequency ω_p exceeds the threshold value ω_t (which is determined by relaxation of magnons), a parametric instability begins to develop in the specimen: the number of magnons with frequency ω_m = ω_t/2 increases. When the density of parametrically excited magnons (PM) reaches a value exceeding the thermal background by two or three orders of magnitude, there occurs a discontinuous increase of the rate of growth of the number of PM, due apparently to saturation of one of the mechanisms of relaxation and turning off of a part of the relaxation. The nature of this turning off is not yet clear. Together with growth of the number of PM, the power absorbed by them also grows. After this pump, the absorbed power reaches a value that can be recorded experimentally (see Ref. 1). Thereafter, the PM system reaches a stationary state with a constant value of the absorbed power.

As was shown in Ref. 4, the limitation of the number of PM and the establishment of a stationary state occur as a result of a shift of phase of the PM with respect to the phase of the pumping. The stationary state is characterized by a new value of the threshold field, h_t < h_t.; (When k ≪ h_p, parametric excitation of magnons stops.) Parametric excitation entailing the presence of two threshold fields is called "hard." [1, 2]

From the values of the threshold fields, one can calculate the corresponding magnon relaxation frequencies Δω and Δω. For CaMnF₃, a specimen of which was studied in the present paper, under the condition ω_p < ω_m = 115 GHz, this relation is

\[ \Delta\omega = 2\gamma/\hbar v, \]

where \( \gamma \) is the external magnetic field and \( v, \) the gyromagnetic ratio. The values of Δω₁ and Δω₂ differ by 5 to 10 times depending on the static magnetic field and the temperature; their order of magnitude is 0.1 MHz, which corresponds to a magnon lifetime \( 1/2\Delta\omega = 1 \) μsec. The occupancy number of the PM in the stationary state exceeds the occupancy number of thermal magnons at \( T = 1 \) to 2 K by five or six orders of magnitude. The wave vector of the PM is determined by the condition of parametric resonance

\[ \langle v_p^2 \rangle = \langle v_m^2 \rangle \]

and the spin-wave dispersion law

\[ \langle v_p^2 \rangle = \left[ (\omega_p^2 - H^2) v_m^2 \right] \]

where \( \omega_p^2 = 6.3/\Gamma [\text{kOe}] \) is the gap caused by hyperfine interaction, and \( \alpha = 0.95 \times 10^5 \text{ cm} \) is the numerical value of the relaxation. By varying \( H \) over a range that permits satisfaction of (2) with allowance for (3), one can excite PM with wave vector from 0 to \( \times 10^5 \text{ cm}^{-1} \) by use of pumping in the centimeter range.

Parametric excitation of magnons is recorded experimentally on the basis of absorption of microwave power in the specimen. For observation of the interaction of magnons of different frequencies, use was made of a target on the specimen by two microwave pumping, from generators operating at different frequencies. The first pumping, at frequency \( v_p \), excited parametrically, in the specimen, magnons of frequency \( v_m = v_p/2 \) in a number greatly exceeding (by a factor of about 10⁶) the level of thermal magnons at this frequency. Interaction with these PM changes the relaxation frequency of the other magnons. The change of relaxation frequency of magnons of frequency \( v_m \) can be detected by measuring the threshold for parametric excitation of them. For this...
the influence of radiation from klystron

parametric excitation

old value of the

on the basis of the signal of detector

Radiation from the klystron

pumping fields with frequencies

circulator

gives reason to expect manifestations of other processes

attenuator

occurs after turning on of klystron

netic field of the

liquid helium in the superfluid state. The microwave

change of the threshold field, \( \Delta \phi \), under the action of the first pumping,

Fig. 1. The specimen was placed at a loop of the mag-

2. METHOD

A schematic of the experimental setup is shown in

Fig. 1. The specimen was placed at a loop of the mag-

field of the \( H_{12} \) mode of a cylindrical resonator

the microwave pumping fields with frequencies \( \nu_p \) and \( \nu_m \) were produced

with the resonator by the klystron generators \( G_p \) and \( G_m \).

Radiation from the klystron \( G_m \) excites in the specimen

PM with frequency \( \nu_m \): the resulting absorption of mi-

crowave power is recorded on the basis of the magni-

tude of the signal of detector \( D_m \). Parametric excitation

of spin waves with frequency \( \nu_m \), which occurs under

the influence of radiation from klystron \( G_m \), is recorded on the basis of the magni-

tude of the signal of detector \( D_m \), and the threshold-

value of the power is determined with the precision

attenuator \( A_1 \). From the scale of \( A_1 \) is read the relative change of the threshold field, \( \delta = (\Delta \phi_m - \Delta \phi_m^0) / \Delta \phi_m^0 \), that

occurs after turning on of klystron \( G_m \) \( (\Delta \phi_m^0) \) is the thresh-

old field under the action of the first pumping, \( \Delta \phi_m^0 \) without it. The ferrite circulators \( FC_1 \) and \( FC_2 \) insure de-

coupling of the waveguide lines and the klystrons. The
circulator \( FC_1 \) is used to decouple the resonator and the
$H = 2.4$, with excitation of parametric spin waves with frequency $\nu_2 - \nu_1/2$, $\nu_2 = 35.73$ GHz. The experimental points correspond to $H$ in kOe: $\Delta \nu = \nu_2 - \nu_1$ in MHz: $\nu_2 = 2.4$, $\Delta \nu = 10$; $H = 3.5$, $\Delta \nu = 50$; $H = 3.6$, $\Delta \nu = 500$; $H = 4.6$, $\Delta \nu = 5000$.

and does not depend explicitly on temperature. Then from (4)-(6) and (1) follows

$$N = \frac{\Delta H}{H} A(H) \left( \frac{h_2}{h_1} \right)^{1/2}.$$

The coefficient of proportionality in (7) for CsMnF$_3$ depends only on the pumping frequency. Formula (7) enables us to calculate $N_2$ from the value of $(h_2/h_1)^{1/2}$ measured in the course of the experiment. The results of the experiment—the relative changes of threshold field $\Delta H = (h_2' - h_2)/h_2$—were plotted as a function of $[(h_2/h_1)^{1/2} - 1]^{1/2} = \nu_2' - \nu_1'$. This dependence is shown in Fig. 2 for several combinations of the pumping frequencies $\nu_2$ and $\nu_1$.

In order to understand the observed phenomenon, it was necessary to explain whether the observed change of the threshold field $h_2'$, which according to (1) is proportional to the relaxation frequency $\Delta \nu = \Delta \nu_1 + \Delta \nu_2$, is determined by the change of the part $\Delta \nu_1$ of the relaxation, of the part $\Delta \nu_2$, or of both of these quantities. The change of the value of $\Delta \nu_2$ upon action of the first pump upon the crystal can be independently determined by measurement of the "second" threshold $h_4'$.

As the result of measurement of the threshold field $h_4'$ with simultaneous measurement of $h_2'$, it was clarified that the decrease of $h_2'$ under the influence of FM when $|\nu_2 - \nu_1| > 3$ GHz, at small excesses over the threshold field of the first pump, is caused by change of $\Delta \nu_1$. When $|\nu_2 - \nu_1| > 3$ GHz, the threshold fields $h_2'$ and $h_4'$ increase equally under the influence of FM, within the limits of measurement error, $z 2\%$. Thus when $|\nu_2 - \nu_1| > 3$ GHz, the result of excitation of FM with frequency $\nu_2$ is that only the relaxation frequency $\Delta \nu_1$ changes, while $\Delta \nu_2$ remains constant. The change of the value of $\Delta H$ with increase of the power of the first pump when $|\nu_2 - \nu_1| < 3$ GHz (see Fig. 2) is obviously caused by both effects: a decrease of $h_2'$ because of decrease of $\Delta \nu_1$ and an increase of $h_4'$ because of increase of $\Delta \nu_2$.

The threshold field $h_4'$ is measured by observation of the signal from detector $D_{m}$ on the oscilloscope screen and can be measured with accuracy no better than 2%. Therefore the determination of the value of $\delta$ by measurement of $h_4'$ is made with accuracy $\pm 0.05\%$. This worsening of the accuracy does not affect the qualitative deductions that follow from this investigation.

As is seen from Fig. 2, when $|\nu_2 - \nu_1| > 3$, over the whole accessible range of power of the first pump, there was observed a linear increase of the relaxation frequency of spin waves at frequency $\nu_2 = \nu_2' > \nu_2$ with the number of PM:

$$\Delta \nu = \phi N_2.$$

The quantity $2\nu_2 > \nu_2'$ is the probability per unit time of an interaction of magnons with frequencies $\nu_1$ and $\nu_2$.

To illustrate the dependence of the effect on the static magnetic field, values of $\delta$ at various magnetic fields, but at a single value of $(h_2/h_1)^{1/2}$, are plotted in Fig. 3.

In order to treat the dependence of $\phi$ on temperature and magnetic field, however, it is necessary to take into account that the curves of Fig. 3 require rescaling, because the coefficient of proportionality between $N_2$ and $[(h_2/h_1)^{1/2} - 1]^{1/2}$ contains $\Delta \nu_2^0/H^2$ and depends on temperature and magnetic field (see (7)). The quantity obtained from experiment is $\delta$, which is connected with $\Delta \nu_2$ in accordance with (1):

$$\Delta \nu_2 = \delta \Delta \nu_2.$$

From (7)-(9) we have

$$\phi = \frac{\delta \Delta \nu_2}{\Delta \nu_2} = \frac{\delta}{\Delta \nu_2} \frac{\Delta \nu_2}{\Delta \nu_2} = \delta \Delta \nu_2 (H, T),$$

that is, for fixed $h_2/h_1$ (the experimental data shown in Fig. 3) were obtained under this condition

$$\phi \Delta \nu_2 = \frac{\delta \Delta \nu_2}{\Delta \nu_2} \Delta \nu_2 = \delta \Delta \nu_2 (H, T).$$

For the specimen under investigation, the field and temperature dependences of $A(H)$, $\Delta \nu_2$, $\Delta \nu_2$, and $\Delta \nu_2$ were measured. From them, values of the function $A(H)$ were calculated and graphs of the function $A(H)$ were plotted (Fig. 4).
4. DISCUSSION OF RESULTS

1. As a result of excitation of PM with frequency $v_{10}$, there occurs an increase of the relaxation frequencies $\Delta v_{1}$ and $\Delta v_{2}$ of magnons with frequency $v_{30}$ by an amount $\Delta v'$ that is proportional to the number of PM. The increase $\Delta v'$ of the relaxation amounts to about 0.1 $\Delta v_{1}$; that is, to about 0.3 MHz.

2. The coefficient of proportionality $\phi$ between $\Delta v'$ and $N_{p}$ is independent of temperature over the interval 1.6 to 2.1 K.

3. The coefficient $\phi$ varies with magnetic field as is shown in Fig. 4.

4. DISCUSSION OF RESULTS

1) We consider the case $|v_{2} - v_{3}| < 3$ GHz.

The decrease of the threshold $h_{0}$ under the influence of the first pump, at small frequency differences $|v_{2} - v_{3}|$, has an appreciable value even at small excesses over the threshold field: $h_{0}/h_{0}=1 \pm 0.1$. When $|v_{2} - v_{3}| < 10$ MHz, $h_{0}^{'}/h_{0}$; that is, the presence of magnons of the second frequency (differing from the frequency of the magnons under study by no more than 5 MHz) leads to practically complete turning off of the part of the relaxation, that is subject to turnover; and when the frequency difference of the magnons is 50–1000 MHz, to a partial turning off. The variation of the turned off part $\Delta v'$ of the relaxation with the frequency of the additional (in relation to the thermal distribution) magnons, for fixed $h_{0}/h_{0}$, is shown in Fig. 5.

The decrease of the "first" threshold for parametric instability, at frequency $v_{30}$, under the influence of PM of frequency $v_{2}=v_{3}/2$, when $|v_{2} - v_{3}| < 3$ GHz, is similar to the decrease of the threshold in the course of the nonstationary stage of parametric excitation of magnons, described in the Introduction: with increase of the number of spin waves, the relaxation decreases, and with it the threshold field also decreases, from $h_{0}$ to $h_{0}^{'}. In the present situation, the threshold field decreases from $h_{0}$ to $h_{0}^{'}, because of the presence of magnons of the second frequency. In Ref. 1, characteristic magnon occupancy numbers were determined, at which the number of excited magnons becomes independent of temperature over the interval 1.6 to 2.1 K.

By use of these data and also of the results shown in Fig. 5, a frequency characteristic of this type can be established for the mechanism of turning off of part of the relaxation. For threshold excesses $h_{0}/h_{0}^{'},$ the number $N_{p}$ of PM in the stationary state corresponds to occupancy numbers $n_{p}=10^{9}$. With such a number of PM of frequency $v_{3}/2$ when $|v_{2} - v_{3}| < 3$ GHz, a part of the relaxation of magnons of frequency $v_{1}/2$ begins to be turned off. For $v_{2}=v_{3}$ (the case of a single pump), this occurs, as follows from Ref. 1, when $n_{p}=10^{3}$. From this it may be concluded that in the presence of detuning of the magnon frequencies by $\sim 1$ GHz, the same decrease of relaxation requires $10^{3}$ as many magnons as when the magnon frequencies coincide.

2) We now consider the results obtained when $|v_{2} - v_{3}| > 3$ GHz, and enumerated at the end of Sec. 3.

The increase of relaxation frequency when a second microwave pump acts on the crystal cannot be caused by superheating of the spin system of the specimen, for the following reason. In the case of superheating, there would occur a sharp increase of $\Delta v'$ with increase of magnetic field (for fixed power $P_{PM}$ absorbed by the spin system), because with increase of the magnetic field a stronger temperature dependence of $\Delta v_{1}$ is observed. But according to the data presented in Fig. 5 and the $\Delta v_{1}(H)$ dependence, a decrease rather than an increase of the value of $\Delta v'$ occurs, when the high-field range. In this case an increase of the relaxation must be caused by interaction of magnons with frequency $v_{3}$ and PM with frequency $v_{2}$.

We shall consider two types of possible processes:

a) These two magnons fuse into a third quasiparticle:

\[ (12a) \]

b) These two magnons form a single quasiparticle:

\[ (12b) \]
b) These two magnons are transformed into two quasiparticles:

\[ \begin{array}{c}
\text{magnon 1} \\
\text{magnon 2}
\end{array} \rightarrow 
\begin{array}{c}
\text{phonon} \\
\text{magnon}
\end{array} \tag{12b} \]

Processes involving participation of a larger number of particles are less probable. Consideration of the probabilities of direct and inverse transitions for the processes (12a) and (12b) gives the following dependences on the number of particles for the frequency \( \Delta \nu ' \) of relaxation occurring because of these processes:

\[ \nu ' = \frac{A}{m} \sum (\Phi_{\text{magnon}} - \Phi_{\text{phonon}})(h\nu_1 + h\nu_2 + h\nu_3 + h\nu_4)/hN, \]

since the thermal occupancy numbers \( n_3 \) are much smaller than \( n_4 \).

b) \( \nu ' = \frac{A}{m} \sum (\Phi_{\text{magnon}} - \Phi_{\text{phonon}})(h\nu_1 + h\nu_2 + h\nu_3 + h\nu_4)/hN \)

for the same reason. Here \( \Phi_{12,36} \), \( \Phi_{23,36} \), and \( \Phi_{36,41} \) are the amplitudes of the corresponding interactions.

Of processes of the type (12a), only fusion of two magnons into a phonon is allowed by the conservation laws. Of processes of the type (12b), the ones allowed are transformations of two initial magnons into two magnons of the low-frequency branch or into two phonons, or into a phonon and a magnon.

Relaxation caused by any of the processes of type (12b) should depend substantially on temperature, since it depends on the thermal occupancy numbers \( n_3, n_4 \) at \( T = 1-2 \) K. The observed constancy of the effect on change of temperature indicates that the main contribution to the phenomenon apparently comes from fusion of two magnons into a phonon.

Satisfaction of the conservation laws for this process is possible when

\[ k + h\nu_4 = \frac{\hbar v_0}{C_1} \]

\[ \text{where } C_1 \text{ is the velocity of sound waves of a definite type; this is satisfied in the magnetic-field range } H > H_f^* \text{ for fusion into a transverse phonon, and in the range } H < H_f^* \text{ for fusion into a longitudinal phonon. The velocities of sound are } C_1 = 2.31 \cdot 10^3 \text{ cm/sec and } C_2 = 4.16 \cdot 10^3 \text{ cm/sec for transverse and longitudinal phonons propagated in the easy plane of the crystal.} \]

Analysis of the magnon-phonon interaction\(^\text{11}\) shows that magnons can interact only with these acoustic oscillations. The fields \( H_f^* \) and \( H_f^* \) are shown on the graphs of Fig. 4. The values of \( H_f^* \) and \( H_f^* \) are determined with accuracy \( \pm 10\% \), since experimental values of \( \alpha \) of \( C_1 \) are used. In the graphs of Fig. 4, we notice a drop of the effect when \( H > H_f^* \).

At magnetic field 3.9 kOe, when \( \nu_3 = 35.73 \) GHz, \( \nu_4 = 26.30 \) GHz, and \( T = 1.6 \) K, a sharp peak is observed in the probability of interaction of magnons with frequencies \( \nu_4 \) and \( \nu_4 \). This value of \( H \) is characteristic of the interaction of the magnons under study when phonons participate, since at this magnetic field the intersection of the magnon spectrum with the spectrum of transverse phonons occurs at frequency \( \nu_4 \), and with the spectrum of longitudinal phonons at frequency \( \nu_4 \); that is, at the frequencies of the magnons under study. This peak is not observed at another combination of \( \nu_3 \) and \( \nu_4 \) (Fig. 4b), when the spectrum intersections mentioned correspond to the frequencies \( \nu_4 \) and \( \nu_4 \) at different values of \( H \) because of the temperature dependence of the magnon spectrum (3). Evidently under the conditions when a peak is observed (Fig. 4a), the probability of four-particle processes in which phonons participate has a resonance maximum.

On the basis of the analysis given and of the data of the experiment described, one can estimate the square of the amplitude of interaction of two magnons when they fuse into a phonon. Since \( \Delta \nu ' = 0.01 \) MHz and in 1 cm\(^3\) of the specimen \( N_p = 10^{11} \), we get for a specimen of volume 1 cm\(^3\)

\[ \left| \Phi_{\text{magnon}} \right|^2 = 10^{-11} \text{ cm}^3. \]

The results obtained enable us to estimate \( \Delta \nu (\nu_1 + \nu_2 - \nu_3) \), the contribution to magnon relaxation from the process of fusion of two magnons into a phonon \( (\nu_1 + \nu_2 - \nu_3) \), and also the value of the spin-lattice relaxation \( \Delta \nu \). Here it should be mentioned that PM of frequency \( \pm 20 \) GHz possess an energy of the order of the mean energy of thermal magnons, and their number is of the order of the total number of thermal magnons, at \( T = 1-2 \) K. Hence the relaxation frequency of magnons in the processes \( \nu_1 + \nu_2 - \nu_3 \), with participation by PM in the role of \( \nu_3 \), must be of the order of the contribution to relaxation from these processes with participation of thermal magnons. Thus

\[ \Delta \nu (\nu_1 + \nu_2 - \nu_3) = 3 \Delta \nu ' = 0.01 \text{ MHz}. \]

In its turn, the spin-lattice relaxation \( \Delta \nu \) is not less than \( \Delta \nu (\nu_1 + \nu_2 - \nu_3) \), since this process is one of the processes that insure transfer of energy from the magnons to the lattice. At the same time, \( \Delta \nu \) does not exceed the total relaxation of magnons \( \Delta \nu ' = 0.1 \) MHz. Therefore the quantity 0.01 MHz correctly characterizes \( \Delta \nu \) in order of magnitude.

The estimate of the contribution to magnon relaxation from processes of fusion of two magnons into a phonon, obtained from the experiment described, is in agreement with a theoretical calculation\(^\text{12}\) in which the probabilities of various magnon-phonon processes in an antiferromagnet and their contribution to the magnon relaxation are considered.

By use of \( \Delta \nu \), one can estimate the superheating of the spin system in the excitation of PM. For the ex- ceeds over the threshold field used, \( \hbar /k_{\text{B}} = 2 \), at \( T = 1.6 \) K, the superheating of the spin system is of order 0.1 K.
Thus in this work there has been observed an interac-
tion of two magnons of different frequencies, consisting
of fusion of them into a phonon. An estimate has been
made of the value of the gap-lattice relaxation and of
the superheating of the spin system in parametric ex-
citation of the waves.

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Interaction between vortices and the surface of a type II
superconductor and the field of a vortex in a cavity
A. T. Abramyan and V. V. Shmidt
Institute of Solid State Physics, Academy of Sciences USSR
(Submitted June 24, 1977)
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The response of a hollow, thin-walled Pb-30 at % In superconducting cylinder in the mixed state to an
external weak alternating magnetic field is investigated experimentally. It is found that vortices located
near the inner surface of the cylinder produce a magnetic field in the cavity of the cylinder. In response to
an external alternating field, the vortices adjacent to the outer surface of the cylinder execute reversible
oscillations until their amplitude reaches a certain critical value, which is proportional to the period of the
vortex lattice. This state corresponds to the flow of a critical current along the outer surface of the
superconductor. This state is illustrated in Fig. 1a, where a vortex is shown, whose core is located near
the plane surface of a superconductor. If the superconductor occupies the halfspace x > 0 then the magnetic
field is equal to zero in the region x < 0. Furthermore, the total magnetic flux which is created in the supercon-
ductor by such a vortex is not equal to the flux quantum \( \phi_0 \) but is equal to \( \phi_0 (1 - e^{-a}) \),

\[
\phi = \phi_0 (1 - e^{-a})
\]

where \( \phi_0 \) is the coordinate of the core of the vortex, \( \lambda \) is the penetration depth of the weak magnetic field. If now we close the open surface of the superconductor at a distance from the vortex that is large in comparison with \( \lambda \) (Fig. 1b), the picture changes radically. A su-
perconducting current immediately passes along the inner surface of the resulting macroscopic cavity, and a
magnetic field induced by the vortex develops inside the cavity. This follows both from the result of the calcu-