Acoustic breakdown in metals

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An analysis is made of the effects accompanying the propagation of sound in metals in which sections of the Fermi surface in different energy bands approach each other closely. It is shown that in such metals the propagation of an intense hypersound wave whose wave vector multiplied by Planck's constant is approximately equal to the distance between the respective sections of the Fermi surface, may appreciably alter the dynamics of the electrons in a magnetic field. In these circumstances open trajectories may become closed ones and vice versa. This phenomenon is termed acoustic breakdown. The energy spectrum of electrons under acoustic breakdown conditions is analyzed and the interaction of acoustic and magnetic breakdown studied. The change in the form of the electron trajectories as a consequence of acoustic breakdown should result in a number of effects, viz. a change in the smooth part of the asymtotic expression for the galvanomagnetic tensor, the appearance of new oscillation periods of the kinetic coefficients, and suppression of the nonlinearity in the absorption of sound. Estimate show that observation of these effects is within the scope of modern experimental techniques.

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I. INTRODUCTION

There are many known metals in which sections of the Fermi surface in different energy bands approach one another closely. The aim of the present work is to show that an appreciable distortion of electron trajectories in a magnetic field occurs under the action of a sound wave whose wave vector $\mathbf{q}$ multiplied by $\hbar$ is approximately equal to the distance between the respective sections of the Fermi surface in $\mathbf{p}$-space. As a result, open trajectories, for example, may be converted into closed ones, and closed ones into open ones. We shall term both phenomena acoustic breakdown. It seems that one of the simplest methods of studying this phenomenon experimentally is to observe the appearance of new Shubnikov oscillation periods under the influence of an intense sound wave, and also to investigate the monotonic part of the asymptotic form of the galvanomagnetic tensor (see[1,2]).

As will be clear from what follows, acoustic breakdown can occur only in combination with magnetic breakdown, since the periodic field of a sound wave cannot on its own give rise to transitions between trajectories. We shall see that in the presence of a sufficiently intense sound wave whose frequency ω is so chosen that $\hbar\mathbf{q}$ is approximately equal to the distance between the electron trajectories in different energy bands, magnetic-breakdown transitions occur in much weaker magnetic fields than in the absence of sound. Acoustic breakdown thus provides a method of quantitatively studying the electron spectra of metals near their singular points.

As is well known, an intense sound wave may lead to splitting of the electron trajectories even within a single energy band. This possibility was pointed out by Brandt *et al.*[3] An analysis of this case should proceed from the results of Keldysh,[4] who examined the problem of the electron spectrum in the periodic field of a sound wave[1)] in the one-energy-band approximation. Keldysh showed that in a coordinate frame bound to the moving field of the sound wave, the electron spectrum is a system of allowed and forbidden one-dimensional acoustic bands. He pointed out that the presence of acoustic bands must have an appreciable effect on the behavior of a crystal in an external electric or magnetic field. The possibility of the existence of a current state in acoustic-band conditions has been discussed by Rakhmanov.[5]

The influence of the periodic field of an intense sound wave on the electron spectrum will be investigated in Sec. 2. For methodological reasons, viz., in order to trace in detail the change from a stationary coordinate frame to a moving one and to elucidate the role of the electric fields arising in this change, it will be convenient for us to start from the one-energy-band case analyzed in[4]. We shall then proceed to the case of two close bands, which is of direct interest to us.[2)]

Everywhere in this paper we confine ourselves to the weak acoustic coupling approximation

$$2U=\lambda_{ik}u_{ik}^{(0)}\ll\hbar^2q^2/8m. \qquad (1.1)$$

The quantity on the left-hand side of this inequality is the characteristic energy of interaction of a conduction electron with the deformation field created by a sound wave (λ_{ik} is the deformation-potential tensor and $v_{ik}^{(0)}$ is the amplitude of the deformation tensor). In the case of a single energy band this quantity is the width of the first forbidden acoustic band. The quantity on the right-hand side of (1.1) is the width of the first allowed acoustic band.

In order to analyze the effects associated with the presence of an acoustic band spectrum it is obviously necessary for the inequality

$$U\gg\hbar/\tau \qquad (1.2)$$

to be fulfilled, where τ is the characteristic relaxation time of the conduction electrons. For $\lambda\sim10^{-11}$ erg and a sound intensity of the order of 1 W/cm^2, the quantity on the left-hand side of the inequality (1.2) amounts to $\sim10^{-16}$ erg. Thus, for relaxation times τ appreciably in excess of 10^{-11} sec the inequality (1.2) is fulfilled.

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2. THE ENERGY SPECTRUM OF THE CONDUCTION ELECTRONS

We shall analyze the problem of the motion of conduction electrons in the periodic field of a crystal lattice $V(\mathbf{r}')$ distorted by a sound wave propagating in the crystal. The corresponding single-electron Hamiltonian of the problem is

$$\mathscr{H}=\hat{\mathbf{k}}'^2/2m_0+V_0(\mathbf{r}'-\mathbf{u})+V'(\mathbf{r}'-\mathbf{u})+V_s(\mathbf{r}'-\mathbf{u}). \qquad (2.1)$$

Here $\hat{\mathbf{k}}'=-i\hbar\,\partial/\partial\mathbf{r}'$ is the electron momentum operator, V_0 is the periodic potential of the undeformed lattice, V' is the change in the periodic potential as a result of the deformation, and V_s is the potential of impurity atoms. It is evident that

$$V'(\mathbf{r}')=V_{ik}'u_{ik}, \qquad (2.2)$$

where $u_{ik}=\frac{1}{2}(\partial u_i/\partial x_k'+\partial u_k/\partial u_i')$ is the strain tensor and $\mathbf{u}(\mathbf{r}',t)$ is the displacement vector of the crystal lattice taking part in the acoustic oscillation.

Analogously as in[6] we shall perform a canonical transformation corresponding to a change to a coordinate frame L_u co-moving with the lattice. We shall choose a canonical-transformation operator of the form

$$\hat{T}=\exp\left(-\mathbf{u}\frac{\partial}{\partial\mathbf{r}'}\right)=\exp\left(-\frac{i}{\hbar}\mathbf{u}\hat{\mathbf{k}}'\right). \qquad (2.3)$$

The displacement $\mathbf{u}$ may depend on the space coordinate $\mathbf{r}'$ and consequently, may not, generally speaking, commute with the operator $\hat{\mathbf{k}}'$. Here and below we shall regard, without further reservations, all such expressions as symmetrized. In addition we shall consider the deformation as small:

$$|\partial u_i/\partial x_k'|\ll 1, \qquad (2.4)$$

assuming $q\ll a_0^{-1}$, where a_0 is the lattice constant, and we shall neglect the derivatives of the strain tensor u_{ik} everywhere.

The transformed energy operator has the form

$$\mathscr{H}_u=\hat{T}^{-1}\mathscr{H}\hat{T}-i\hbar\hat{T}^{-1}\frac{\partial\hat{T}}{\partial t}=\frac{\hat{\mathbf{k}}^2}{2m_0}+V_0(\mathbf{r})+V'(\mathbf{r})+V_s(\mathbf{r}) -\frac{1}{m_0}u_{il}\hat{k}_i\hat{k}_l-\dot{\mathbf{u}}\hat{\mathbf{k}}, \qquad (2.5)$$

where $\mathbf{r}=\mathbf{r}'-\mathbf{u}$ is the coordinate in the co-moving frame and $\hat{\mathbf{k}}=-i\hbar\partial/\partial\mathbf{r}$. The energy of interaction of the electrons with impurities thus becomes stationary after the transformation, i.e., it depends on the coordinate $\mathbf{r}$. We can define the amplitude f of electron scattering by impurities in this frame and this amplitude will be considered a known quantity in what follows. We shall therefore disregard the potential $V_s(\mathbf{r})$ in further transformations of the Hamiltonian, but the scattering at impurities can be accounted for directly in the collisional term of the kinetic equation.

We wish further to change to the well-known generalized-effective-mass method. The essence of this is that if the motion of an electron under the influence of perturbations which vary slowly in time and space is considered, the operator $\hat{\mathbf{k}}^2/2m_0+V_0(\mathbf{r})$ can be replaced by the operator $\varepsilon(-i\hbar\nabla)$, where $\varepsilon(\mathbf{p})$ is the dispersion law for an electron in the given energy band and $\mathbf{p}$ is its quasimomentum. The perturbation produced by the deformation is usually of this type. It can therefore be described by introducing a deformation potential $\lambda_{ik}(-i\hbar\nabla)u_{ik}$, where $\lambda_{ik}(\mathbf{p})$ is the average of the operator $-u_{ik}\hat{\mathbf{k}}_i\hat{\mathbf{k}}_k/m_0+V_{ik}'(\mathbf{r})$ over the Bloch wave function. The motion of an electron in the given band is thus described by the one-band Hamiltonian

$$\mathscr{H}_u=\varepsilon(\mathbf{p})+\lambda_{ik}(\mathbf{p})u_{ik}-m_0\dot{u}_iv_i; \qquad (2.6)$$

the strain tensor u_{ik} and the displacement vector $\mathbf{u}$ are functions of $\mathbf{r}-\mathbf{w}t$, where $\mathbf{w}$ is the phase velocity of the sound. Thus, in the L_u coordinate frame the Hamiltonian (2.6) is nonstationary. However, the Hamiltonian turns out to be stationary on changing to the L_w coordinate frame moving together with the sound wave with velocity w. Such a change can be effected by means of a canonical transformation with the operator

$$\hat{T}_1=\exp\left(\mathbf{w}t\frac{\partial}{\partial\mathbf{r}}\right). \qquad (2.7)$$

The Hamiltonian of the generalized-effective-mass method transformed to the L_w frame has the form

$$\mathscr{H}_w^{(e)}=\varepsilon(\hat{\mathbf{p}}_R)-\hat{\mathbf{p}}_R\mathbf{w}-m_0\mathbf{v}\dot{\mathbf{u}}(\mathbf{R})+\lambda_{ik}(\hat{\mathbf{p}}_R)u_{ik}(\mathbf{R}), \qquad (2.8)$$

where $\mathbf{R}=\mathbf{r}-\mathbf{w}t$, $\hat{\mathbf{p}}_R=-i\hbar\partial/\partial\mathbf{R}$. The term $-m_0\mathbf{v}\cdot\dot{\mathbf{u}}$ describes the Stewart-Tolman effect in the nonstationary field of a sound wave. Usually the contribution of this term is small compared to the last term, which is due to the deformation potential. In estimating the magnitude of the interaction of sound with the electrons below we shall therefore consider that it is determined principally by the last term.

Like the original Hamiltonian (2.1), the Hamiltonian (2.8) does not allow for the solenoidal electromagnetic fields which may arise during propagation of the sound. These fields can be neglected for sufficiently high-frequency sound with wavelength much less than the anomalous skin length.[7] We shall assume below that ultrasound frequencies satisfy this condition.

Satisfaction of inequality (1.1) means that one acoustic band is under examination, viz. the lowest forbidden one; the widths of all the other bands are small in terms of the parameter (2.8).

The presence of perturbations of the type $U(e^{i\mathbf{q}\cdot\mathbf{R}}+e^{-i\mathbf{q}\cdot\mathbf{R}})/2$ leads to interaction of the states $|\mathbf{p}-\hbar\mathbf{q}/2\rangle$ and $|\mathbf{p}+\hbar\mathbf{q}/2\rangle$. The Hamiltonian of the effective-mass method is therefore a 2×2 matrix whose nondiagonal elements are equal to $U(\mathbf{p})/2$. Now, in the framework of the generalized-effective-mass approximation, we may perform a reverse transformation to the L_u frame by means of the operator

$$\hat{T}_1^{-1}=\begin{pmatrix}1&0\\0&1\end{pmatrix}\exp\left(-\mathbf{w}t\frac{\partial}{\partial\mathbf{R}}\right). \qquad (2.9)$$

In such a transformation the nondiagonal matrix elements remain unchanged and we can thus investigate the spectrum of the Hamiltonian $\mathscr{H}_u$.

So far we have not allowed for the magnetic field in the Hamiltonian. We shall show that such an allowance will yield, inasmuch as we wish to trace how the trajectories of the electrons are distorted under the action of sound in a magnetic field. For this purpose the substitution

$$\hat{\mathbf{k}}'\to\hat{\mathbf{K}}'=\hat{\mathbf{k}}'-\frac{e}{c}\mathbf{A}(\mathbf{r}'), \tag{2.10}$$

should be made in the Hamiltonian, where $\mathbf{A}(\mathbf{r}')$ is the vector potential. If the same substitution is also made in the canonical-transformation operator (2.3), the result will be that the transformed Hamiltonian is obtained from (2.5) by a substitution analogous to (2.10), but with the difference that $\mathbf{A}$ depends on the argument $\mathbf{r}+\mathbf{u}$. It is, however, more convenient for us to deal with a Hamiltonian in which $\mathbf{A}$ is a function of $\mathbf{r}$. We consider a constant, homogeneous magnetic field: $\mathbf{A}$ is a linear function of its argument, and in that case a gauge transformation may be performed, as a result of which $\mathbf{A}$ becomes a function of $\mathbf{r}$, but the term $e\varphi_u$, which describes the induced electric field

$$\mathbf{E}_u=-\nabla\varphi_u=c^{-1}[\dot{\mathbf{u}}\mathbf{H}].$$

is added to the Hamiltonian. This field, although nonstationary in the L_u frame, becomes stationary in the L_w frame. In the latter frame, however, there appears an additional electric field

$$\mathbf{E}_w=c^{-1}[\mathbf{w}\times\mathbf{H}], \tag{2.11}$$

due to the Lorentz transformation. In the estimates that follow we shall assume that the inductive interaction of the electrons with the sound does not exceed the deformation interaction in order of magnitude. As regards the field $\mathbf{E}_w$, it vanishes in the reverse transformation (2.9). The effective Hamiltonian which describes the interaction of the electrons with the sound thus turns out to be $\mathscr{H}_u$.

As is known,[1,2] the rule for finding an electron trajectory in $\mathbf{p}$-space in a magnetic field amounts to the following: after the Hamiltonian of the system has been found in the absence of a magnetic field, it is necessary to find the intersections of the constant-energy surfaces with the constant p_z planes ($\mathbf{H}$ is parallel to the Z axis). The spectrum of the Hamiltonian $\mathscr{H}_u$ in our approximation is determined by the eigenvalues of the matrix

$$G=\begin{pmatrix}\varepsilon(\mathbf{p}-\hbar\mathbf{q}/2), & U(\mathbf{p})/2\\ U(\mathbf{p})/2, & \varepsilon(\mathbf{p}+\hbar\mathbf{q}/2)\end{pmatrix}, \tag{2.12}$$

which are equal to

$$E_{1,2}={}^1/_2[\varepsilon(\mathbf{p}+\hbar\mathbf{q}/2)+\varepsilon(\mathbf{p}-\hbar\mathbf{q}/2)]\pm\{U^2+{}^1/_4[\varepsilon(\mathbf{p}+\hbar\mathbf{q}/2)-\varepsilon(\mathbf{p}-\hbar\mathbf{q}/2)]^2\}^{1/2}. \tag{2.13}$$

(The Stewart-Tolman effect and the inductive interaction may easily be allowed for, if desired, by adding the corresponding term to U in this expression).

It is seen that the electron trajectories are appreciably distorted in regions where the difference $\varepsilon(\mathbf{p}+\hbar\mathbf{q}/2)-\varepsilon(\mathbf{p}-\hbar\mathbf{q}/2)$ is small. For simplicity we confine ourselves to the case $\mathbf{q}\perp\mathbf{H}$ and direct the X axis along $\mathbf{q}$. Inasmuch as the above difference vanishes at $p_x=0$, and $\hbar q\ll p$ (where p is the characteristic value of the electron momentum), the region of significant trajectory distortion corresponds to low values of p_x. If the vector $\mathbf{p}_1(0, p_{1y}, p_z)$ is introduced, where p_{1y} is connected with E and p_z by the relation

$$E={}^1/_2[\varepsilon(\mathbf{p}_1+\hbar\mathbf{q}/2)+\varepsilon(\mathbf{p}_1-\hbar\mathbf{q}/2)],$$

then the shape of the trajectory in the vicinity of the point $\mathbf{p}_1$ is determined by the expression

$$v_y(\mathbf{p}_1)(p_y-p_{1y})=\pm\{U^2+(m_{xx}^{-1}\hbar qp_x)^2\}^{1/2},\qquad m_{xx}^{-1}=\left.\frac{\partial^2\varepsilon}{\partial p_x^2}\right|_{\mathbf{p}=\mathbf{p}_1}. \tag{2.14}$$

Figure 1a shows the electron trajectories in this region, and Fig. 1b shows the general appearance of trajectories which have been distorted by the perturbing action of the sound wave.

We shall now turn to an analysis of the case of two close energy bands. For simplicity we shall confine ourselves to the case of weak energy coupling, in which the Hamiltonian of the $\mathscr{H}_u$ type is a 2×2 matrix of the following form:

$$\mathscr{H}_u=\begin{pmatrix}\frac{1}{2m_0}\left(\mathbf{p}+\frac{\mathbf{b}}{2}\right)^2, & V_{12}\\ V_{12}^*, & \frac{1}{2m_0}\left(\mathbf{p}-\frac{\mathbf{b}}{2}\right)^2\end{pmatrix}. \tag{2.15}$$

Here V_{12} is the interband matrix element of the pseudopotential and $\mathbf{b}$ is the reciprocal-lattice vector. We shall consider this vector to be directed along the Y axis. m_0 is the free-electron mass. The energy spectrum corresponding to the Hamiltonian (2.15) has the form

$$E=\frac{\mathbf{p}^2}{2m_0}+\frac{\mathbf{b}^2}{8m_0}\pm\left\{|V_{12}|^2+\left(\frac{bp_y}{m_0}\right)^2\right\}^{1/2}. \tag{2.16}$$

It is evident that the interband matrix elements are important in the vicinity of points where the difference $(\mathbf{p}+\mathbf{b}/2)^2-(\mathbf{p}-\mathbf{b}/2)^2$ is small. The trajectory of an electron in the neighborhood of such a point is determined by the expression

$$p_x-p_{2x}=\pm p_0\{1+(bp_y/2p_0p_{2x})^2\}^{1/2}, \tag{2.17}$$

where the quantities $p_{2x}(E,p_z)$ and $p_0(E,p_z)$ are defined by the relations

$$\frac{1}{2m_0}\left(p_{2x}^2+p_z^2+\frac{b^2}{4}\right)=E,\qquad p_0=\frac{m_0|V_{12}|}{p_{2x}}. \tag{2.18}$$

The minimum distance between trajectories is thus

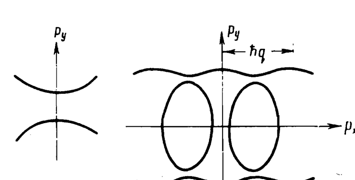

FIG. 1. Trajectories of an electron near the boundary of an acoustic band.

equal to $2p_0$. If the difference $\hbar q - 2p_0$ is small, then the acoustic perturbation intermixes the states $|1, \mathbf{p} + \hbar\mathbf{q}/2\rangle$ and $|2, \mathbf{p} - \hbar\mathbf{q}/2\rangle$ in the vicinity of the point p_{2x}, since the corresponding matrix element differs from zero. In its turn, the state $|1, \mathbf{p}+\hbar\mathbf{q}/2\rangle$ is formed from plane waves with momenta $\mathbf{p}+\hbar\mathbf{q}/2+\mathbf{b}/2$ and $\mathbf{p}+\hbar\mathbf{q}/2-\mathbf{b}/2$. The energy spectrum in the region of interest to us is thus determined by equating to zero the determinant of a 4×4 matrix of the form

$$\begin{pmatrix} \mathscr{P}(+,+), & V^{(0)}, & 0, & U \\ V^{(0)}, & \mathscr{P}(+,-), & U, & 0 \\ 0, & U, & \mathscr{P}(-,+), & V^{(0)} \\ U, & 0, & V^{(0)}, & \mathscr{P}(-,-) \end{pmatrix} = 0, \quad (2.19)$$

where $V^{(0)} = \langle 1, \mathbf{p}|V_0|2, \mathbf{p}\rangle$ is the interband matrix element of the periodic lattice potential

$$\mathscr{P}(\beta,\beta') = \frac{1}{2m_0}\left(\mathbf{p}+\beta\frac{\hbar\mathbf{q}}{2}+\beta'\frac{\mathbf{b}}{2}\right)^2 - E, \quad \beta,\beta'=\pm1.$$

We shall examine below the case where the difference $|\hbar q - 2p_0|$ is small. We assume that the ratio $\gamma = (\hbar q - 2p_0)/2p_0$ is a small parameter of the subsequent theory.

Equation (2.19) is a fourth-order equation. At constant p_z it describes four trajectories in $\mathbf{p}$-space. By virtue of the smallness of $|\hbar q - 2p_0|$ two of them approach each other closely. The other two are separated from the point of closest approach by a distance $\geq 2p_0$.[3] This fourth order equation has accordingly two neighboring roots which we shall try to find. In doing so we shall describe the shape of the electron trajectories in the vicinity of the point of closest approach in a region much smaller than p_0 in size. Thus to find the shape of the trajectory in the region of interest to us, we must, in the lowest approximation, solve a second-order and not a fourth-order equation. In doing this we in fact make use of the standard procedure of perturbation theory for a doubly degenerate state. We shall cite the result, leaving out the cumbersome but straightforward algebraic transformations. The shape of the trajectory is defined by the equation

$$p_x - p_{2x} = \pm p_0\left\{\left(-\gamma+\frac{1}{2}\xi\right)^2 + \frac{\delta}{2}(1+\xi)^2\right\}^{1/2}, \quad (2.20)$$

$$\xi = bp_y/2p_0p_{2x}, \quad \delta = 2(m_0U/p_0p_{2x})^2.$$

The trajectories near the point of closest approach in $\mathbf{p}$-space are shown schematically in Fig. 2. The shape of the trajectory depends on the sign of γ. If $\gamma<0$ ($\hbar q < 2p_0$), then there is one point of closest approach at $p_y = 0$, and the minimum distance between the trajectories is

$$2p_0(\gamma^2+\delta/2)^{1/2}. \quad (2.21)$$

If, however, $\gamma>0$ ($\hbar q > 2p_0$), then two cases are possible

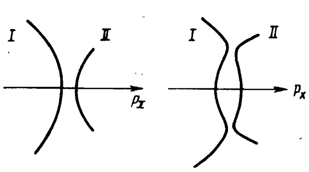

FIG. 2. Electron trajectories near the point of closest approach of the energy bands.

depending on the ratio of the γ and δ. The parameter δ is a relative measure of the distortion of the electron trajectories under the influence of the periodic field of the sound wave. The parameters γ and δ are small, and in that case the shape of the trajectory proves to be dependent on their ratio. If $\gamma<\delta$, there is one point of closest approach at $p_y=0$, and the distance between the trajectories at that point is determined by formula (2.21) as before. If, however, $\gamma>\delta$, there are two such points at $\xi = 2(\gamma-\delta)$, which corresponds to

$$p_y = \pm\frac{2p_0p_{2x}}{b}(2(\gamma-\delta))^{1/2};$$

the minimum distance between the trajectories is equal to

$$2p_0(\delta/2)^{1/2} = 2p_0(m_0U/p_0p_{2x}). \quad (2.22)$$

3. THE INTERACTION OF THE ACOUSTIC AND MAGNETIC BREAKDOWNS

As is known, in a magnetic field an electron moves along a trajectory in $\mathbf{p}$-space like a classical particle. Exceptions are those regions of $\mathbf{p}$-space where the classical trajectories approach each other closely. To be precise, if the magnetic field is such that the quantity $\hbar/a_L$ (where $a_L = (c\hbar/eH)^{1/2}$ is the so called Landau magnetic length) is comparable with the distance between the trajectories, tunnel transitions from one classical trajectory to another occur.[4] This phenomenon has been termed magnetic breakdown.

We shall start from an investigation of interband acoustic breakdown. In the absence of a sound wave let there be a region in $\mathbf{p}$-space where the classical trajectories in different energy bands approach one another to a distance of $2p_0$. The dynamics of an electron in magnetic breakdown conditions are determined by the so called "joining" matrix[8] whose elements are the amplitudes of the transition between the classical sections of the trajectoıy. The square of the nondiagonal matrix element, which has the meaning of the probability of a transition from a trajectory in one band to a trajectory in the other, may be written in the form[9]

$$W = \exp(-H_M/H), \quad (3.1)$$

where for the energy spectrum we used (cf.[8])

$$H_M = \frac{8c\pi|V^{(0)}|^2m_0^2}{e\hbar p_{2x}b} = \frac{4\pi cp_0^2}{e\hbar}\left(\frac{2p_{2x}}{b}\right). \quad (3.2)$$

Thus, if $H<H_M$, the probability of interband tunnel transitions is exponentially small and the electron moves along a classical trajectory in one hand.

The existence of acoustic perturbation, as we have seen, leads to a significant reduction in the distance between the trajectories. The possibility is therefore created of tunnel transitions at magnetic fields appreciably less than H_M. A concrete calculation of the elements of the "joining" matrix cannot be performed by using the existing magnetic breakdown theory. This is connected with the significant difference between spectrum (2.20) and spectrum (2.16), which permits a matrix whose elements contain only quadratic terms in p_y to be used for the solution of the quantum-mechanical problem. In our case, the dependence of the matrix elements on p_y is more complicated and we shall confine ourselves solely to a classical breakdown-probability calculation in the spirit of the procedure set forth in Abrikosov's book.[2] The solutions of the dispersion equation $p_y(p_x)$ are complex in the classically inaccessible region. As is known, the quasiclassical probability of a tunnel transition is equal to

$$W=\exp\left(-\frac{2}{\hbar}\int \operatorname{Im} p_y dy\right), \tag{3.3}$$

where the integral is taken over the classically inaccessible section in coordinate space. The projection of the trajectory in coordinate space on the XY plane is similar to the trajectory in **p**-space with a similarity factor of $\hbar/a_L^2$, one trajectory being rotated through 90° with respect to the other. The integral in (3.3) is therefore rewritten in the form

$$-\frac{2a_L^2}{\hbar^2}\int \operatorname{Im} p_y dp_x. \tag{3.4}$$

Using (2.20) and (3.4), we obtain for the breakdown probability

$$W_A=\exp\left[-\frac{8a_L^2p_0^2p_{2x}}{b\hbar^2}\left(\frac{p_{min}}{p_0}\right)^{3/2} f\left(\frac{\gamma-\delta}{(\delta/2)^{1/2}}\right)\right], \tag{3.5}$$

where $2p_{min}$ is the minimum distance between the trajectories in the presence of sound, and

$$f(x)=\begin{cases} 2\sqrt{2}/3 & x\ll -1\\ 2\sqrt{\pi}/5 & |x|\ll 1.\\ \sqrt{2}/3\sqrt{x} & x\gg 1\end{cases}$$

This probability may be written in the form $\exp(-H_A/H)$, where $H_A/H\sim f\cdot(p_{min}/p_0)^{3/2}\ll 1$, the case where $\gamma-\delta \gg (\delta/2)^{1/2}$ being the most favorable. This is essentially the principal result of the work. It means that if the field H is $\gtrsim H_A$, then the probability of breakdown is of the order of unity, although the condition $H\ll H_M$ may be fulfilled at the same time, i.e., acoustic perturbation stimulates magnetic breakdown.

From formula (3.5) in conjunction with (2.20) it is evident that even at exact resonance $\gamma=\hbar q-2p_0=0$ the field H_A (and the minimum distance between the trajectories) remains finite and depends on the sound intensity, which must in turn satisfy the condition

$$U\tau/\hbar\gg 1.$$

From this we obtain the following criterion for the magnetic field H_A:

$$H_A\gg H_M(\hbar/p_0l)^{3/4}. \tag{3.6}$$

In particular, in the example we analyzed in the Introduction, the dimensionless parameter $(\hbar/p_0l)^{3/4}$ is $\sim 10^{-3}$. This means that for $H_M\approx 10^5$ Oe, the field must appreciably exceed 100 Oe.

What observable consequence can interband acoustic breakdown lead to? Acoustic breakdown leads essentially to a new system of electron trajectories, or, in different language, to a new system of quasiclassical electron levels. In the presence of acoustic breakdown new periods must therefore arise in the "thermodynamic"[5] and kinetic quantities.

We shall now discuss the case of a single energy band, which was analyzed in[4]. As is evident from Fig. 1, under the action of acoustic perturbation two types of trajectories arise, viz., closed and open ones in the direction of propagation of the sound wave. At the same time the character of the movement along the trajectories depends on the magnitude of the magnetic field. Owing to the magnetic breakdown, a sufficiently strong field may lead to suppression of the influence of the sound. In contrast to the interband breakdown examined above, such intraband breakdown of the acoustic bands is described by the standard theory of magnetic breakdown.[8] An estimate made in accordance with that theory gives the value

$$H_M{}^b=\frac{8c\pi U^2}{e\hbar^2 v_y(\mathbf{p}_1)qm_{xx}^{-1}} \tag{3.7}$$

for the critical field H_M^b in this case. Of greatest interest to us is the case where $H<H_M^b$, but $\Omega(H)\tau\gg 1$ (where $\Omega(H)$ is the cyclotron frequency corresponding to the magnetic field H). These estimates may not contradict each other since

$$\Omega(H_M{}^b)\tau=\frac{U}{\hbar q v_F}\frac{U\tau}{\hbar}.$$

In the purest bismuth with a mean free path of 3 mm at a sound intensity of 3 W/cm^2 and a frequency of 1 GHz this parameter may be of the order of 10–20. In that case the electrons move along trajectories created by the sound, some of which are open, and this must change the asymptotic form of the galvanomagnetic tensor in a radical way.

An important criterion for the possibility of observing the effects examined above is the requirement of low damping of sound over the length of the sample. At the frequencies necessary for observing acoustic breakdown, the linear damping length is, as a rule, small. Under nonlinear conditions the absorption coefficient for sound can be appreciably smaller than under linear conditions. In the case where one energy band is important and $\Omega\tau \ll 1$, the solution of the problem of nonlinear absorption of sound is contained in the Laĭkhtman and Pogorel'skiĭ paper.[10] According to that work, under nonlinear conditions $\Gamma\sim\Gamma_0\hbar/U\tau$, where Γ_0 is the linear absorption coefficient, and at $H<H_M^b$ the magnetic field has practically no influence on the absorption. When the value of H_M^b is exceeded by the magnetic field, magnetic breakdown of the acoustic bands occurs and absorption should increase sharply. This sharp increase in absorption with increase in the magnetic field can also be used to study breakdown of the acoustic bands experimentally. In the case where two bands are important, it is also natural to expect that $\Gamma\sim\Gamma_0\hbar/U\tau$ in the region of magnetic fields below breakdown. If it is assumed that Γ_0

$\propto q$ and the damping length at a frequency of 1 GHz is equal to 1 cm, then at the frequency of 20 GHz needed to observe acoustic breakdown the damping length is of the order of 5×10^{-2} cm. The nonlinearity parameter $U\tau/\hbar$ may reach several hundreds. There is therefore reason to suppose that the phenomena we have examined may be observed experimentally in samples of reasonable dimensions. Let us note that in the case where the acoustic momentum $\hbar q$ is appreciably smaller than $2p_0$, it is not possible to observe acoustic breakdown in the literal sense of the word. It is, however, possible to observe the sound-stimulated magnetic breakdown phenomenon which consists in a reduction of the breakdown field under the action of a sound wave. This situation has the advantage from the experimental point of view that it requires sound of lower frequencies which undergo less damping.

1)We shall adhere to the following terminology. Allowed bands which arise as a result of the periodic field of the crystal lattice we shall term energy bands. Under the influence of the periodic field of a sound wave they break up into allowed and forbidden acoustic bands.

2)In the present work we shall not touch on the change in the electron spectrum in the field of a standing sound wave: this problem is more complicated than ours because it cannot be reduced to a stationary one by any transformation of coordinates.

3)We recall that the electron spectrum is periodic in **p**-space, with a period $\hbar\mathbf{q}$, and it is not therefore necessary to consider the two distant trajectories: their existence is a consequence of this periodicity.

4)This estimate is only applicable for the so called interband magnetic breakdown.[8] Only the latter is realized in our situation.

5)We have put the word "thermodynamic" in quotation marks because the state of a metal containing a traveling sound wave is not, strictly speaking, an equilibrium one.

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Surface impedance of cadmium in a magnetic field

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The behavior of the surface impedance of cadmium is investigated theoretically and experimentally in the range of Doppler-shifted cyclotron resonance. It is found that the reactance of the metal has minima in the neighborhood of the thresholds of various dopplerons. The experimental data agree with the theoretical results for the case of diffuse reflection of the electrons by the surface.

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Doppler-shifted cyclotron resonance (DSCR) leads to the appearance of singularities in the variation of the surface impedance of a metal with the value of the applied magnetic field **H**. The behavior of the surface resistance R (the real part of the impedance) of cadmium, for $\mathbf{H}\parallel[0001]$, has already been investigated.[1,2] A kink in the function $R(H)$ was observed experimentally in the neighborhood of a doppleron threshold. Theoretical investigation showed that the function $R(H)$ should in fact experience a kink for diffuse reflection of the electrons by the surface. Unfortunately, in the experiments cited[1,2] the imaginary part X of the impedance (the reactance) was not studied; and in regard to the resistance, the measurements gave only the functional dependence $R(H)-R(0)$ on an unknown scale.

In the present paper, the variation of the surface impedance of cadmium with magnetic field is investigated theoretically and experimentally. The experimental method used made possible measurement of the absolute changes both of the real and of the imaginary parts of the impedance. It was found that, in contrast to the resistance, the variation of the reactance with field has a nonmonotonic character: there are minima on the $X(H)$