$$F(\theta)=-\frac{4(\alpha_1\alpha_2)^{3/2}(\alpha_1+\alpha_2)}{(qk_1k_2)^2}\frac{1}{(z-\cos\theta)^2}. \qquad (21)$$

This expression also has a peak for forward scattering. Examining Eqs. (18), (19), and (21), we can find the nature of the dependence of the amplitude on the scattering angle for small θ:

$$F(\theta)\sim(z-\cos\theta)^{-\nu-3/2}. \qquad (22)$$

4. These small-angle maxima of the differential cross section for exchange scattering, with sharpness increasing with the energy, may give the explanation for similar maxima observed in experiments on exchange excitation[8,9]

$$e+\mathrm{He}(1\,{}^1S)\rightarrow e+\mathrm{He}(2\,{}^3S). \qquad (23)$$

For a detailed comparison with experiment there is a need for more accuracy both in the experimental data, which at present provide only a point or two in the region of the maximum of the differential cross section, and in the theory—the calculation of an accurate coefficient of the exponential, which is a rather complicated problem. Nevertheless the results of the present paper reveal a qualitative agreement with the available experimental data, enable us to identify the characteristic parameter describing the process, and are a natural extension of the results found previously in the case of charge transfer, in which the two particles between which the exchange took place could be treated classically.

[1)] The interaction $1/r_1$ with the nucleus does not contribute to the partial amplitude F_L for $L\neq 0$ because of the orthogonality of the angular functions.

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Anisotropy of the polarization of hot photoluminescence in gallium arsenide crystals

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The theoretically predicted strong angular dependence of the degree of linear polarization ρ_L of hot photoluminescence in GaAs crystals, due to undulation of the equal-energy surfaces in the valence band, has been observed experimentally. The measurements were performed for samples cut parallel to the planes (100), (110), and (111) at an energy in the luminescence-spectrum 1.93 eV, which is close to the excitation energy (1.96 eV) when the relaxation effects can be neglected. The obtained dependences of ρ_L on the angles between the polarization vector of the exciting radiation and the crystallographic directions are in good agreement with the results of the theoretical calculation. The temperature dependence of ρ_L at different excitation polarizations is discussed.

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The spectrum of the hot photoluminescence of cubic *p*-GaAs crystals excited by linearly polarized light has revealed a noticeable linear polarization of the recombination radiation.[1] At frequencies close to the frequency $\hbar\omega_{ex}=1.96$ eV of the exciting line, the degree of linear polarization ρ_L^0 obtained in[1] was 0.14–0.18 and decreased to zero with energy relaxation of the photoexcited electrons. At the same time, anomalously large values of the degree of circular polarization were observed when the excitation was with circularly polarized light. The polarization dependences in the spectrum of hot luminescence in GaAs are described in greater detail in[2]. Dymnikov, D'yakonov, and Perel'[3] have shown that these singularities of hot photoluminescence are due to the fact that the momentum distribution of the photoexcited electrons is anisotropic in the case of interband absorption of light in semiconductors having the band structure of GaAs. Thus, in excitation from the heavy-hole band, electrons are produced with momenta that are predominantly perpendicular to the electric vector of the exciting light. The light thus aligns the momenta of the electrons after a fashion. Since the selection rules in radiative recombination and photoexcitation are identical, anisotropy of the momentum distribution of the electrons leads to linear polarization of the radiation, too. The distribution function becomes isotropic in the course of the energy and momentum relaxation, and this decreases the degree of polarization of the recombination radiation. The calculations in[3] were made in an approximation in which spherical symmetry of the valence band was assumed. Dymnikov[4] has shown, by taking into account the undulations of the equal energy surfaces in the valence band, that for this mechanism one should expect a strong dependence of the degree of linear polarization of the recombination radiation on the orientation of the polarization vector of the exciting light relative to the crystallographic axes. The values of ρ_L^0 lie in the interval from 0 to 0.25. The main cause of this strong anisotropy of ρ_L is connected with the following circumstance.[4]

Consider the high-frequency section formed in the luminescence spectrum when the electrons excited from the heavy-hole band return to this band (see Fig. 1). This maximum value of the effective mass in the heavy-hole band corresponds to a quasimomentum directed along [111], and the minimum value to a direction along [100]. The use of the GaAs band parameters given in[5] leads respectively to the values $0.92m_0$ and $0.355m_0$. In view of this large anisotropy at not very high temperatures, $T < \Delta\mathscr{E}_{vh}$ (see Fig. 1), the diagonal directions of the type [111] become predominantly populated in the heavy-hole band and this determines also the direction of the quasimomentum of the recombining electrons in the direct transitions. Taking this into account, Dymnikov[4] calculated for the recombination radiation the Stokes parameters $\xi_1 = l \sin\psi$ and $\xi_3 = l\cos 2\psi$, which characterize, as is well known, the polarization properties of the radiation.[6] The parameter ξ_2, which characterizes the degree of circularity, is equal to zero in this case. The Stokes parameters were calculated in[4] in a rectrangular coordinate system $e_1e_2e_3$, where $\mathbf{e}_1$ coincides with the direction of the polarization of the exiting light, and $\mathbf{e}_3$ is directed along the beam, (in the case when the directions of the exciting radiation and the luminescence coincide). The quantity l has the meaning of the maximum linear polarization, while ψ is the angle between the direction of the maximum linear polarization and $\mathbf{e}_1$. The parameter ξ_3 is numerically equal to the degree of linear polarization ρ_L relative to the axes $\mathbf{e}_2$ of the selected coordinate system:

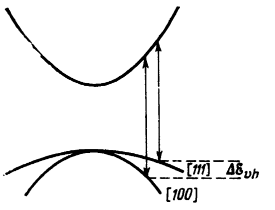

FIG. 1. Conduction band and heavy-hole band for two directions of the quasimomentum. The length of the arrows correspond to the excitation energy. $\Delta\mathscr{E}_{vh}$ is the corresponding energy gap between the directions [111] and [100] in the Brillouin zone for heavy holes. In the case of excitation with $\hbar\omega = 1.96$ eV the value of $\Delta\mathscr{E}_{vh}$ in GaAs amounts to 0.04—0.05 eV (at $T = 0$—300 °K) and depends on the temperature because of the $E_g(T)$ dependence.

$$\xi_3 = \rho_L = (I_\parallel - I_\perp)/(I_\parallel + I_\perp),$$

where $I_{\parallel,\perp}$ is the luminescence intensity polarized parallel (perpendicular) to the polarization of the exciting light $\mathbf{e}_1$. The quantities ρ_L, l, and ψ turn out to be, as already mentioned, functions of the angle φ between the direction $\mathbf{e}_1$ of the polarization of the exciting light and the principal crystallographic axis, which lies in the plane of the surface of the sample. These functions take the following forms.[4]

1. *Luminescence from the* (100) *plane.* The exciting beam is directed along [100]; in this case

$$\rho_L = \sin^2 2\varphi/4, \quad l = |\sin 2\varphi|/4, \quad \mathrm{tg}\, 2\psi = \mathrm{ctg}\, 2\varphi. \tag{1}$$

From the last equality it follows that $\psi + \varphi = 45°$, i.e., the direction of the maximum polarization always makes an angle of 45° with the [001] axis regardless of the direction of the polarization vector of the exciting light.

2. *Luminescence from the* (110) *plane.* The exciting beam is directed along [110]; in this case

$$\rho_L = (4 - \cos 2\varphi - 3\cos^2 2\varphi)/(17 - \cos 2\varphi). \tag{2}$$

The expressions for l and ψ in this geometry are given in[4].

3. *Luminescence from the* (111) *plane.* The exciting beam is directed along [111]. In this case the degree of polarization of the luminescence does not depend on the direction of the excitation polarization and amounts to $2/13 \approx 0.154$.

RESULTS OF EXPERIMENT AND DISCUSSION

The luminescence spectra were registered with a DFS-24 spectrometer and a photon-counting system. The excitation was by an LG-36 He-Na laser (1.96 eV). The (100) and (111) surfaces were finished by polishing and bright-dipping in a solution $H_2SO_4 : H_2O_2 : H_2O = 5:1:1$. The (110) surfaces were obtained by cleavage. The procedure for measuring the degree of polarization was described in detail in[2]. When the functions $\rho_L(\varphi)$ were plotted the angle φ was varied by rotating the sample around the normal to the surface.

Figure 2 shows in polar coordinates the experimental and calculated plots of $\rho_L(\varphi)$ for exciting light directed along [100], [110], and [111]. The measurements were made at a photon energy 1.93 eV, when the luminescence spectrum is formed exclusively by interband (conduction band–heavy-hole band) transitions. The experimental angular dependences are in all cases in good agreement with the calculation.

Figure 3 shows, the experimental plots of the luminescence intensity against the direction of the polarization plane of the analyzer for exciting light directed along [100] and $T = 100$ °K. The intensities were mea-

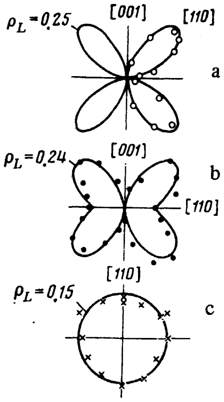

FIG. 2. Angular dependences of the degree of polarization $\rho_L(\varphi)$ in polar coordinates; φ is the angle between the polarization vector of the exciting light and the principal crystallographic axis, which lies in the plane of the surface as a sample. Solid curves—calculation, point—experiment. $\hbar\omega = 1.93$ eV, $T = 300$ °K. a, b, c) Luminescence from planes (100), (110), and (111) respectively. The carrier density for p-GaAs samples is 2×10^{19}, 1.6×10^{18}, and 2×10^{18} cm^{-3} for a, b, and c, respectively.

sured with the exciting-light polarization corresponding to angles φ equal to $-5°$, $15°$, and $45°$. The curves are ellipses whose major and minor semiaxes (a and b, respectively) determine the degree of maximum polarization of the luminescence $l = (a-b)/(a+b)$. As seen from Fig. 3, for all the angles φ, in accord with[4], the major semiaxis of the ellipse makes an angle of 45° with the [001] axis (and lies in the same quadrant as the polarization vector of the exciting light). The values of the degree of the maximum polarization given in Table I are well described by formula (1).

The measurements whose results are shown in Figs. 2 and 3 were made for a luminescence photon energy 1.93 eV, i.e., near the excitation line, when the relaxation effects are negligible. Figure 4 shows the values of ρ_L at two angles, $\varphi = 45°$ and $\varphi = 0°$, for excitation along [100] in a broad spectral interval that covers the entire region of energy relaxation. The figure shows also the luminescence spectrum. In the region of the luminescence of the thermalized electrons we have $\rho_L = 0$, i.e., there is no luminescence polarization.

We proceed to discuss the temperature dependence of the degree of linear polarization. The expression for

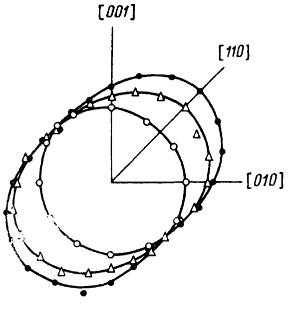

FIG. 3. Intensity of luminescence from the (100) plane as a function of the angle between the plane of polarization of the analyzer and the [001] axis for different values of the angle φ: $-5°$ (●), $15°$ (△), $45°$ (○); $\hbar\omega = 1.93$ eV, $T = 100$ °K, $p = 2\times10^{19}$ cm^{-3}.

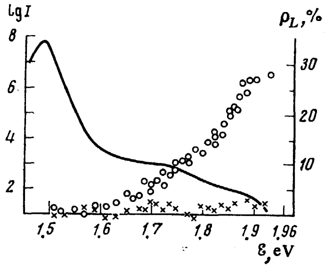

FIG. 4. Spectral dependence of the degree of linear polarization for two values of the angle φ: $\varphi = 45°$ (○) and $\varphi = 0°$ (×). Luminescence from the (100) plane. $p = 10^{18}$ cm^{-3}, $T = 100$ °K. Solid curve—spectrum of luminescence intensity I.

the Stokes parameters were obtained in[4] under the assumption that only "diagonal" electrons and holes, i.e., those with quasimomenta directed along [111], take part in the recombination. This is true for sufficiently low temperatures, when $T \ll \Delta_{vh}$. With increasing temperature it is necessary to take into account the fact that a certain contribution is made to the recombination by electrons and holes with quasimomenta in all directions. The parameter A in the expression for the Stokes parameters (formulas (11) and (12) of[4]) then turns out to depend on the temperature and takes the form

$$A = \left(\int \exp\left(-\frac{\mathscr{E}_{vh}}{T}\right)(\mu_{ch})^{3/2}\frac{1}{g^2}\,d\Omega\right)\Big/\int \exp\left(-\frac{\mathscr{E}_{vh}}{T}\right)(\mu_{ch})^{3/2}\,d\Omega, \quad (3)$$

where

$$\mathscr{E}_{vh} = \frac{\mu_{ch}}{m_h}(\hbar\omega - E_g), \quad \mu_{ch} = \frac{m_c m_h}{m_c + m_h}, \quad m_h = \frac{m_0}{\gamma_1 - 2|\gamma_2| g},$$
$$g = [1 + 3\gamma(\nu_x^2\nu_y^2 + \nu_y^2\nu_z^2 + \nu_x^2\nu_z^2)]^{1/2}, \quad \gamma = (\gamma_3^2 - \gamma_2^2)/\gamma_2^2,$$

ν is a unit vector along the quasimomentum; γ, γ_1, γ_2, γ_3 are the parameters of the valence band. The parameter A was calculated in accordance with formula (3) and in accordance with the degree of linear luminescence polarization ρ_L for an exciting beam directed along [100] and at angles 0 and 45° corresponding to the minimum and maximum values of ρ_L (see (1)). The calculation was made for temperatures 100, 300, and 400 °K, and also in the limit $T \gg \Delta\mathscr{E}_{vh}$ (we assumed $T = 10^5$ °K in the calculation). Account was taken also of the temperature dependence of the width E_g of the forbidden band. The values of the band parameters were taken from[5]. The calculation results and the experimental data are given in Table II. The experimental values of ρ_L and $\varphi = 0$ are somewhat smaller than the calculated ones. It appears that a somewhat better agreement between experiment and calculation can be obtained by allowing for the increased mass of the elec-

TABLE I. Degree of maximum polarization l when the exciting light is directed along [100].

φ, deg	l, %	
	experiment $T=100$ K	calculation $T=0$ K
−5	5±1	4,3
15	11±1	12,5
45	24±1	25,0

TABLE II. Temperature dependence of the degree of polarization ρ_L for an exciting beam directed along [100].

φ, deg	T, K	$\rho_\text{л}$, % (calculation) I	II	III	ρ_L, % (experiment)
0	100	4.7	3.6	3.2	1,5±1.0
	300	7.4	6.3	5.6	2,0±1.0
	400	7.8	6.8	6.1	4.5±1.0
	10^5	10.5	–	–	–
45	100	21.4	22.3	22.6	24.0
	300	19.5	20.3	20.8	22.0
	400	19.1	19.9	20.4	20.0
	10^5	17.1	–	–	–
	Values of parameters used in the calculation.				
	m_c/m_0	0.0655	0.11	0.0665	–
	γ	0.85	0.85	1.23	–

Note. In the calculations we used the values $\gamma=0.85$, $\gamma_1=7.65$, $|\gamma_2|=2.41$, $\gamma_3=3.28$[6].

trons due to the non-parabolicity of the conduction band (see, e.g., [7]). To illustrate this, column II to Table II shows the results of the calculations $m_c=0.1m_0$. A noticeable increase of γ (Table II, column III) does not improve significantly the agreement between the calculated and experimental data at $\varphi=0$. The reason for some discrepancy between them still remains unclear.

In the high-temperature limit, $T\gg\Delta\mathscr{E}_{vh}$, the populations for the different directions become equalized. However, as shown by the corresponding calculations (see the data for $T=10^5$ °K in Table II), the degrees of polarization for different excitation directions still differ noticeably. This "residual" anisotropy is obviously due to the anisotropy of the matrix elements for the interband transitions and leads to values of ρ_L equal to 0.105 and 0.17 for 0 and 45°, respectively.

We note in conclusion that the experimental results obtained in this paper serve as a direct confirmation of the correctness of the interpretation proposed in[3] for the polarization effects in the hot-photoluminescent spectrum.[1]

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Modulational instability of magnetohydrodynamic waves in a plasma

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We study the interaction between magnetohydrodynamic waves and a plasma with $\beta<1$. We show that this leads to a modulational instability for the fast magnetosonic waves at a well defined level of oscillation energy. As a result of the development of this instability the particle density and wave energy density start to increase in a certain region. The modulational instability of a beam of almost parallel waves can be stabilized in the weakly non-linear stage and a wave channel is then formed with an increased plasma density to which the waves are confined due to refraction. In the case of an isotropic wave distribution the compression of the plasma may proceed until the energy of the oscillations and of the particles becomes comparable with the energy of the stationary magnetic field. We discuss the possibility of observing the modulational instability of fast magnetosonic waves under natural conditions.

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Modulational instabilities play an important role in the formation of plasma turbulence. They affect the spectrum of the turbulent oscillations, the acceleration and heating of particles, the emission of electromagnetic waves, and so on. It is of interest in that connection to consider the possibility of the development of the modulational instability of magneto-hydrodynamic waves, since MHD turbulence is widespread in cosmic and laboratory plasmas. The modulational instability of MHD waves must qualitatively differ strongly from the modulational instability of Langmuir waves which has been studied in much more detail.[1-4]