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Langmuir turbulence and dissipation of high-frequency energy

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We construct a theory of three-dimensional Langmuir turbulence. We give the turbulence spectra in the inertial and the absorption ranges and we analyze the role of possible absorption mechanisms for the short-wavelength plasmons. We find the effective collision frequency which characterizes the power dissipated from the pumping wave. We study the effect of the build-up of short-wavelength acoustic oscillations on the turbulence. We solve the problem of the dynamics of plasma turbulence excited by an electromagnetic wave with a frequency differing from the plasma frequency.

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§1. INTRODUCTION

The interest in the problem of strong turbulence is to a large extent connected with practical applications—the necessity for establishing effective collisionless mechanisms for energy dissipation when one uses lasers of beams to initiate a thermonuclear reaction. On the other hand, at the present there is no sufficiently consistent formulation of a strong turbulence theory which would enable us to connect the collapse of an isolated caviton with plasmons as the microscopic manifestation of the modulational instability with such macroscopic characteristics as the power dissipated from the pumping wave, the hot-particle distribution function, and others.

The idea of the collapse as the non-linear stage, discovered by Vedenov and Rudakov,[1] of the modulational instability of Langmuir waves is due to Zakharov.[2] He showed that the localization of the plasma waves caused by the modulational instability ultimately can lead to the formation of cavitons (regions of lowered plasma density with plasmons trapped in them) collapsing to dimensions where some mechanism for the dissipation of the plasmon energy is switched on.

One of the authors of the present paper (see, e.g., Ref. 3) suggested simple considerations which could be used to study the possibility for collapse depending on the number of dimensions of the caviton. These considerations are based on the relation between the density well depth and the wavelength of the plasmons trapped in them:

$$|\delta n|/n_0 \sim k^2\lambda_D^2 \tag{1.1}$$

(the kinetic energy of the trapped plasmons is of the order of the potential energy), and on the condition that the plasmon number in each separate caviton is constant:

$$\frac{1}{4\pi\omega_p}\int d\mathbf{r}|\mathbf{E}|^2=\text{const.} \tag{1.2}$$

As the depth of the density modulation in the cavitons is usually small ($\delta n \ll n_0$), $\omega_p \approx$ const and it follows from the last condition that the radiation pressure in the center of the caviton leading to a displacement of the plasma and to collapse will during the collapse increase inversely proportionally to its volume $|E|^2 \propto l^{-s}$($l \sim 1/k$ is the characteristic size of the caviton, $s = 1, 2, 3$ is its dimensionality). For collapse, at the same time, one must overcome the pressure of the displaced plasma δnT which, according to (1.1), increases as l^{-2}. Hence it follows that the one-dimensional case is a special one—for some l the balance between the pressures and the collapse necessarily ceases. When $s = 2$ the possibility of a collapse depends on the initial conditions—if initially the high-frequency pressure in the caviton is larger than the gas-kinetic pressure, collapse will not set in at a later stage. And finally, in the three-dimensional case collapse is inevitable.

Recently self-similar solutions describing the collapse of a caviton have been obtained[4,5] and the approach to the self-similar solution has been considered using a computer (see Ref. 5) so that notwithstanding the absence of a rigorous mathematical proof the fact of the collapse of an isolated caviton with plasmons is indisputable.

Our aim in the present paper is to make the transition

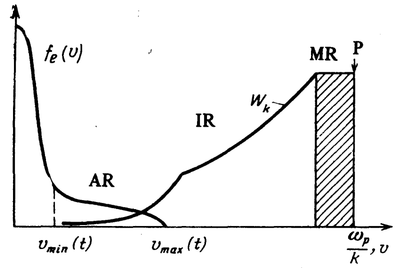

FIG. 1. Wave and particle spectra in strong Langmuir turbulence (AR—absorption region, IR—inertial range, MR—modulational instability region, P—pump).

from the collapse of isolated cavitons to a theory of three-dimensional plasma turbulence[1)] which includes collapse only as one of the mechanisms for short-wavelength plasmon transfer. Such a theory must describe the intense plasmon gas in which a large number of randomly oriented cavitons which are in various stages of collapse are formed by virtue of the modulational instability. It is convenient to characterize the turbulence which thus arises in the **k**-representation language and it then turns out to be possible to distinguish three regions: the long-wavelength region where energy is pumped into the turbulence, the inertial range in which energy is transferred to shorter lengthscales, and the short-wavelength absorption region of the plasma turbulence (see Fig. 1).

Some features of the strong turbulence of plasma waves were already elucidated in Ref. 4, and the present paper is devoted to a more consistent exposition of the theory. The plan of the paper is the following. In the second section we briefly give the results of studying turbulence spectra in the inertial range and the absorption region and we analyze the role of possible mechanisms for the absorption of short-wavelength plasmons. The third section is devoted to the long-wavelength region of the source and we obtain here an expression for the effective collision frequency which determines the power dissipated from the pump wave. In the fourth section we analyze the features of the turbulence which arise for rather large pumping amplitudes and which are connected with the build-up of sound oscillations in the short-wavelength region of plasmon absorption. The presence of strong short-wavelength sound produces an additional channel for the transfer of plasmons to the absorption region by virtue of their conversion into sound and it can thereby stabilize the collapse and appreciably change the effective collision frequency. Finally in the fifth section we solve the problem which is important for applications of the dynamics of the plasma turbulence which is excited by an electromagnetic wave with a frequency which rather strongly differs from the plasma frequency. In that case the spectrum of the plasmons arising from the pump wave turn out to be modulationally stable and there arises a strong transfer of plasmons according to weak turbulence and the formation of a long-wavelength condensate. We solve the problem of the dynamics of such a condensate and the problem of the rate of dissipation under those conditions.

§2. INERTIAL RANGE AND ABSORPTION REGION

The characteristic wavelengths of the plasmons produced in the source region are determined by the threshold of the modulational instability

$$l_0 \sim \lambda_D \left[\frac{n_0 T}{W}\right]^{1/2}, \quad W = \frac{1}{8\pi}\sum_{\mathbf{k}} |\mathbf{E}_{\mathbf{k}}|^2,$$

where W is the energy of the plasma oscillations (for details see §3).

In the present paper we consider the case which is most important for applications when $W/n_0T \gg m/M$ and we assume at the same time that the turbulence is not too strong so that plasmons produced in the source region do not immediately fall into the absorption region and that there exists an inertial range between large length-scales $\sim l_0$ and small length-scales $\sim k_*^{-1}(k_* \sim 1/3\lambda_D$ is the characteristic value of k in the absorption region; (2.8) below). The short-wavelength transport of plasmons through the inertial range is caused by the collapse. Simple scale estimates in the equations which describe strong plasma turbulence:

$$\operatorname{div}\left(i\frac{\partial \mathbf{E}}{\partial t} + \frac{3}{2}\omega_p\lambda_D^2\nabla \operatorname{div}\mathbf{E}\right) = \frac{\omega_p}{2n_0}\operatorname{div}\delta n\mathbf{E}, \tag{2.1}$$

$$\frac{\partial^2\delta n}{\partial t^2} - \frac{T_e}{M}\nabla^2\delta n = \frac{1}{16\pi M}\nabla^2|\mathbf{E}|^2, \tag{2.2}$$

enable us to obtain the law for the self-similar collapse of a caviton.

In the equations given here $\mathbf{E}(t, \mathbf{r})$ is the complex amplitude of the electric field in the plasma oscillations

$$\mathbf{E}_p = {}^1/_2\mathbf{E}(t, \mathbf{r})e^{-i\omega_p t} + \text{c.c.},$$

and $\delta n(t, \mathbf{r})$ is the slow density variation. If we assume that the collapse speed of the caviton is larger than the sound speed we get from (2.2) and (1.1) the way the high-frequency pressure at the center of the caviton increases:

$$|\mathbf{E}|^2 \sim 1/(t_0 - t)^2. \tag{2.3}$$

We get the way the plasma density variation in the caviton changes from Eq. (1.2) which corresponds to the constancy of the number of plasmons trapped in the caviton (we shall show below that we can assume a three-dimensional caviton in the final stage of its collapse to be isolated from the pumping). In the three-dimensional case which is of most interest we then have from (1.1), (1.2), and (2.3)

$$\delta n \sim 1/(t_0 - t)^{4/3}. \tag{2.4}$$

Hence it follows in particular that the collapse speed $dl/dt \propto 1/(t_0 - t)^{1/3}$ increases as the size of the caviton diminishes so that in the three-dimensional case the caviton necessarily gets into the supersound collapse regime.

We study the dynamics of the turbulence when there is a source present which maintains the average field of the plasma oscillations at a given level. The role of such a source can be played by an electron beam which excites plasma oscillations with a wavelength much larger than l_0 which act as the long-wavelength pumping for the development of the modulational instability. The presence of such pumping corresponds to satisfying the following integral condition for **E**:

$$\langle \mathbf{E}\rangle = \mathbf{E}_0$$

(the brackets correspond to an average over the plasma volume, $\mathbf{E}_0 = \text{const}$ is the pumping amplitude). We then get from Eq. (2.1) the integral which determines the change in the plasmon number due to the pumping:

$$\frac{d}{dt}\int d\mathbf{r}\frac{|\mathbf{E}|^2}{8\pi} = i\frac{\omega_p}{2n_0}\mathbf{E}_0\int\frac{\mathbf{E}\delta n}{8\pi}d\mathbf{r} - \text{c.c.} \qquad (2.5)$$

Applying this relation to a separate, isolated caviton and using the self-similar solution (2.3), (2.4) we find

$$\int_K d\mathbf{r}\frac{|\mathbf{E}|^2}{8\pi} = \text{const} + O(t_0 - t)^{2/3},$$

i.e., the pumping indeed becomes detached from the caviton as it collapses.

The turbulence spectrum in the inertial range (see Fig. 1) can be found from the equation expressing the constancy of caviton flux along the spectrum:

$$N_k dk/dt(k) = \text{const},$$

where $N_k dk$ is the number of cavitons in the range of characteristic scales $(k, k+dk)$ while the time-dependence for the transition of cavitons from larger sizes to smaller ones is determined by the self-similar solution given above. As all cavitons with length scales $\sim l_0$ are produced with approximately the same energy content which after that is conserved in the collapse process, the spectral density of the plasmon energy is proportional to the number of cavitons in a given range of turbulence length scales:

$$|\mathbf{E}_k|^2 k^2 dk \sim N(k)\,dk \sim 1/k^{5/2}. \qquad (2.6)$$

The approximation used to obtain this spectrum is in fact equivalent to the Kolmogorov hypothesis that energy flux is constant over the spectrum in the inertial range.

We now dwell upon the problem of the dissipation mechanisms for the plasmon energy in the short-wavelength sub-region. Possible dissipation mechanisms are resonance absorption by plasma electrons and the characteristic non-linearity of the plasma oscillations and the intersection of electron trajectories connected with it. As the transfer of plasmons in turbulence occurs from large scales to smaller ones, it is clear that resonance absorption must primarily be included for "tail" electrons with velocities well above the thermal one, i.e., in the absorption range we must have $k\lambda_D < 1$. This made it possible to assume already in the initial stage of the study of strong plasma turbulence (see Ref. 6) that Landau damping is the basic mechanism for the dissipation of short-wavelength plasmons.

The characteristic wavenumber value k_* for which damping becomes important can be determined from the condition for balancing in the short-wavelength range—the energy flux along the spectrum is compensated by absorption by particles

$$\gamma_{\text{mod}} W = \Gamma_{k^*} W'. \qquad (2.7)$$

In this equation $W' = W(k_0/k_*)^{3/2}$ is the energy of the short-wavelength $(k \gtrsim k_*)$ plasma noise, determined by (2.6), $k_0 \sim l_0$ is the wavenumber in the source region; Γ_{k_*} is the Landau damping rate, $\gamma_{\text{mod}} \approx \omega_p(mW/Mn_0T)^{1/2}$ is the modulational instability growth rate determining the characteristic rate of short-wavelength energy transfer during collapse. We then get from (2.7)

$$\Gamma_{k^*}/\omega_p = \left(\frac{m}{M}k_*^3\lambda_D^3\right)^{1/2}\left(\frac{n_0T}{W}\right)^{1/4}, \qquad (2.8)$$

and even if we use to estimate Γ_{k_*} the formula obtained for a Maxwellian plasma,

$$\Gamma_{k^*} = \omega_p\left(\frac{\pi}{8}\right)^{1/2}\frac{\exp(-{}^1/{}_2k^{*2}\lambda_D^2 - {}^3/{}_2)}{k_*^3\lambda_D^3},$$

we find that the Landau damping of plasmons becomes appreciable for sufficiently small k_* $(k_*\lambda_D \sim \frac{1}{3}$ to $\frac{1}{4})$.

Indeed, when we take the formation of electron tails into account Landau damping is switched on for even smaller[2)] k.

The characteristic non-linearity of the Langmuir oscillations is characterized by the occurrence of a non-linear correction to the frequency:

$$\Delta\omega \sim \omega_p k^2 \lambda_E^2$$

($\lambda_E = eE/m\omega_p^2$ is the amplitude of the high-frequency displacement of the electrons). The non-linearity is important if the correction given here is comparable to the dispersion correction,

$$\lambda_E^2 \approx \lambda_D^2, \text{ i.e., } E^2/8\pi n_0 T \sim 1. \qquad (2.9)$$

As the collapse of a three-dimensional caviton is supersonic, the short-wavelength transfer of plasmons during collapse occurs more slowly than the field growth and when condition (2.9) is satisfied the parameter $k\lambda_D$ remains small. One can easily estimate that quantity by using the self-similar solution (2.3), (2.4):

$$k^2\lambda_D^2 \sim \frac{W}{n_0T}\left(\frac{t_0}{t_0 - t}\right)^{4/3} \sim \frac{W}{n_0T}\left(\frac{E^2}{8\pi W}\right)^{4/5} \sim \left(\frac{W}{n_0T}\right)^{1/5}. \qquad (2.10)$$

For the case considered, $W/n_0T \gg m/M$, this quantity is large compared to $k_*^2\lambda_D^2$ which makes it possible to assume the Landau damping to be the basic mechanism for the dissipation of short-wavelength plasmons.[3)]

The spectrum of the plasmons in the absorption region must then be found at the same time as the spectrum of the resonant particles. The corresponding set of equations consists of the quasi-linear equation for the electron distribution function and the equation for the spectral density of the noise in which both the energy transfer along the spectrum caused by the collapse of the cavitons with plasmons and the resonance absorption by electrons are taken into account. Assuming the wave and particle spectra to be isotropic we get the following set of equations:

$$\frac{\partial f}{\partial t}=\frac{e^2\omega_p^2}{4\pi m^2}\frac{1}{v^2}\frac{\partial}{\partial v}\left[\frac{1}{v}\int\limits_{\omega_p/v}^{\infty}\frac{dk}{k^3}W_k\frac{\partial f}{\partial v}\right], \qquad (2.11)$$

$$\frac{\partial W_k}{\partial t}=-\frac{\partial}{\partial k}\left(W_k\frac{dk}{dt}\right)+2\Gamma_k W_k,$$
$$W_k=k^2|\mathbf{E}_k|^2, \quad \Gamma_k=\frac{4\pi^3 e^2\omega_p^2}{mk^3}f\left(\frac{\omega_p}{k}\right), \qquad (2.12)$$

Γ_k is the resonance absorption damping rate. The set of equations given here has a quasi-stationary solution with a constant particle flux along the spectrum in the large velocity region

$$\frac{\partial J}{\partial v}=0 \text{ when } v\leqslant v_{max}(t),$$

$$J=\frac{e^2\omega_p^2}{m^2 v}\frac{\partial f}{\partial v}\int\limits_{\omega_p/v}^{\infty}\frac{dk}{k^3}W_k, \qquad (2.13)$$

J is the particle flux, v_{max} the upper limit of the "tail" of resonant electrons.

Using the law for the self-similar collapse of the cavitons in (2.12) which is determined from (2.4): $k(t)\propto 1/(t_0-t)^{2/3}$ we find the following power-law spectra of plasmons and particles first obtained in Ref. 4 in such a solution:

$$f(v)=A/v^{9/2}, \quad W_k=B/k^{5/2},$$
$$v_{min}(t)<v<v_{max}(t), \quad k>k^*=\omega_p/v_{max}. \qquad (2.14)$$

Generally speaking, when finding the law for the self-similar collapse one must in the short-wavelength scales of the absorption region take into account that the cavitons lose energy due to Landau damping. The corresponding balance equation has the form

$$\frac{d}{dt}\left(\frac{E^2}{k^3}\right)\approx\Gamma_k\frac{E^2}{k^3}\approx Ck^{3/2}\frac{E^2}{k^3}. \qquad (2.15)$$

As long as the collapse of the caviton remains supersonic, the growth of the field at the center of the caviton is, as before, determined by Eq. (2.3): $E\propto 1/t_0-t)$ and we have then from (2.15) the following relation for the collapse[8]:

$$k\sim\frac{1}{(t_0-t)^{2/3}}\left[1+\frac{C}{2}\ln\frac{t_0}{t_0-t}\right]^{-2/3},$$

which is with logarithmic accuracy the same as the collapse rule $k\propto 1/(t_0-t)^{2/3}$ which leads to the spectrum (2.14).

To find the constant in the formula for the plasmon spectral density we use the normalization condition

$$\frac{1}{16\pi^3}\int W_k\,dk=W$$

and the condition that the spectrum be continuous at the boundary of the absorption region $k=k_*$:

$$B=24\pi^3 k_0^{1/2}k_*^2 W.$$

We can express the constant A in terms of the number of particles in the tail:

$$n'=\frac{8\pi}{3}A\left(\frac{1}{v_{min}^{3/2}}-\frac{1}{v_{max}^{3/2}}\right).$$

At the same time we find from the particle number and energy balance equations the following relations:

$$n_0\left(\frac{2m}{\pi T}\right)^{1/2}v_{min}\exp\left\{\frac{-mv_{min}^2}{2T}\right\}=\frac{8\pi}{3}A\left(\frac{1}{v_{min}^{3/2}}-\frac{1}{v_{max}^{3/2}}\right),$$
$$mA[v_{max}^{1/2}-v_{min}^{1/2}]=\frac{Qt}{4\pi}, \quad Q=\nu_{\text{eff}}\frac{E_0^2}{8\pi}, \qquad (2.16)$$

Q is the power absorbed from the pump into plasma turbulence. In the problem considered the stationary state of the turbulence is reached because the energy dissipated into turbulence at long length scales is transferred to short scales, is absorbed by particles, and finally leads to acceleration of the tail. At large t, when $v_{max}\gg v_{min}$ the upper limit of the tail increases according to the rule $v_{max}\propto t^2$, the lower limit $v_{min}\approx$ const, and the law for the saturation of the number of particles in the tail has the form $n'=n'_\infty(1-a/t^3)$ which agrees with the time dependence, obtained from (2.13), (2.15), for the particle flux into the tail: $J\propto 1/v_{max}^2\propto 1/t^4$. The flow of energy transferred to the "fast" particles can also be put in the form $\frac{1}{2}mv_{max}^2 J$. Using (2.13) and (2.15) and equating this flux to the quantity $\gamma_{mod}W$ which gives the energy flux into plasma turbulence we get a relation connecting the constant A with the plasmon energy:

$$A=n_0\frac{13}{27\pi^2}\left(\frac{\omega_p}{k_0}\right)^{7/2}\left(\frac{m}{M}\frac{W}{n_0T}\right)^{1/2}.$$

Using this relation we find the following final formula for the maximum number of particles in the tail:

$$n_\infty'=\frac{1}{4}n_0\left(\frac{m}{M}\right)^{1/2}\left(\frac{n_0T}{W}\right)^{1/4}\frac{1}{\xi^{3/4}}, \quad \xi=\frac{mv_{min}^{\infty\,2}}{2T}. \qquad (2.17)$$

The lower limit of the tail v_{min}^{∞} as $t\to\infty$ is given by the equation

$$e^{-\xi}\xi^{5/4}=\frac{\sqrt{\pi}}{8}\left(\frac{m}{M}\right)^{1/2}\left(\frac{nT}{W}\right)^{1/4}, \qquad (2.18)$$

and the asymptotic relation for the growth of the upper limit of the tail has the form

$$v_{max}=\left(\frac{T}{m}\right)^{1/2}\left(\frac{\pi}{2}\nu_{\text{eff}}t\frac{E_0^2}{8\pi n_0T}\left(\frac{M}{m}\right)^{1/2}\left(\frac{W}{n_0T}\right)^{1/4}\right)^2. \qquad (2.19)$$

The analysis in the present paper, which is based upon Eqs. (2.1) and (2.2), is valid so long as we can neglect the effect of the tail upon the dispersion of the Langmuir oscillations. The appropriate condition has the form

$$\int\frac{mv^2}{2}f\,dv<n_0T$$

and is satisfied for times

$$t<8\pi n_0 T\nu_{\text{eff}}^{-1}/E_0^2.$$

§3. SOURCE REGION. ENERGY DISSIPATION IN PLASMA TURBULENCE

Just as in hydrodynamic turbulence, the description of the long-wavelength region, where energy is pumped into the turbulence, is very complicated. In the present section we expound a model theory of plasma turbulence in the source region; this theory is based upon the assumption that long-wavelength plasmons produced from the pumping wave stochastically randomize their phase when scattered by cavitons. In that case the effect of energy dissipation in plasma turbulence turns out to be to a considerable extent analogous to the stochastic heating of particles.[9]

The analysis of long-wavelength plasma turbulence will be based upon the set of Eqs. (2.1), (2.2) for the modulational instability given in the preceding section. In the analysis it turns out to be convenient to use the following Fourier expansions for the amplitude of the electric field and the long-wavelength density variations:

$$\mathbf{E}(t,\mathbf{r})=\mathbf{E}_0+\sum_{\mathbf{k}}\mathbf{E}_{\mathbf{k}}(t)\exp\{i\mathbf{kr}-i\Phi_{\mathbf{k}}(t)-i\delta_{\mathbf{k}}t\},$$
$$\frac{\delta n}{n_0}=\sum_{\mathbf{k}}\eta_{\mathbf{k}}(t)e^{i\mathbf{kr}}+\text{c.c.} \tag{3.1}$$

Here $\delta_{\mathbf{k}}=\frac{3}{2}k^2\lambda_D^2\omega_p$; $\Phi_{\mathbf{k}}$ are the random phases of the plasmons which possess the following correlation function:

$$\langle\exp(i\Phi_{\mathbf{k}})\rangle=0,\ \langle\exp\{i\Phi_{\mathbf{k}}(t)-i\Phi_{\mathbf{k}'}(t')\}\rangle=\delta_{\mathbf{kk}'}\exp\{-\nu_{\text{corr}}|t-t'|\} \tag{3.2}$$

(in this case the angle brackets correspond to averaging over the ensemble of random phases of the plasmons). The characteristic time for the decoupling of the phase correlations in (3.2) is determined by the frequency scattering of the long-wavelength plasmons by the density cavitons. Thus, $\nu_{\text{corr}}\sim v_g/l_0$, $v_g\sim k_0\lambda_D^2\omega_p$ is the group velocity of a plasmon, k_0 is a characteristic wavenumber in the source region, and l_0 the distance between the cavitons. It is clear that $l_0\sim 1/k_0$ (see Fig. 2) and the final formula for ν_{corr} takes the form

$$\nu_{\text{corr}}\approx k_0^2\lambda_D^2\omega_p. \tag{3.3}$$

We find the Fourier coefficients of the electric field from Eq. (2.1):

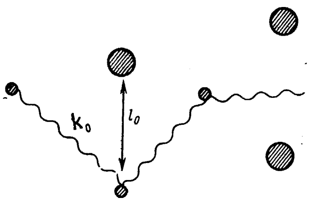

FIG. 2.

$$\mathbf{E}_{\mathbf{k}}=-\frac{\omega_p}{2}\int_{-\infty}^{t}dt'\Big[\eta_{\mathbf{k}}\mathbf{E}_0\exp\{i\Phi_{\mathbf{k}}(t')+i\delta_{\mathbf{k}}t'\}+\sum_{\varkappa}\eta_{\varkappa}\mathbf{E}_{\mathbf{k}-\varkappa}\exp\{i(\delta_{\mathbf{k}}-\delta_{\mathbf{k}-\varkappa})t'\}\times\exp\{i\Phi_{\mathbf{k}}(t')-i\Phi_{\mathbf{k}-\varkappa}(t')\}\Big]. \tag{3.4}$$

The growth in the energy of the plasma oscillations is determined by the integral of the equations for the modulational instability (Eq. (2.5)). Substituting into the right-hand side of the integral the Fourier expansion (3.1) for the electric field with the coefficients (3.4) and averaging over the random phases of the plasmons we get

$$\frac{d}{dt}\sum_{\mathbf{k}}|\mathbf{E}_{\mathbf{k}}|^2=\frac{\omega_p^2}{2}\sum_{\varkappa}\frac{\nu_{\text{corr}}}{\delta_\varkappa^2+\nu_{\text{corr}}^2}\eta_\varkappa^2E_0^2\approx\frac{\omega_p^2}{2\nu_{\text{corr}}}\sum_{\varkappa}\eta_\varkappa^2E_0^2,$$
$$\varkappa\lambda_D\lesssim(\nu_{\text{corr}}/\omega_p)^{1/2}. \tag{3.5}$$

We get the equation for the Fourier components of the long-wavelength density variations from (2.2) if we assume that the characteristic time for changes in the Fourier amplitudes $\eta_\varkappa$ and $E_{\mathbf{k}}$ is large compared to ν_{corr}^{-1}:

$$\frac{\partial^2\eta_{\mathbf{k}}}{\partial t^2}-\omega_s^2\eta_{\mathbf{k}}=\frac{\omega_p^2k^2}{16\pi n_0M}\eta_{\mathbf{k}}\Big\{\frac{E_0^2}{2(\delta_{\mathbf{k}}+i\nu_{\text{corr}})}+\sum_{\varkappa}|E_\varkappa|^2\frac{1}{\delta_{\mathbf{k}+\varkappa}-\delta_{\mathbf{k}}+i\nu_{\text{corr}}}+\text{c.c.}\Big\}. \tag{3.6}$$

If we neglect the last term Eq. (3.6) leads to the well known dispersion relation of the linear theory (see Ref. 10) modified to take account of the stochastic change of the plasmon phases. The two last terms in Eq. (3.6) correspond to the obvious fact that the role of the main wave, whose amplitude is modulated as a result of the instability, can be performed by the long-wavelength part of the plasma noise spectrum.

For the plasmon spectral distributions (2.6) and (2.15) which decrease rather fast with increasing k it is just the long-wavelength part of the spectrum ($k\sim k_0$) which gives the main contribution to the plasma oscillation energy. We shall assume that the energy of the plasma oscillations

$$W=\frac{1}{8\pi}\sum_{\mathbf{k}}|\mathbf{E}_{\mathbf{k}}|^2$$

is appreciably larger than the pump wave energy $E_0^2/8\pi$. We can then in Eq. (3.6) neglect the contribution from the pump and the characteristic values of the wavelengths of the plasmons formed in the modulational instability and the growth rate of that instability are given by the relations

$$k_0\approx\lambda_D^{-1}\left(\frac{W}{n_0T}\right)^{1/2},\quad \gamma_{\text{mod}}\approx\omega_p\left(\frac{m}{M}\frac{W}{n_0T}\right)^{1/2}. \tag{3.7}$$

One can find the amplitude of the long-wavelength density fluctuations from the condition that the plasmons are trapped in the density well:

$$\frac{|\delta n|}{n_0}\sim k_0^2\lambda_D^2\sim\frac{W}{n_0T}. \tag{3.8}$$

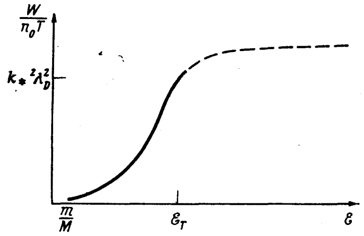

FIG. 3. Plasma noise energy as function of pump amplitude ($W/n_0T=\nu_{eff}/\alpha\omega_p$, $\mathcal{E}=E_0^2/8\pi n_0T$, $\mathcal{E}_T=(m/M)^{1/2}k_*\lambda_D/\alpha$).

Using (3.7) and (3.8) we can finally write Eq. (3.5), which determines the growth of energy of the plasma oscillations under the action of the pump, in the following form:

$$\frac{dW}{dt}=\nu_{eff}\frac{E_0^2}{8\pi}, \tag{3.9}$$

the quantity ν_{eff} which characterizes the dissipation rate equals

$$\nu_{eff}=\alpha\omega_pW/n_0T, \tag{3.9$'$}$$

α is a numerical coefficient which cannot be found from the qualitative considerations given here. Simulation of the process of the development of the modulational instability from the pumping wave (see, e.g., Ref. 11) confirms Eqs. (3.9), (3.9′) and gives the value $\alpha\approx 0.3$.

The value of ν_{eff} enables us to round off the theory of plasma turbulence considered in the present paper and to estimate the rate of absorption of high-frequency energy in the plasma. For stationary turbulence one must satisfy not only the balance condition (2.7) for short length scales but also a balance condition at long length scales—the power dissipated from the pump wave determines the energy flow along the spectrum in the collapsing cavitons

$$\nu_{eff}\frac{E_0^2}{8\pi}=\gamma_{mod}W. \tag{3.10}$$

This equation gives together with Eqs. (3.7) and (3.9′) the possibility to express the energy of the stationary plasma waves in terms of the pump wave amplitude

$$\frac{W}{n_0T}=\frac{M\alpha^2}{m}\left(\frac{E_0^2}{8\pi n_0T}\right)^2. \tag{3.11}$$

The analysis given in the present section of the long-wavelength plasma turbulence in which the energy of the oscillations and the effective collision frequency depend strongly on the pump wave amplitude is valid only in a well defined range of pumping:

$$\frac{m}{M}\ll\frac{E_0^2}{8\pi n_0T}<\left(\frac{m}{M}\right)^{1/2}\frac{k^*\lambda_D}{\alpha}=\mathcal{E}_T.$$

The lower limit of this interval is obtained from the condition $W\gg E_0^2/8\pi$ and the upper limit corresponds to the condition for the existence of an inertial range $k_0\ll k_*$. For larger pumps the oscillations arising as a result of the modulational instability fall at once into the absorption region. A detailed analysis of that case is very difficult and a modeling of the process on a computer is here very important. We indicate merely that it follows from the balance equation which we now write in the form

$$\nu_{eff}\frac{E_0^2}{8\pi}=\Gamma\left(k\lambda_D=\left(\frac{W}{nT}\right)^{1/2}\right)W, \tag{3.12}$$

that for large pumps the increase in W must be stabilized (in the opposite case we are shifted rather deeply into the absorption region which leads to a steep increase in the damping rate Γ).

The way ν_{eff} depends on the pumping energy $\sim E_0^2$ is shown in Fig. 3, and the dotted line corresponds to pumps for which there is no inertial range.

§4. BUILD-UP OF SHORT-WAVELENGTH SOUND AND DYNAMICS OF PLASMA TURBULENCE IN A NON-ISOTHERMAL PLASMA

An analysis of numerical experiments[12,13] enabled us to draw attention to an important feature of the collapse of Langmuir waves. When a caviton with plasmons trapped in it reaches sufficiently small dimensions when absorption of plasma waves is starting, the equilibrium between the high-frequency and the gas-kinetic pressure is violated. The caviton collapses and the excess density variation is discarded in the form of sound waves emitted from the caviton (Fig. 4).

In a separate caviton the fraction of plasmon energy which is transformed into the energy of low-frequency motions is small in the ratio $k^2\lambda_D^2$. Indeed, when $k\lambda_D\ll 1$ the energy of the low-frequency motions is mainly given by the ion oscillations

$$W_i\approx\frac{Mn_0}{2}\int dr\left(\frac{\partial\xi}{\partial t}\right)^2$$

(ξ is the displacement in the low-frequency motions, $\mathrm{div}\,\xi=-\delta n/n_0$) and using (2.2) we easily get the following estimate:

$$W_i=\frac{Mn_0}{2}\int\left(\frac{\partial\xi}{\partial t}\right)^2dr\approx Ml^2(t)\int\frac{\delta n}{n_0}\frac{\partial^2\delta n}{\partial t^2}dr$$
$$\approx\frac{1}{16\pi}\int\frac{\delta n}{n_0}|\mathbf{E}|^2dr\approx k^{*2}\lambda_D^2\int\frac{|\mathbf{E}|^2}{16\pi}dr. \tag{4.1}$$

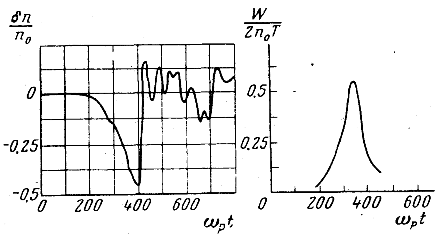

FIG. 4. Density variation and electrical field amplitude at the center of a caviton as functions of time in a numerical experiment.[12] When $\omega_pt\sim 400$ the energy of the Langmuir field is damped and then the caviton becomes a source for short-wavelength sound oscillations.

In an isothermal plasma ($T_e = T_i$) the sound emitted from a collapsing caviton is fast absorbed by ions and cannot appreciably affect the dynamics of plasma turbulence. The results of the preceding two sections are applicable just in that case. In a non-isothermal plasma with hot electrons ($T_e \gg T_i$) the process of the absorption of the sound by resonance electrons becomes slow. In that case there occurs a build-up of short-wavelength sound, and the conversion into that sound completely alters the dynamics of the turbulence. The present section of this paper is devoted to considering this problem.

We determine the stationary level of short-wavelength sound from the balance equation

$$\gamma_{\rm mod} W k^{*2}\lambda_D^2 \approx \gamma_s W_s, \quad W_s \approx \frac{n_0 T}{2}\sum_k \frac{|\delta n_k|^2}{n_0^2},$$

W_s is the energy of the short-wavelength sound pulsations, $\gamma_s \approx \omega_p m k_* \lambda_D / M$ is the damping rate due to their resonance absorption by electrons. As a result we get a very high level of sound:

$$\sum_k \frac{|\delta n_k|^2}{n_0^2} \approx 2\left(\frac{M}{m}\right)^{1/2}\left(\frac{W}{n_0 T}\right)^{1/2} k_*^2\lambda_D. \tag{4.2}$$

The presence of strong short-wavelength sound leads to a stabilization of the collapse. This effect is connected with the occurrence of an additional channel for transferring the plasmons to the short-wavelength absorption region as a result of direct conversion through sound pulsations.

If the acoustic turbulence is isotropic and weak, i.e.,

$$\sum_k \frac{|\delta n_k|^2}{n_0^2} \ll (3k_*^2\lambda_D^2)^2,$$

the characteristic growth rate which determines the conversion speed turns out to be equal to

$$\gamma_{\rm conv} = \frac{1}{18}\sum_k \frac{|\delta n_k|^2}{n_0^2}\frac{\Gamma_k}{k^4\lambda_D^4}, \tag{4.3}$$

where Γ_k is the damping rate for the absorption of short-wavelength plasmons, given by Eq. (2.8). A study of the dynamics of collapsing cavitons, taking conversion into account, has shown[14] that stabilization of the collapse occurs when the damping rate of the plasma waves caused by conversion is larger than the growth rate of the modulational instability. Indeed, when one takes conversion into account the plasmon energy in the caviton is damped according to the rule $E^2 \propto \exp(-2\gamma_{\rm conv} t)$, and using Eq. (2.2) one can show that the density well produced by the radiation pressure for $\gamma_{\rm mod} \ll \gamma_{\rm conv}$ turns out to be insufficiently deep to trap the plasmons:

$$\frac{|\delta n|}{n_0} \sim \frac{\gamma_{\rm mod}}{\gamma^2_{\rm conv}} k_0^2\lambda_D^2 \ll k_0^2\lambda_D^2.$$

As a result we apply the analysis of plasma turbulence neglecting the build-up of short-wavelength sound in a non-isothermal plasma for sufficiently small W:

$$W \leqslant W_{\rm thr} = n_0 T\left(\frac{81m}{M}\right)^{1/3} k_*^2\lambda_D^2, \tag{4.4}$$

when the level of the short-wavelength sound given by Eq. (4.2) corresponds to the condition $\gamma_{\rm conv} < \gamma_{\rm mod}$. One can use Eq. (3.10) to show easily that inequality (4.4) corresponds to a pumping

$$E_0^2 \leqslant E^2_{0\,\rm thr} = 8\pi n_0 T\left(\frac{6m}{M}\right)^{1/4}\frac{k_*\lambda_D}{\alpha}. \tag{4.4'}$$

At large pump strengths the sound appreciably affects the dynamics of Langmuir turbulence. The complete stabilization of the collapse can then not occur as sound waves are produced just in collapsing cavitons. As a result a regime is established in which the energy of the sound pulsations is maintained at a level determined by the threshold condition $\gamma_{\rm conv} = \gamma_{\rm mod}$:

$$\sum_k \frac{|\delta n_k|^2}{n_0^2} = 18\left(\frac{W}{n_0 T}\right)^{3/4}(k_*\lambda_D)^{5/2}. \tag{4.5}$$

The condition for the acoustic turbulence determined by this relation to be weak

$$\frac{1}{9}\sum_k \frac{|\delta n_k|^2}{n_0^2} \ll (k_*\lambda_D)^4,$$

is the same as the condition $k_0 \ll k_*$ for the existence of an inertial range. In the case $W > W_{\rm thr}$ the conversion on sound is the main mechanism for the transfer of plasmons from the modulational instability region $k \sim k_0$ into the short-wavelength absorption region $k \sim k_*$. The energy of the short-wavelength Langmuir oscillations produced in the conversion,

$$W' = \frac{1}{8\pi}\sum_{k \geqslant k^*} |E_k|^2 \approx \frac{W}{9}\sum_k \frac{|\delta n_k|^2}{n_0^2 k^4\lambda_D^4},$$

turns out, when one takes (4.5) into account, to be the same as in the small W regime (collapse regime):

$$W' \approx W(k_0/k_*)^{3/2},$$

with the only difference that when conversion is taken into account the oscillations fall directly from the source region into the short wavelength region, omitting the inertial range. The collapse itself as mechanism for short-wavelength transfer of plasmons through the length scales of the inertial range is permitted only to the degree as it is necessary to maintaining the level (4.5) of the sound pulsations. In that case the balance Eq. (4.2) for a known sound level determines the flow of plasmon energy to short length scales proceeding through collapsing cavitons:

$$I \approx \omega_p\left(\frac{81m}{M}\right)^{1/2}(k_*\lambda_D)^{3/2}\left(\frac{n_0 T}{W}\right)^{1/4} W.$$

As the rate of collapse of a separate caviton remains of the order of $\gamma_{\rm mod}$, we can also write this flux in the form $\gamma_{\rm mod} W_{\rm cav}$ and we find that only a small fraction of the long-wavelength plasmons goes to short length scales along the collapse channel. Their energy is

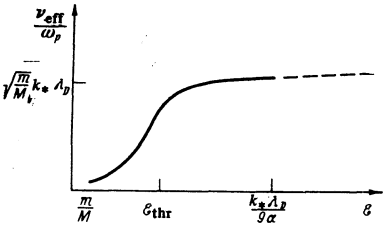

FIG. 5. Effective collision frequency as function of the pump amplitude in a non-isothermal plasma when the conversion due to short-wavelength sound is important ($\mathscr{E} = E_0^2/8\pi n_0 T$).

$$W_{cav} \approx W^{1/4} W_{thr}^{3/4}. \qquad (4.6)$$

The build-up of short-wavelength sound leads also to a change in the quantity ν_{eff}. Such a change is connected with two different mechanisms. On the one hand, the conversion of the pump by short-wavelength sound pulsations produces a new channel for dissipation and in ν_{eff} there appears a term equal to γ_{conv}. On the other hand, the dissipation mechanism considered in the preceding section is connected with scattering by long-wavelength density fluctuations produced by the modulational instability, and in Eq. (3.9′) for ν_{eff} there enters therefore only that part of the energy of the plasma oscillations which proceeds through the cavitons and leads to a modulation of the plasma density. In the regime of the developed conversion ($W > W_{thr}$) this energy is given by Eq. (4.6) and is small compared to W. We thus finally get the following formula for ν_{eff}:

$$\nu_{eff} \approx \alpha\omega_p \frac{W_{thr}^{3/4} W^{1/4}}{n_0 T} + \omega_p \left(\frac{m}{M}\frac{W}{n_0 T}\right)^{1/2}. \qquad (4.7)$$

The last term in Eq. (4.7) for ν_{eff} is usually small. Neglecting it and using the balance Eq. (3.10) (the dissipation of energy from the pump compensates for the short-wavelength transfer, caused in the present case by the conversion) we get the following equation for the connection between the energy of the plasma oscillations and the pump:

$$\frac{W}{n_0 T} = \left[9\alpha (k_* \lambda_D)^{3/2} \frac{E_0^2}{8\pi n_0 T}\right]^{4/5}. \qquad (4.8)$$

Using that equation we can express the effective collision frequency in terms of the pump wave amplitude:

$$\nu_{eff} \approx 9\alpha\omega_p \left(\frac{m}{M}\right)^{1/2} (k_*\lambda_D)^2 \left[9\alpha \frac{1}{k_*\lambda_D} \frac{E_0^2}{8\pi n_0 T}\right]^{1/5}. \qquad (4.9)$$

We show in Fig. 5 ν_{eff} as function of E_0. The fast growth of ν_{eff} for small pumps ($\nu_{eff} \propto E_0^4$ for $E_0 < E_{0thr}$) is practically saturated in the regmine of the developed conversion ($\nu_{eff} \propto E_0^{2/5}$). The discussion given here is valid under the condition that there exists an inertial range between the source and the absorption regions: $W/n_0 T \ll (k_* \lambda_D)^2$, i.e., $E_0^2/8\pi n_0 T < k_* \lambda_D/9\alpha$. The second term in ν_{eff} (equal to γ_{conv}) which we neglected when obtaining (4.8), (4.9) grows faster with W than the first one, but it can become appreciable only for pumps $E_0^2/8\pi n_0 T \sim k_* \lambda_D/9\alpha$ for which there is no inertial range.

On the whole the conversion through short-wavelength sound diminishes the effective collision frequency considerably and its magnitude turns out in that case to be less than the ion plasma frequency.

§5. LANGMUIR TURBULENCE FOR LARGE "MISMATCHES" OF PUMP AND PLASMA FREQUENCIES

So far we have studied the dynamics of Langmuir turbulence in the case when the frequency of the pump wave equals the plasma frequency. For many applications, for instance, for the problem of the heating of a laser pellet the study of mechanisms for the dissipation of high-frequency energy for large mismatches between the pump and the plasma frequencies is of interest. In that case the parametric instability of the electromagnetic pump, the linear theory of which one can find, e.g., in Ref. 10, leads to the production of rather short-wavelength plasma oscillations with $k\lambda_D \approx (\frac{2}{3}\Delta)^{1/2}$ ($\Delta = (\omega_0 - \omega_p)/\omega_p$ is the mismatch of the pump wave and plasma frequencies).

If

$$\Delta < \Delta_0, \quad \Delta_0 = \frac{M}{m}\left(\frac{E_0^2}{8\pi n_0 T}\right)^2,$$

the plasma oscillations are accumulated in the region of the spectrum in resonance with the pump up to the turning on of the modulational instability. The stationary level of the energy of the plasma oscillations is in that case determined by Eq. (3.11) and the dynamics of the turbulence is the same as that considered in the preceding sections—if collapse is taken into account the oscillations are transferred to the short-wavelength region and are absorbed by the particles.

The opposite case of large mismatches, $\Delta > \Delta_0$, when the spectrum of the plasma oscillations arising from the pump wave is modulationally stable and the spectral transfer to small k according to weak turbulence is important, is more complicated to analyze In the present section we shall consider in detail the case of an isothermal plasma, $T_e = T_i$, when sound waves are strongly damped. In that case the production of plasma oscillations from the pump wave is connected with the process of induced scattering by ions. The characteristic growth rate of such a process is[10]

$$\gamma_i \approx \omega_p \frac{E_0^2}{64\pi n_0 T}.$$

As usual in weak turbulence, the build-up of plasma oscillations in the region in resonance with the pump $\delta k \approx \lambda_D^{-1}(m/M)^{1/2}$ proceeds to a level comparable with it

$$\int dk\, W_k \sim \frac{E_0^2}{8\pi}.$$

The scattering of the plasma oscillations by ions leads to a spectral transfer to the region of small wavenumbers. In each scattering process the plasmon wavenumber is changed by an amount δk. The energy of the plasma oscillations in the region δk is of the order $E_0^2/8\pi$ so that the induced scattering leads to an approximately

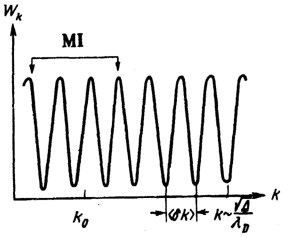

FIG. 6. Spectral transfer of plasma oscillations in the long-wavelength region according to weak turbulence and occurrence of the modulational instability (MI) at large "mismatches" $\Delta>\Delta_0$.

uniform distribution of the plasmon energy over the spectrum with a spectral density (Fig. 6)

$$W_k\approx\frac{E_0^2}{8\pi}\lambda_D\left(\frac{M}{m}\right)^{1/2}. \tag{5.1}$$

If the total width of the spectrum is large compared to δk, its evolution is with rather good accuracy described by the equation for differential transfer[10]:

$$\frac{\partial W_k}{\partial t}\approx\frac{16}{27\sqrt{2\pi}}\frac{W_k}{n_0 M v_{Te}\lambda_D^3}\frac{\partial W_k}{\partial k}. \tag{5.2}$$

If one takes (5.1) into account, it turns out that the flux of plasmon energy into the small k region, described by Eq. (5.2), is equal to $\gamma_i E_0^2/8\pi$. This flux reaches the long-wavelength region where the modulational instability is important and is there dissipated in collapsing cavitons. It is clear that the quantity ν_{eff} which determines the rate of dissipation of the pump wave energy in this case turns out to be equal to γ_i:

$$\nu_{eff}\approx\omega_p\frac{E_0^2}{64\pi n_0 T}. \tag{5.3}$$

The characteristic k for the long-wavelength region of the spectrum where the modulational instability arises are determined from the instability condition (see (3.7)):

$$W_0\sim\frac{E_0^2}{8\pi}k_0\lambda_D\left(\frac{M}{m}\right)^{1/2}\sim n_0 T k_0^2\lambda_D, \tag{5.4}$$

i.e.,

$$k_0\lambda_D\sim\left(\frac{M}{m}\right)^{1/2}\frac{E_0^2}{8\pi n_0 T}.$$

The energy of the long-wavelength plasma oscillations W_0 is then, as before, connected with E_0^2 through Eq. (3.11) with $\alpha=1$. The total energy of the plasma waves is appreciably larger:

$$W\approx\frac{E_0^2}{8\pi}\left(\frac{M}{m}\Delta\right)^{1/2}.$$

Owing to the large magnitude of the growth rate of the modulational instability the oscillations in the long-wavelength region of the spectrum remain all the time at the threshold of instability. The energy of the plasmons going to short length scales in the collapsing cavitons can be estimated from the balance equation:

$$W_{cav}\simeq\frac{\nu_{eff}}{\gamma_{mod}}\frac{E_0^2}{8\pi}\sim\frac{E_0^2}{8\pi}\ll W_0.$$

The fact that the ratio W_{cav}/W_0 is small explains the jump in ν_{eff} when one changes from the small mismatch regime to the large mismatch regime (Fig. 7). If $\Delta<\Delta_0$ a free development of the modulational instability is possible and $W_{cav}=W$. If $\Delta>\Delta_0$ only a small part of the energy of the long-wavelength plasmons goes into the cavitons. The magnitude of ν_{eff}, which is proportional to the energy W_{cav} in the cavitons, then discontinuously reduces to the growth rate of the parametric instability γ_i. The equality of ν_{eff} and γ_i is usual for weak turbulence and the jump in ν_{eff} when one goes over to the large mismatch regime where the long-wavelength transfer according to weak turbulence is important is given by the relation

$$\frac{\nu_{eff}^{<}}{\nu_{eff}^{>}}\approx\frac{W_{cav}}{W}\approx\left(\frac{m}{M\Delta}\right)^{1/2}\ll 1. \tag{5.5}$$

We show in Fig. 7 the "effective collision frequency" as function of the energy of a pump with a frequency which differs from the plasma frequency. The number 1 indicates the region of large pump amplitudes (small mismatches) when the plasmon spectrum in resonance with the pump turns out to be modulationally unstable, and the number 2 the region of small pump amplitudes (large mismatches) when the dynamics of the plasma turbulence is in two stages—a long-wavelength transfer to the modulational instability region due to scattering by ions and a subsequent transfer of energy by collapsing cavitons to the absorption region.

In a non-isothermal plasma, $T_e\gg T_i$, for large mismatches, $\Delta>\Delta_0$, the production of plasma oscillations from the pump wave is caused by decay of the pump into a Langmuir wave and sound. The energy flow along the spectrum to small k is in that case connected with a stagewise transfer during the decay of Langmuir waves and ceases, as before, on reaching the long-wavelength region where the modulational instability is important. As a whole, the picture of the evolution of the spectrum is to a considerable extent analogous to that expounded above for the case of an isothermal plasma with the only difference that now the quantity ν_{eff} is the same as the growth rate of the decay process

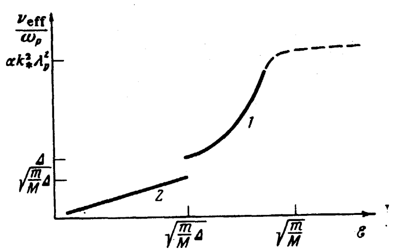

FIG. 7. Effective collision frequency in the case when the frequency "mismatch" of the pumping frequency is larger than the plasma frequency: $\Delta>m/M$.

$$\nu_{\rm eff} \approx \omega_p \left[\frac{E_0^2}{64\pi n_0 T}\left(\frac{m}{M}\Delta\right)^{1/2}\right]^{1/2}. \qquad (5.6)$$

For the energy of the plasma oscillations in the long-wavelength region of the modulational instability W_0 we have again Eq. (3.11) with $\alpha = 1$. In the large mismatch regime the energy of the oscillations going over into the cavitons is small compared to W_0:

$$W_{\rm cav} \approx \frac{\nu_{\rm eff}}{\gamma_{\rm mod}}\frac{E_0^2}{8\pi} \approx \frac{E_0^2}{64\pi}\left[\left(\frac{m}{M}\Delta\right)^{1/2}\frac{64\pi n_0 T}{E_0^2}\right]^{1/2} \ll W_0,$$

and, as before the jump in $\nu_{\rm eff}$ when we change to the large mismatch regime is connected with this.

The basic distinguishing feature of the case considered is that the long-wavelength sound produced in the decay of the Langmuir waves must lead to a randomization of the phases of the plasmons scattered by it and, hence, to the occurrence of an additional dissipation mechanism characterized by an effective frequency

$$\nu'_{\rm eff} \approx \omega_p \sum_{k_0\lambda_D} \frac{|\delta n_{k_0}|^2}{n_0^2}\frac{1}{3k_0^2\lambda_D^2}, \quad k_0\lambda_D < \Delta^{1/2}.$$

In the stationary state we have $\nu'_{\rm eff} \sim \nu_{\rm eff}$ by analogy with the problem of the conversion considered in the preceding section. Indeed, for large levels of long-wavelength sound, when $\nu_{\rm eff} < \nu'_{\rm eff}$ the dissipation mechanism connected with the stagewise transfer during the decay of Langmuir waves must cease, as it is impossible since just in such a transfer sound is generated. Both dissipation channels give therefore in the case considered approximately the same contribution to the effective collision frequency which is given by Eq. (5.6).

1)One-dimensional Langmuir turbulence has specific peculiarities connected with the absence of collapse and is the subject of a separate discussion.

2)A more exact definition ofthe limits of the absorption region will be given at the end of the present section.

3)One should bear in mind that the formula for the non-linear frequency shift was given for the case of an isotropic spectrum which is the most favorable one in the sense of the occurrence of electron non-linearities. In the one-dimensional case such a correction is totally absent. Numberical simulation of the collapse process[5] shows that a dipole caviton formed in the collapse is strongly anisotropic: $\theta^2 \sim 1/10$. The plasman spectrum in such a caviton is nearly one-dimensional and a small factor $\sim \theta^2$ appears in the formula for the non-linear correction to the frequency (see Ref. 7). In that case the threshold for the occurrence of an electron non-linearity increases substantially is compared to (2.9), (2.10):

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