Electron-hole liquid and the metal-dielectric phase transition in layered systems

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A phase transition of the gas-liquid type in a system of electrons and holes in a quasi-two-dimensional (layered) semiconductor is considered. The phase diagram, critical temperature and density for the transition are obtained. It is shown that near the critical point the transition is of a purely plasma nature.

In other words, bound states of the exciton type are absent in both (gas and liquid) phases.

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It was shown in a previous work\textsuperscript{11} that the role of correlation effects in electron-hole plasma (EHP) turns out to be anomalously large in semiconductors and semimetals possessing extremely strong anisotropies of the electron spectrum, in particular, layered systems. A significant decrease in the energy associated with interelectron correlations creates a tendency to "self-compression" of such a plasma, i.e., the formation of an electron-hole-liquid (EHL) with particular density \( n \gg E_0^2 \) and binding energy per electron-hole pair \( |E_{\text{BH}}| \gg E_0 \) where \( E_0 \) and \( \alpha \) are the binding energy and the effective radius of the hydrogen-like exciton. Formation of an EHL at low temperatures takes place through a first-order phase transition, in which the concentration of the carriers, free and bound in excitons, reaches some critical value \( n_{\text{cr}}(T) \) that depends on the temperature—the density of the saturated vapor. This situation can be realized both at thermodynamic equilibrium for a semiconductor with a sufficiently narrow forbidden band \( E_g \) upon increase in the temperature or decrease in \( E_g \) and under essentially nonequilibrium conditions, in which the critical concentration is achieved by introduction of selected carriers in intense excitation of the semiconductor.

In the first case, the formation of the EHL means a discontinuous change in the width of the forbidden band \( E_g \) to some negative value (overlapping of the bands), i.e., a transition of the initial semiconductor to a semimetal state. The second case corresponds to the so-called condensation of nonequilibrium carriers (or excitons) into EHL drops. With the same accuracy with which the nonequilibrium carriers can be assumed to be thermalized, the thermodynamics of both these transitions is identical and we shall consider them here using an idealized model in which the motion of the electrons and holes is two-dimensional, i.e., it takes place only in the plane of the layers without transitions between them.

For simplicity, the dispersion laws of the electrons and holes \( E(E_{\text{BH}}) \) will be assumed to be quadratic and isotropic in the plane of the layer: \( c_n(E) = \pm \sqrt{2m_nE} \). We shall also assume the permittivity tensor of the crystal to be isotropic in the plane of the layers. This tensor is characterized by two principal values \( c_{\alpha} \) and \( c_{\beta} \), for the directions along and perpendicular to the layers, respectively.

We emphasize that we are speaking of a permittivity without any contribution of free carriers. We shall use a system of units defined by the relations

\[
\epsilon = \frac{e^2}{4\pi\varepsilon_0}; \quad b = \frac{\hbar}{m^*}.
\]

where \( e \) and \( b \) are the charge on the electron and Planck's constant, and \( m^* \) is the reduced mass, \( m^* = m_e^{-1} + m_h^{-1} \).

In this work, along with the parameter of the ratio of the effective masses of the electron and the hole, \( s = m_e/m_h \) that is usually employed, we will find it convenient to use another parameter \( s = (1 - s)/\sqrt{s} \).

The system of electrons and holes in the considered model is described by the Hamiltonian

\[
H = \mu_n - \mu_h - \frac{1}{2} \sum_{q} \left( \frac{1}{2} \left( \frac{\gamma^2}{2} - \mu \right) \right) n_\alpha(q) \epsilon_{\alpha}\langle \nu\nu\rangle + \frac{1}{2} \sum_{q} V_{\alpha\beta}(q) \epsilon_{\alpha}\langle \nu\nu\rangle \epsilon_{\beta}\langle \nu\nu\rangle + \frac{1}{2} \sum_{q} \omega_\alpha(q) \epsilon_{\alpha}\langle \nu\nu\rangle + \frac{1}{2} \sum_{q} \omega_\beta(q) \epsilon_{\beta}\langle \nu\nu\rangle + \frac{1}{2} \sum_{q} V_{\alpha\beta}(q) \epsilon_{\alpha}\langle \nu\nu\rangle \epsilon_{\beta}\langle \nu\nu\rangle + \frac{1}{2} \sum_{q} \omega_\beta(q) \epsilon_{\beta}\langle \nu\nu\rangle + \frac{1}{2} \sum_{q} V_{\alpha\beta}(q) \epsilon_{\alpha}\langle \nu\nu\rangle \epsilon_{\beta}\langle \nu\nu\rangle.
\]

where \( n_{\alpha\beta}(q) \) and \( n_{\alpha\beta}(q) \) are the electron and hole creation operators in the layer \( l \) \((0,1,2,3,4,\ldots)\), and spin projections \( s_{\alpha} \) and momentum \( p \) in the plane of the layer; \( p_{\alpha} \) and \( q \) are two-dimensional vectors; the normalized area of the layer is set equal to unity:

\[
V_{\alpha\beta}(q) = \frac{1}{2\pi} \sum_{l} \epsilon_{\alpha}\langle \nu\nu\rangle \epsilon_{\beta}\langle \nu\nu\rangle.
\]

\[c^* = cE_\text{BH}/E^2_{\text{BH}}; \quad \epsilon \] is the distance between the neighboring planes; \( \mu_n \) and \( \mu_h \) are the chemical potentials of the electrons and holes, calculated from the bottom of the respective bands and connected by the condition of electric neutrality

\[\sum_{\alpha} \langle \epsilon_{\alpha}\nu\nu\rangle \alpha = -n_c \]

where \( n_c \) is the volume density of the number of electron-hole pairs; \( n_c \) is the surface density in a single layer.

The introduction of the chemical potentials \( \mu_n \) and \( \mu_h \) is necessary for the description of nonequilibrium systems, when the electrons and holes assumed to be dif-
ferent types of particles and their concentration \( n \) is specified arbitrarily. In a thermodynamically equi-
ilibrium situation, the chemical potential of the electrons in both bands should be the same, which adds an additional condition for \( \mu_p \) and \( \mu_h \):

\[
\mu_p + \mu_h = -E_{\text{F}}^p.
\]

where \( E_{\text{F}}^p \) is the width of the forbidden band, i.e., the distance between the initial values of the energies of the electrons and holes in (1). It should be noted that \( E_{\text{F}}^{p(0)} \) is the unrenormalized width of the forbidden band, a width that enters into the bare dispersion law. The ac-
count given below of the interaction of electrons and holes leads to a significant difference between \( E_{\text{F}}^{p(0)} \) and the real width \( E_{\text{F}}^p \) is the Green's function of the electrons and \( \epsilon_k^+ = \epsilon_k^0 + \Sigma_k^+(\epsilon_k) \) is the renormalized width of the forbidden band right up to the transition from the semiconductor unrenormalized con-
tinuous spectrum \( E_{\text{F}}^{p(0)} > 0 \) to the semimetal \( E_{\text{F}}^p < 0 \) after renormalization.

The investigation of the thermodynamics of the con-
sidered models is most conveniently carried out by start-
ing from the dependence of the particle number density on the temperature and the chemical potential

\[
n_{\mu_p, \mu_h, T} = \frac{1}{\zeta}(\int d\epsilon \rho_{\mu_p}(\epsilon)\rho_{\mu_h}(\epsilon))^{-1} \int d\epsilon \rho_{\mu_i}(\epsilon) \epsilon \rho_{\mu_h}(\epsilon) - \epsilon \rho_{\mu_p}(\epsilon) + \Sigma_k^+(\epsilon) - \Sigma_k^-(\epsilon),
\]

where \( \rho_{\mu_i}(\epsilon) \) is the Green's function of the electrons and \( \epsilon \rho_{\mu_h}(\epsilon) - \epsilon \rho_{\mu_p}(\epsilon) \) is the self-energy parts, \( \Sigma_k^+ \) and \( \Sigma_k^- \) depend on the radius of the two-dimensional exciton, i.e., with account of the scales employed,

\[
c = 1
\]

We shall show below (formulas (43)–(49)) that upon sat-
sisfaction of the condition (6), in the region of concentra-
tions and temperatures defined by the inequalities

\[
c^i \leq c \leq c^f
\]

the self-energy parts \( \Sigma_{k^i} \) do not depend in first approxi-
mation on either the temperature or the arguments \( p \) and \( \epsilon \), and have the form

\[
\Sigma_{k^i} = -\epsilon(\epsilon/c)\xi(\epsilon/c)^2
\]

where the plus sign corresponds to electrons, and the minus to holes,

\[
\xi(\epsilon/c) = \frac{\Sigma_{k^i}(\epsilon)}{\partial^2 \rho_{\mu_p}(\epsilon)/\partial \epsilon^2}
\]

Thus, in this region of concentrations and tempera-
tures, the interaction in the electron-hole plasma leads to a narrowing of the forbidden band without any appreci-
able change in any other of the parameters of the electron spectrum. We shall also show that the entire re-
gion of existence of the electron-hole liquid, including the critical point, falls in the interval of concentrations and temperatures satisfying the conditions (7) and (8). In order to establish this, we consider first thermodynamics of the electron-hole plasma, starting out from the formulas (9) and (10), and then give the basis of the formulas themselves. Carrying out the summation and integration in (8) with account of the independence of \( \Sigma \) of \( p \) and \( \epsilon \), we obtain

\[
n_{\mu_p, \mu_h, T} = \frac{1}{\zeta}(\int d\epsilon \rho_{\mu_p}(\epsilon)\rho_{\mu_h}(\epsilon))^{-1} \int d\epsilon \rho_{\mu_i}(\epsilon) \epsilon \rho_{\mu_h}(\epsilon) - \epsilon \rho_{\mu_p}(\epsilon) + \Sigma_k^+(\epsilon) - \Sigma_k^-(\epsilon),
\]

Solving the relation (11) for \( \mu_{\mu_p, \mu_h, T} \) and combining the equa-
tions thus obtained, we find the dependence of the chemical potential of a single electron-hole pair \( \mu_p + \mu_h \) on \( n \) and \( T \):

\[
\mu(n, T) = E_{\text{F}}^p + T \ln \left\{ \frac{\exp(\frac{\mu(n, T)}{T})}{\exp(\frac{\mu(n, T)}{T} - \mu_{\mu_p, \mu_h, T})} \right\}
\]

where the coefficients \( A = \alpha(\beta) \) and \( \alpha(\beta) \) are connected in the following fashion with the function \( f(\beta) \) introduced pre-
viously by

\[
A = \frac{2\pi^2}{\zeta(3/2)^{1/2}} f(\beta).\]

The relation (12) is essentially the equation of state of the electron-hole plasma, written in the variables \( \mu \) and \( n \) instead of the more customary pressure and volume. Several typical curves, described by these relations for the simplest case \( \sigma = 1 \) (\( s = 0 \)), are given in Fig. 1. It is seen that they have the typical van der Waals character, i.e., at temperatures \( T > T_c \) there are two stable branch-
s \( n_{\mu_p, \mu_h, T} \) and \( n_{\mu_{\mu_p, \mu_h, T}} \) of the solutions. The lesser of the two stable solutions \( n_{\mu_{\mu_p, \mu_h, T}} \) corresponds to the gas-
ous phase, the greater \( n_{\mu_p, \mu_h, T} \) corresponds to the elec-
tron-hole liquid. The critical point \( (T_c, n_c) \) is determined by the two equa-
tions

\[
\rho_{\mu_p}(\beta) = \frac{(n_c - \beta n_c)}{n_c},
\]

which reduce to a single transcendental equation for the parameter \( n_c \) and \( T_c \):

\[
\mu_{\mu_p, \mu_h, T_c} = \frac{1}{T_c} \frac{\beta}{1 + \beta} \frac{\partial \mu_{\mu_p, \mu_h, T}}{\partial \beta} - \frac{1}{T_c} \frac{\beta}{1 + \beta} \frac{\partial \mu_{\mu_p, \mu_h, T}}{\partial \beta} + \frac{1}{T_c} \frac{\beta}{1 + \beta} \frac{\partial \mu_{\mu_p, \mu_h, T}}{\partial \beta}.
\]

The critical parameters of the liquid are expressed

\[
\frac{\mu_{\mu_p, \mu_h, T_c}}{T_c} = \frac{1}{2} \frac{\beta}{1 + \beta} \frac{\partial \mu_{\mu_p, \mu_h, T}}{\partial \beta} - \frac{1}{2} \frac{\beta}{1 + \beta} \frac{\partial \mu_{\mu_p, \mu_h, T}}{\partial \beta} + \frac{1}{2} \frac{\beta}{1 + \beta} \frac{\partial \mu_{\mu_p, \mu_h, T}}{\partial \beta}.
\]

FIG. 1. Dependence of the chemical potential on the density in relative units at various temperatures: curves 1) \( T = 0 \), 2) \( T = 0.25T_c \), 3) \( T = T_c \), 4) \( T = 2T_c \). The semi-axis \( n = 0 \) also belongs to the isotherm 1 at \( T = 0 \).
By virtue of (10), \(z_2\) depends only on a single parameter—the ratio of the masses of the electron and hole \(u\) (or \(s\)). The dependence of \(z_2\) on \(u\) is very weak:

\[z_2 = \frac{1}{2} \left( 1 + \frac{u}{2} \right) \left( 1 + \frac{u}{2} \right)\]  

With accuracy to one hundredth, the function \(z_2(u)\) can be obtained in explicit form by expansion of the right side of (15) in a series in \(z_2\) up to terms of third order:

\[z_2(u) = \frac{3}{2} \left( 1 + \frac{u}{2} \right) \left( 1 + \frac{u}{2} \right)\]  

At temperatures less than critical, one of the two stable solutions \(n_{\text{eq}}(n, T)\) and \(n_{\text{liq}}(n, T)\) is generally metastable. Only for a single value of the chemical potential \(n(T)\), determined for each temperature \(T\), is the existence of both phases possible with balanced densities \(n_{\text{eq}}(T) = n_{\text{liq}}(n(T), T)\) and \(n_{\text{liq}}(T) = n_{\text{eq}}(n(T), T)\).

The quantities \(n(T), n_{\text{eq}}(T)\) and \(n_{\text{liq}}(T)\) are determined by the conditions that the chemical potentials and pressures be equal in both phases:

\[\mu(T) = \mu(n_{\text{eq}}(T), T) = \mu(n_{\text{liq}}(T), T)\]  

At low temperatures \(T < T_c\):

\[n_{\text{eq}} = \frac{1}{2} \left( \frac{1}{2} \right) \left( 1 + \frac{u}{2} \right) \left( 1 + \frac{u}{2} \right)\]  

\[\mu_{\text{eq}} = \frac{1}{2} \left( \frac{1}{2} \right) \left( 1 + \frac{u}{2} \right) \left( 1 + \frac{u}{2} \right)\]  

where

\[n_{\text{eq}} = \frac{1}{2} \left( \frac{1}{2} \right) \left( 1 + \frac{u}{2} \right) \left( 1 + \frac{u}{2} \right)\]  

are respectively the equilibrium density and minimum densities of the ground state of the EHL at \(T = 0\). In this limiting case, the expressions (29), (30) go over into the formula (10) of Ref. 11. It is seen from Eqs. (21)-(23) and (30), (31) that the region of existence of the EHL satisfies the inequalities (7) and (8).

It should be noted that the expansion used in (20) and subsequently assures an accuracy to within a few percent in all the formulas. This accuracy will obviously be quite sufficient when a comparison is made with the experimental data, the more so that, in the region of real values of the distance between the layers \(c\), the expressions (21)-(23), (21) and (22) will be satisfied only qualitatively. The problems of the applicability of the present consideration are discussed at the end of the article.

Figure 2 shows a phase diagram in reduced units \(n/\rho_n\) and \(T/T_c\) constructed numerically according to (24), (25) for the case \(\sigma = 1\). The form of the phase diagram does not depend on the parameter \(c\) and depends weakly on the parameter \(\sigma\). It follows from the calculation that for all values of the parameters, the following relation between the critical temperature \(T_c\) and the binding energy of the EHL \(E_{\text{bind}}\) at \(T = 0\) is well satisfied:

\[T_c = 1.1 E_{\text{bind}}\]

This relation turns out to be quite general. Thus, it holds approximately for the experimentally observed phase diagrams of the EHL in germanium and silicon, which are never layered materials. The experimental data are for germanium (52), \(T_c = 6.5 \pm 0.1\) K, \(|E_{\text{bind}}| = 5.65 \pm 0.15\) meV; for silicon (53), \(T_c = 28.5 \pm 0.2\) K, \(|E_{\text{bind}}| = 22.5 \pm 0.2\) meV. Calculations of the critical point in germanium and silicon were also carried out in other researches (54). A similar relation evidently exists for liquid-gas transitions generally; see the note on page 288 of the book of Landau and Lifshitz.

However, we now return to layered system. In the considered model, the gas phase is a nondegenerate plasma, and the excitons in it do not play any significant role. Even at very low temperatures \(T < T_c\) their concentration \(\exp(-\mu(T)/|T|)\) is much less than the concentration of the free carriers, which is of the order of

\[
\text{FIG. 2. Phase diagram of the EMP-EHL transition on the (n, T) plane in relative units, } \sigma = 1.
\]
exp(βμ/2T), by virtue of the condition |μ/μ_c| e^{-βμ/2} < 2.
At temperatures that are close to critical, the density
of the carriers in the gas phase is so large that the exis-
tence of excitons in it is generally impossible.

We also give the formulas for the symmetric case n = 1.
The equation for x_n is materially simplified:
\[ y_n = \exp(-x_n) - 1. \] (33)
its solution is x_n = 0.5501 and Eqs. (21)-(23) reduce to
\[ n_n = \frac{1}{\beta} \exp(-x_n). \] (34)
\[ F_n = \frac{1}{\beta} \exp(-x_n). \] (35)
\[ m_n = -2 \ln \left( \frac{1}{\beta} \exp(-x_n) \right). \] (36)

Up to now we have had in mind a nonequilibrium
situation, in which the total number of electrons and holes in
the sample is given by the external source of the excita-
tion, although in all the other parameters, these carriers
are in equilibrium with the crystal lattice and with one
another. In the case of complete thermodynamic equilib-
rium, i.e., in the absence of an excitation, the formulas
(12)-(36) obtained above remain in force, but their mean-
ing is modified somewhat. The condition that the chem-
ical potentials of the electrons be the same in the va-
lence band and the conduction band (4) leads to the re-
placement in Eq. (12) of μ by \( E_c \), the unrenormalized
width of the forbidden band:
\[ E_c = -\frac{1}{\beta} \ln \left( \frac{1}{\beta} \exp(-x_n) \right). \] (37)

Equation (37) determines the dependence of the electron
and hole concentration \( n \) on the temperature \( T \) and on \( E_c \).

The actual (renormalized) width of the forbidden band \( E_c \),
by virtue of (9) and (10), is equal to
\[ E_c(T) = \frac{1}{\beta} \ln \left( \frac{1}{\beta} \exp(-x_n) \right). \] (38)

The gas-liquid phase transition means a discontinuous
increase in \( n(T) \) and therefore a decrease in \( E_c(T) \), i.e.,
semiconductor—semimetal transition at sufficiently
small values of the bare forbidden band. In the plane of the
variables \( (T, E_c) \) (Fig. 3) the curve \( E_c(T) = \mu(T) \),
described by the formulas (22)-(23), (27), (29), (31),
divides the regions of existence of the semimetallic (SM)
and semiconductor (SC) phases. In the region \( T < T_\mu \),
\[ |μ| < E_c \] the carrier concentrations \( n \) and the
width of the forbidden band \( E_c \) depend in discontinuous
fashion on \( T \) and \( E_c \):
\[ n = \omega_0(T), \quad E_c(T) = μ(T) = A(e^{βn})^{-1/2}(T). \] (39)

The general form of the expression (39) for \( E_c \) reduces in this approxima-
tion to \[ E_c(T) = \frac{1}{\beta} \ln \left( \frac{1}{\beta} \exp(-x_n) \right). \] (40)

The polarization operator
\[ \Pi(n, \omega_n) = \sum \frac{f_{\omega_n}(g_{\omega_n})}{(2\pi)^m} \] (41)

where the screened interaction \( f_{\omega_n}(g_{\omega_n}) \) is determined by the
(42) equation
\[ f_{\omega_n}(g_{\omega_n}) = \sum f_{\omega_n}(g_{\omega_n}) \] (43)

The form of the approximation (42) is the usual one (see, for
example, Ref. 3, p. 250); in the summation, however, it is now assumed that \( \Sigma_n \) is independent of its argu-
ments, an assumption discussed below.

The expression (39) for \( E_c \) reduces in this approxima-
tion to
\[ \Sigma_n(g_{\omega_n}) = \sum \frac{\phi_{\omega_n}(g_{\omega_n})}{(2\pi)^m} \] (44)

where \( f_{\omega_n}(g_{\omega_n}) \) is the Fermi distribution function. The
form of the expression (42) is the usual one (see, for
example, Ref. 3, p. 250); in the summation, however, it is now assumed that \( \Sigma_n \) is independent of its argu-
ments, an assumption discussed below.

The expression (39) for \( E_c \) reduces in this approxima-
tion to
\[ \Sigma_n(g_{\omega_n}) = \sum \frac{\phi_{\omega_n}(g_{\omega_n})}{(2\pi)^m} \] (45)

where the screened interaction \( f_{\omega_n}(g_{\omega_n}) \) is determined by the
equation
\[ f_{\omega_n}(g_{\omega_n}) = \sum f_{\omega_n}(g_{\omega_n}) \] (46)

The form of the expression (42) is the usual one (see, for
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tion to
\[ \Sigma_n(g_{\omega_n}) = \sum \frac{\phi_{\omega_n}(g_{\omega_n})}{(2\pi)^m} \] (47)

FIG. 3. Phase diagram of the semiconductor—semimetal transi-
tion on the plane of the variables \( (T, E_c) \), with both variables
in units of \( 1 E_m = |q_\mu| \). The continuous curve separates the
semiconducting (SC) from the semimetallic (SM) phase.
where \( V_0 \) is obtained by solution of the Eq. (40):

\[
V_0(q,\omega_n) = \frac{2\pi}{\nu} \left( 1 - \frac{\omega_n}{q} \right) \partial V(q,\omega_n) / \partial q = \frac{4\pi}{\nu} \left( \partial V(q,\omega_n) / \partial q \right) ^{1/2}.
\]  

(44)

The first term in the right side of (43) is the exchange correction to the dispersion of the electrons; the second is the correlation correction. The exchange part is obviously \(-\gamma / D\) and, as we shall see below, is small in comparison with the correlation part at concentrations satisfying the inequalities (7) and (8). For estimates of the correlation contribution \(\Sigma_{\text{corr}}\), we consider (44) in more detail.

For the behavior of the polarization operators, a knowledge of the quantity

\[
\langle q^{(1)} (\omega_n \gamma) \rangle
\]

is essential, where \(\gamma\) is the average momentum of the particles, \(\gamma = \langle \gamma \rangle (\text{in units of } \Gamma / 2\nu^{1/2})\). In the two limiting cases, we have

\[
\Pi(q,\omega_n) = -\frac{1}{\pi} \int \frac{dq'}{q'^2} \frac{1}{(\omega_n - \omega')^2 - \nu^2}, \quad \Pi(q,\omega_n) = -\frac{1}{\pi} \int \frac{dq'}{q'^2} \frac{1}{(\omega_n - \omega')^2 - \nu^2} \leq 1,
\]

(45)

In the case \(\omega_n \ll 1\) (7), we have \(\gamma = c \ll 1\). Therefore, the transferred momenta \(q \gg c\) correspond to the asymptotic form (46) of the polarization operator,

\[
\gamma^{-1} \ll \omega_n \ll 1
\]

and the contribution of the region \(q \ll 1\) to the integral for the correlation part \(\Sigma\) is very small. The basic contribution is made by the region \(q \gg 1\), and for \(p,\omega_n\) corresponding to (46) and the definition of \(\gamma\), we get

\[
\Sigma_{\text{corr}}(p,\omega_n) = -\frac{2\pi}{\nu} \int \frac{dq}{q^3} \left[ 1 - \frac{\omega_n}{q^2} \right] \partial V(q,\omega_n) / \partial q = \partial V(q,\omega_n) / \partial q = \frac{1}{\nu} \left( \partial V(q,\omega_n) / \partial q \right) ^{1/2}.
\]

(47)

Substituting (45) and (46) successively in (47), we can easily establish the fact that at \(\gamma > \gamma^{1/2}\) the basic contribution to (47) is made by \(q \gg \gamma^{1/2}\), which corresponds to the asymptotic form (46). But the inequality \(\gamma > \gamma^{1/2}\) by virtue of the definition of \(\gamma\) is equivalent to the inequalities (7) and (8), so that in the region of concentrations and temperatures of interest to us we should use the polarization operator in the form (46). Moreover, in (47) we can omit \(\gamma\) in the argument of \(G_0\), because of the inequality \(q > \gamma\) and, consequently, for real momenta of the particles \(\Sigma_{\text{corr}}\) does not depend on \(p\). In the summation over the frequencies, the basic contribution under these conditions is made by \(\omega_n \ll \gamma^{1/2}\); as is easily understood from (45) and (46). Therefore, by virtue of (8) and of the definition of \(\omega_n = 2\gamma\nu / \gamma\), the summation over \(\gamma\) can be replaced by the integration

\[
\sum_{\gamma} \rightarrow \int d\omega_n / 2\pi
\]

which is equivalent to neglect of the temperature dependence, i.e., the replacement of \(\Sigma_{\text{corr}}\) by its value at \(\gamma = 0\).

Finally, the dependence on \(\omega_n\), in the argument \(G_0\), in (47) also turns out to be unimportant for \(\omega_n \ll \gamma^{1/2}\), i.e., right up to energies significantly greater than the mean energy of the particles \(\gamma^2/2\nu\). With account of all these simplifications, (47) takes the form

\[
\Sigma_{\text{corr}}^2 = -\frac{2\pi}{\nu} \left( \frac{1 + \cos \left( \frac{1}{\omega_n} \right) \left( \omega_n^2 - \nu^2 \right) ^{1/2} - 1}{1 + \cos \left( \frac{1}{\omega_n} \right) \left( \omega_n^2 - \nu^2 \right) ^{1/2} - 1} \right)
\]

\[
\int dq \left[ 1 + \cos \left( \frac{1}{\omega_n} \right) \left( \omega_n^2 - \nu^2 \right) ^{1/2} - 1 \right]
\]

(48)

or, after the introduction of the new integration variables

\[
x = \frac{\omega_n}{\nu}, \quad y = \sin \left( \frac{1}{\omega_n} \right),
\]

(49)

The system of equations (11) and (49) together determines the dependence of \(\mu_{\text{corr}}\) and \(\Sigma_{\text{corr}}\) on \(n\). With relative accuracy \(\sim \gamma^{1/4}\) the terms \(\omega_n \approx \gamma^{1/4}\) from the right side of (49) and, after integration over \(\gamma\), (49) reduces to (9) and (10). In connection with this solution, it is necessary to make the following remark. The approximation that we have used is, strictly speaking, not the random phase approximation, although it is similar to it. In the approach that is usual for the RPA, one must use in (5), (39) and (41) the zeroth approximation for the Green’s function

\[
G_0^2(p,\omega_n) = (\omega_n - \Sigma_{\text{corr}})^{-1},
\]

(50)

Neglect of \(\Sigma_{\text{corr}}\) in the integrand of (49) actually corresponds to this. However, the total Green’s functions (in the principal approximation to

\[
G_{\gamma}(p,\omega_n) = (\omega_n - \Sigma_{\text{corr}})^{-1}
\]

(51)

must be used in (5), (11), (41) and (42). This difference is very important, since the Matsubara diagram technique is developed at fixed \(\gamma\) and not at fixed \(n\). Therefore, substitution of \(G_{\gamma}^\infty(p,\omega_n) = \langle \gamma \rangle / 2\nu^{1/2}\) and, correspondingly, exponentially small screening of the interaction \(\Sigma_{\text{corr}}\) in (49) (in other words, we would have found for each \(\gamma\) only a single solution corresponding at low temperatures to the gas phase and generally not suitable to \(T = T_\gamma\). The analysis given above shows the existence for the nonlinear system of Eqs. (39)-(41), (51) of a second solution of the form (41), (0), corresponding to high concentrations \(\gamma > \gamma^{1/2}\) to a liquid phase.

In conclusion, we discuss the limits of applicability of the results. The model used in the present work and

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A model introduced by Potts back in 1952 has recently become the subject of intensive study, with particular attention paid to the question of the order of the phase transition. The point is that the Hamiltonian of this different real systems: electric circuits at wave functions of the electrons on neighboring layers. The consideration given above, which does not take into account the dependence of $c_{\text{ex}}$ on $p_{\text{ex}}$, is therefore valid only so long as all the energies entering into the calculations significantly exceed the width $W$ of transverse-motion band, i.e., $W = c_{\text{ex}}^{-1/2}$. In the case of opposite sign of this inequality, the motion of the electrons is essentially three-dimensional and for the description of the EHL we must use the approach based on the study of the effective masses for motion in the layer $(m_{\text{t}})$ and perpendicular to it $(m_{\text{q}})$. The role of the small parameter of the theory is played in this case by $m_{\text{t}}/m_{\text{q}} < 1$ instead of $c$. 

The coefficient $A_0$ of Ref. 1, which determines the correlation contribution to the total energy, differs by the factor $\tilde{d}$ from the $A$ coefficient used in the present work in the self-energy parts of the electrons and holes. Moreover, in formulas (10) from Ref. 1, there are errors. The correct form of these formulas agrees with formulas (30) and (31) of this paper.

8. Translated by R. T. Bayer

Phase transitions in the Potts three-component model

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It is shown by the renormalization-group method that the Potts three-component model has a second-order phase transition. The dependence of the critical exponent of the susceptibility on the dimensionality of space is determined.

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A model introduced by Potts back in 1952 has recently become the subject of intensive study, with particular attention paid to the question of the order of the transition. The point is that the Hamiltonian of this model contains invariants of third order, and calculation within the framework of the self-consistent field yields a first-order transition at a number $q = 3$. The obtained critical dimensionality $d_{\text{c}} = 4.58$ greatly exceeds $3$, and in all probability a more exact calculation will not lead to $d_{\text{c}} < 3$.

The Potts model is a generalization of the Ising model, in which each site can be in one of the $q$ states, the interaction energy assuming only two values, depending on whether the nearest sites are in identical or different states. The Potts model with various $q$ describes different real systems: electric circuits at $q = 0$, percolation at $q = 1$, and nematic liquid crystals at $q = q_{\text{c}}$, or crystals with cubic symmetry at $q = 3$. Of course, at $q = 0$ we obtain the usual Ising model.

In the $q$-component Potts model, a phase transition is described by a tensor order parameter $Q_{\mu}$. The tensor $Q_{\mu}$ has no traces, is symmetric, and has the dimensionality $q$. In particular, in the phase transition from a nematic liquid crystal into an isotropic liquid, the role of the order parameter can be played by the anisotropic part of the dielectric tensor. The Hamiltonian of the model is of the form

$$k = \sum \epsilon_{\mu} Q^\mu + \zeta Q^0$$

where $Q^0, Q^\mu$, and $Q^\lambda$ denote respectively invariants of second, third and fourth order in $Q$. 


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