Geometric and cyclotron resonances of the velocity of sound in metals

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(Submitted June 26, 1976)
Zh. Eksp. Teor. Fiz. 71, 2389–2401 (December 1976)

Acoustic cyclotron resonance and geometric oscillations of the sound velocity in metals with an arbitrary conduction-electron dispersion law are investigated theoretically. Special attention is paid to the role of the electromagnetic waves accompanying the sound propagation. A resonance is predicted which is due to coupling between the sound oscillations and the shortwave branches of the transverse cyclotron waves. In some cases, resonant coupling of the waves causes the solenoidal electromagnetic field to play the main role in the dispersion and absorption of the sound. The contribution of the fields is predominant in the case of an isotropic dispersion law for transverse sound.

PACS numbers: 76.40.+b, 72.50.+b

1. INTRODUCTION

Among the many methods of study of the interaction of electrons with acoustical vibrations in metals, a significant place is occupied by the investigation of the dispersion of the sound velocity. The various magnetoacoustic resonances of the sound absorption coefficient are well studied. Thanks to the significant progress in obtaining super-pure materials, the observation of a number of resonance effects for the dispersion of the sound velocity have also become possible.[1–5]

Upon satisfaction of the inequalities

$$ql,\ qR>1 \tag{1.1}$$

in a magnetic field **H** perpendicular to the direction of sound propagation (**q** is the sound wave vector; l and R are the free path length and the cyclotron radius of the electron), geometric oscillations of the absorption coefficient are observed in metals.[6] Geometric oscillations of the sound velocity s were discussed by Rodrigues for the model of free electrons in the low frequency region ($\omega \ll \nu$)[7] (ω is the sound frequency; $\nu = \tau_0^{-1}$ is the reciprocal of the relaxation time). Oscillations with relative amplitude $\Delta s/s \sim 10^{-4}$–10^{-5} were predicted. This effect was discovered experimentally[1,2] in Al, Cd, Cu. Recently, anomalously large oscillations of the longitudinal sound velocity were reported in Ga ($\Delta s/s \sim 6\%$).[5] The oscillations take place both upon change in the value of the magnetic field and in the change of its orientation in the plane $\mathbf{H} \perp \mathbf{q}$. The measurements were carried out in weak magnetic fields at high frequencies, when $\omega \sim \nu \sim \Omega$ (Ω is the cyclotron frequency).

In the range of frequencies

$$\omega \sim \Omega > \nu \tag{1.2}$$

the geometric oscillations of the absorption coefficient Γ are modulated by the acoustic cyclotron resonance (ACR).[8,9] A similar effect should take place also for the dispersion of the sound velocity.

Near the cyclotron resonance, weakly damped electromagnetic waves should exist in the metal, viz., two transverse, ordinary and extraordinary, and one longitudinal wave, propagating transverse to the magnetic field.[10] These waves can be excited by an external sound wave and turn out to have a significant effect on the character of its propagation. In the present paper, the velocity dispersion and the sound absorption are studied theoretically in metals with an arbitrary Fermi surface, with account of the electric fields in the region (1.1).

2. STATEMENT OF THE PROBLEM. GENERAL FORMULAS

For the calculation of the absorption coefficient $\Gamma(H)$ and of the change in the velocity $\Delta s(H)$, it is necessary to solve the complete set of equations describing the sound propagation in an unbounded metal. It consists of the equations of vibrations of the lattice, the Maxwell equations, and the kinetic equation for the conduction electrons:

$$\rho \ddot{u}_i - \lambda_{izzl} \frac{\partial^2 u_l}{\partial z^2} = \frac{\partial}{\partial z} \sum \frac{2}{h^3} \int m\,dp_x \oint d\tau \Lambda_{iz} \chi + \frac{1}{c} [\mathbf{j} \times \mathbf{H}]_f, \tag{2.1}$$

$$\frac{\partial^2 E_\alpha}{\partial z^2} = -\frac{4\pi i \omega}{c^2} j_\alpha, \quad j_z = 0 \quad (\alpha = x, y), \tag{2.2}$$

$$(\nu - i\omega)\chi + v_z \frac{\partial \chi}{\partial z} + \Omega \frac{\partial \chi}{\partial \tau} = e\left\{\mathbf{E} + \frac{1}{c}[\dot{\mathbf{u}} \times \mathbf{H}]\right\}\mathbf{v} + \Lambda_{iz} \dot{u}_{iz}, \tag{2.3}$$

$$\mathbf{j}(z) = \frac{2e}{h^3} \sum \int m\,dp_x \oint d\tau\, \mathbf{v}(\tau) \chi(\mathbf{p}, z). \tag{2.4}$$

Here ρ is the density of the metal; $\mathbf{u} = \mathbf{u}_0 \exp\{iqz - i\omega t\}$ is the displacement vector, u_{ik} is the strain tensor, λ_{iklm} is the elastic modulus tensor; the sign Σ denote summation over groups of carriers; m, $\mathbf{v}$, $\mathbf{p}$, e are respectively the cyclotron mass, the velocity, momentum and charge of the electron; τ is the dimensionless time of its motion along the orbit; $\Lambda_{ik}(\mathbf{p}) = \Lambda_{ik}(-\mathbf{p})$ is the renormalized deformation potential tensor ($\Sigma \int m\,dp_x \oint d\tau \Lambda_{ik} = 0$); $\mathbf{E}$ is the intensity of the electric field excited by the sound; $\mathbf{j}$ is the current density; $-\chi(\mathbf{p}, z)\delta(\varepsilon - \varepsilon_F)$ is the nonequilibrium contribution to the distribution function. The x axis is directed along the vector **H** and the z axis along the vector **q**. The direction of propagation of the wave is identical with one of the principal axes of the tensor λ_{izzl}, which allows us to consider purely longitudinal and purely transverse vibrations.

The right side of the equation of the vibrations (2.1) represents the force of deformation and induction origin, with which the electrons act on the lattice. This force, which governs the electron absorption and renormalization of the sound velocity, is small in comparison with the lattice elasticity force. Therefore, we can use the method of successive approximations to find the functions $\Gamma(H)$ and $D(H)=\Delta s/s$ ($s\equiv s_i\equiv(\lambda_{izzi}/\rho)^{1/2}$ is the unperturbed value of the sound velocity):

$$\Gamma_i(H)=\omega^{-1}\operatorname{Re}\Phi_i,\quad D_i(H)=\omega^{-1}\operatorname{Im}\Phi_i; \qquad (2.5)$$

i is the polarization number; Φ_i is a complex increment to the sound frequency. The induction terms in Eqs. (2.1) and (2.3) are smaller by a factor of at least $(qR)\gg 1$ than the deformation terms, and we neglect them.

It is convenient to represent the frequency increment Φ_i in the form of a sum

$$\Phi_i=\Phi_{ei}+\Phi_{\Lambda i}. \qquad (2.6)$$

The term Φ_{ei} is due to the solenoidal electric field, the Fourier components of which are equal to

$$\mathscr{E}_k(q)=Q_{ki}u_i,\quad Q_{zi}=-\sigma_{zz}^{-1}[\omega qP_{zi}+\sigma_{z\alpha}Q_{\alpha i}]\quad(k=x,y,z),$$
$$Q_{\alpha i}=\frac{4\pi i\omega^2 q}{c^2}\left\{q^2\delta_{\beta\alpha}-\frac{4\pi i\omega}{c^2}\left[\sigma_{\beta\alpha}-\frac{\sigma_{\beta z}\sigma_{z\alpha}}{\sigma_{zz}}\right]\right\}^{-1}\left[P_{\beta i}-\frac{\sigma_{\beta z}}{\sigma_{zz}}P_{zi}\right]\ (\alpha,\beta=x,y).$$

The field increment to the frequency has the form

$$\Phi_{ei}=(2\rho s)^{-1}\sum_{k=x,y,z}K_{ik}Q_{ki}. \qquad (2.7)$$

The symbol $\{\cdots\}^{-1}$ denotes the reciprocal matrix; $\sigma_{\beta\alpha}$ is the conductivity tensor; $P_{\beta i}$ and $K_{i\alpha}$ are the transformation coefficients of the sound wave into the electromagnetic and the electromagnetic into the sound:

$$\sigma_{\beta\alpha}=e^2L[v_\alpha,v_\beta],\quad P_{\beta i}=eL[v_\beta,\Lambda_{iz}],\quad K_{i\alpha}=eL[\Lambda_{iz},v_\alpha]. \qquad (2.8)$$

Here we have introduced the notation $L[f,\varphi]$ for the following quantity:

$$L[f,\varphi]=\frac{2}{h^3}\sum\int\frac{m\,dp_x}{\Omega}\oint d\tau\,f(\tau)\int_{-\infty}^{\tau}d\tau'\,\varphi(\tau')\exp\left\{\int_{\tau}^{\tau'}\left[\gamma+\frac{iqv_z}{\Omega}\right]d\tau''\right\},$$
$$\gamma=(\nu-i\omega)/\Omega. \qquad (2.9)$$

Using the condition (1.1) and applying the stationary-phase method, we obtain an asymptotic expression for L. To simplify the final expressions, we assume that the sections of the Fermi surface by the plane $p_x=\text{const}$ are convex, i.e., there are at most two points of stationary phase τ_1 and τ_2, for which $v_z(\tau_1)$ and $v_z(\tau_2)=0$, $v_z'(\tau_1)<0$, $v_z'(\tau_2)>0$, $\tau_2>\tau_1$. We have

$$L[f,\varphi]=\frac{1}{q}\frac{2\pi}{h^3}\sum\int m\,dp_x\left\{\sum_{m=1,2}\frac{f(\tau_m)\varphi(\tau_m)}{|v_z'(\tau_m)|}\operatorname{cth}\left[\pi\frac{(\nu-i\omega)}{\Omega(p_x)}\right]\right.$$
$$+\frac{1}{\operatorname{sh}\pi\gamma|v_z'(\tau_1)v_z'(\tau_2)|^{1/2}}[(f(\tau_1)\varphi(\tau_2)+f(\tau_2)\varphi(\tau_1))\sin qd(p_x)$$
$$\left.+i(f(\tau_1)\varphi(\tau_2)-f(\tau_2)\varphi(\tau_1))\cos qd(p_x)]-\frac{i}{\pi}\int_0^{2\pi}\frac{d\tau f(\tau)\varphi(\tau)}{v_z(\tau)}\right\}. \qquad (2.10)$$

Here $d(p_x)=(c/eH)[p_y(\tau_1)-p_y(\tau_2)]$. The latter term takes into account the contribution to the integral (2.9) from the "noneffective" portions of the electron orbits ($v_z(\tau)\neq 0$). Its appearance is due to the fact that, in the asymptotic expression for the double integral (2.9), the principal role is played by the "boundary" of the integration region $\tau'=\tau$. The finite upper limit in the integral with respect to τ' gives the value of the imaginary part of the integral (2.9) at $H=0$. The contribution from the "effective" points $\tau=\tau'=\tau_m$ is taken into account by the first two terms in Eq. (2.10). In order to obtain expressions for the conductivity tensor and the transformation coefficients (2.8), it is necessary to take into account the symmetry properties of the functions $\Lambda_{iz}(\mathbf{p})$ and $\mathbf{v}(\mathbf{p})$:

$$\Lambda_{iz}(\tau_1,p_x)=\Lambda_{iz}(\tau_2,-p_x),\quad \mathbf{v}(\tau_1,p_x)=-\mathbf{v}(\tau_2,-p_x).$$

The second term in Eq. (2.6) will be called the deformation term and will be written in the form of a sum

$$\Phi_{\Lambda i}=\frac{\omega^2}{\rho s^2}L[\Lambda_{iz},\Lambda_{iz}]=\Phi_{\Lambda i}^{(1)}+\Phi_{\Lambda i}^{(2)}, \qquad (2.11)$$

where

$$\Phi_{\Lambda i}^{(1)}=\Phi_0\sum\int_0^1 dx\,C_i^{(1)}(x)\operatorname{cth}\left[\pi\frac{\nu-i\omega}{\Omega(x)}\right],$$
$$\Phi_{\Lambda i}^{(2)}=\Phi_0\sum\int_0^1 dx\,C_i^{(2)}(x)\frac{\sin qd}{\operatorname{sh}\pi\gamma},\quad \Phi_0=\frac{4\pi\omega\Lambda^2m_0^2}{\rho sh^3}.$$

Λ is the characteristic value of the deformation potential, $\Lambda_{iz}(\mathbf{p})=\Lambda g_{iz}(\mathbf{p})$, v is the Fermi velocity, $\mathbf{v}=v\mathbf{n}(\mathbf{p})$, $m_0=p/v$, $p=p_{x\,\max}$,

$$C_i^{(1)}(x)=\frac{m(x)}{m_0}\sum_{m=1,2}\frac{g_{iz}^2(\tau_m,x)}{|n_z'(\tau_m,x)|}$$
$$C_i^{(2)}(x)=\frac{m(x)}{m_0}\frac{2g_{iz}(\tau_1,x)g_{iz}(\tau_2,x)}{|n_z'(\tau_1,x)n_z'(\tau_2,x)|^{1/2}}.$$

3. REGION OF HIGH FREQUENCIES

We consider first the region (1.2) of high frequencies at which ACR takes place. The first terms in the expressions (2.10) and (2.11) contain the resonance factor $\operatorname{coth}[\pi(\nu-i\omega)/\Omega(p_x)]$. Near the ACR, integration over p_x separates groups of electrons with extremal masses ($m'(p_l)=0$, where l is the number of the section with the extremal mass) and of electrons situated near the elliptical turning point.[1)] A condition for closeness to ACR is the smallness of the relative detuning from resonance $\Delta_l=(\omega-n\Omega_l)/\omega$ in comparison with unity. The form of the ACR lines for the absorption and dispersion coefficients is described by the formulas

$$\Gamma_{\Lambda i}^{(1)}=\omega^{-1}\operatorname{Re}\Phi_{\Lambda i}^{(1)}=\omega^{-1}\Phi_0\alpha C_i^{(1)}(p_l)\operatorname{Re}I(p_l), \qquad (3.1)$$
$$D_{\Lambda i}^{(1)}=\omega^{-1}\operatorname{Im}\Phi_{\Lambda i}^{(1)}=\omega^{-1}\Phi_0\alpha C_i^{(1)}(p_l)\operatorname{Im}I(p_l), \qquad (3.2)$$

where

$$\operatorname{Re}I(\Delta_l,\bar\nu)=\frac{1}{n}\left|\frac{(\bar\nu^2+\Delta_l^2)^{1/2}+\Delta_l\operatorname{sign}b}{2b(\bar\nu^2+\Delta_l^2)}\right|^{1/2},$$
$$\operatorname{Im}I(\Delta_l,\bar\nu)=-\frac{\operatorname{sign}b}{n}\left|\frac{(\bar\nu^2+\Delta_l^2)^{1/2}-\Delta_l\operatorname{sign}b}{2b(\bar\nu^2+\Delta_l^2)}\right|^{1/2}.$$

Here n is the number of the resonance; $\bar{\nu}=\nu/\omega$; $b = -(p^2/2m)(d^2m/dp_x^2)$; $\alpha=\frac{1}{2}$ at $p_l=0$; $\alpha=1$ at $p_l\neq 0$. If the ACR is due to electrons at the turning point, the formulas for Γ_Λ and D_Λ agree, with accuracy to a numerical factor, with the expressions (3.1) and (3.2) (here $b=-(p/m)(dm/dp_x)$).

The formula for the absorption coefficient Γ_Λ has been obtained and studied in Ref. 9. The dispersion $D_\Lambda^{(1)}$, as also the absorption coefficient, changes abruptly with change in the magnetic field. The maximum or minimum of D_Λ, which depends on the sign of b, is reached at $\Lambda_l=-3^{-1/2}\bar{\nu}$ sign b:

$$|D_{\Lambda i}^{(1)}|_{ext}=\frac{3^{1/4}\Phi_0 C_i^{(1)}(p_x)}{2^{5/4}\omega|b|^{1/2}}\bigg|_{p_x=p_l}\frac{1}{n\bar{\nu}^{1/2}}. \qquad (3.3)$$

The absorption coefficient $\Gamma_\Lambda^{(1)}$ is maximal at $\Delta_l=3^{-1/2}\bar{\nu}$ sign b and is equal to $|D_{\Lambda i}^{(1)}|_{ext}$.

The locations of the resonance peaks $\Gamma_\Lambda^{(1)}$ and $D_\Lambda^{(1)}$ are shifted in the magnetic field relative to the values at which $\Omega(H_n)=\omega/n$, by a value that is proportional to ν/n. The shift decreases as the number of the resonance increases. It occurs at high or low fields depending on the sign of the derivative d^2m/dp_x^2. The resonance lines have a symmetric shape. At $|\Delta_l|\gg\bar{\nu}$, the values of $D_\Lambda^{(1)}$ on the right and left wings differ by a factor $|\bar{\nu}/\Delta_l|$. The widths of the resonance curves $\Gamma_\Lambda^{(1)}$ and $D_\Lambda^{(1)}$ are determined by the parameter $\bar{\nu}$. The shift in the positions of the resonances of Γ has been observed experimentally in Ga. [11]

Far from resonance $|\Delta_l|\sim 1$, the dispersion $D_\Lambda^{(1)}$ is $\Omega/\pi\nu$ times greater than the absorption coefficient $\Gamma_\Lambda^{(1)}$:

$$\Gamma_{\Lambda i}^{(1)}=\frac{\pi\Phi_0}{2\omega}\sum\int_0^1\frac{\nu}{\Omega}\frac{C_i^{(1)}(x)}{\sin^2(\pi\omega/\Omega)}dx,$$

$$D_{\Lambda i}^{(1)}=\frac{\Phi_0}{2\omega}\sum\int_0^1 C_i^{(1)}(x)\,\mathrm{ctg}\frac{\pi\omega}{\Omega}dx.$$

In the calculation of the integrals

$$G=\int_0^1 dx\,F(x)\exp\{iqd(x)\}\mathrm{sh}^{-1}\pi\frac{\nu-i\omega}{\Omega(x)}, \qquad (3.4)$$

containing oscillating factors, competition develops between the rapidly changing functions $\sinh^{-1}[\pi(\nu-i\omega)/\Omega(x)]$ and $\exp\{iqd(x)\}$, which separate the sections with extremal masses and extremal diameters. Since the exponential function oscillates more rapidly than the resonance factor, greatest interest attaches to the contribution to the integral (3.4) from the sections with extremal values of $d(x)$. Here we must distinguish two cases, depending on whether or not the extremum of the diameter coincides with the extremum of the cyclotron frequency. If both extrema are the same, the functions $d(x)$ and $1/\Omega(x)$ can be expanded in series near their common extremum point up to quadratic terms, inclusive, and Eq. (3.4) takes the form

$$G=\alpha F e^{iqd}\int_{-\infty}^{\infty}dx\frac{\exp\{iqd''x^2/2\}}{\mathrm{sh}\,\pi[(\nu-i\omega)/\Omega+i\omega bx^2/\Omega]}, \qquad (3.5)$$
$$d''=d^2d/dx^2.$$

The values of the functions F, d, Ω, d'' are taken at the extremum point. In the range of magnetic fields $|b|^{-1}q|d''||\Delta+i\bar{\nu}|\gg 1$, the term with x^2 in the argument of the hyperbolic sine can be neglected and

$$G=\frac{\alpha F\exp\{iqd+\delta\}(2\pi)^{1/2}}{\mathrm{sh}\,\pi[(\nu-i\omega)/\Omega](q|d''|)^{1/2}}\qquad \delta=\frac{\pi}{4}\mathrm{sign}\,d''. \qquad (3.6)$$

At $\bar{\nu}q|d''|/|b|\gg 1$, this formula is valid for all values of the detuning Δ. If the latter inequality is violated, then, over a narrow region near resonance, when $q|d''\Delta|/|b|\ll 1$, the exponential in the numerator under the integral sign in (3.5) can be replaced by unity, and then

$$G=\alpha(-1)^n F e^{iqd} I(\Delta,\bar{\nu}). \qquad (3.7)$$

In this case, when the extrema of the diameter and the mass are not identical, then ACR in the quantity G will take place only under the condition $q|d''|(\Delta^2+\bar{\nu}^2)\gg 1$. It is then obvious that the formula (3.6) is applicable for the quantity G.

It follows from all of the above that, in the range of frequencies

$$\omega\ll v^2\nu/\Omega s,\quad\text{or}\quad \bar{\nu}\gg(qd)^{-1/2}, \qquad (3.8)$$

the function $G(\omega,\nu,\Omega)$ is determined by all the cross sections $p_x=p_\mu$ with the extremal diameters. The contribution to the frequency $\Phi_{\Lambda i}^{(2)}$ represents the superposition of the geometric oscillations modulated by the ACR:

$$\Gamma_{\Lambda i}^{(2)}=\frac{\Phi_0(2\pi)^{1/2}}{\omega}\sum_\mu\left[\alpha C_i^{(2)}(p_x)\frac{\sin(qd+\delta)}{(q|d''|)^{1/2}}\mathrm{Re}\frac{1}{\mathrm{sh}\,\pi\gamma}\right]_{p_x=p_\mu}, \qquad (3.9)$$

$$D_{\Lambda i}^{(2)}=\frac{\Phi_0(2\pi)^{1/2}}{\omega}\sum_\mu\left[\alpha C_i^{(2)}(p_x)\frac{\sin(qd+\delta)}{(q|d''|)^{1/2}}\mathrm{Im}\frac{1}{\mathrm{sh}\,\pi\gamma}\right]_{p_x=p_\mu}. \qquad (3.10)$$

The resonance values of the amplitudes $\Gamma_\Lambda^{(2)}$ and $D_\Lambda^{(2)}$ are achieved at $\Delta_\mu=0$ and $\Delta_\mu=\pm\bar{\nu}$ respectively, and are smaller by a factor of $(qd\bar{\nu})^{1/2}$ than the extremal values of $\Gamma_\Lambda^{(1)}$ and $D_\Lambda^{(1)}$. Far from the resonances, the amplitude of the oscillations of $D_\Lambda^{(2)}$ is $\bar{\nu}^{-1}$ times larger than the amplitude of $\Gamma_\Lambda^{(2)}$.

In the range of frequencies and magnetic fields

$$v\nu/s\omega\ll\Omega/\nu\ll v/s,\quad\text{or}\quad (qd)^{-1/2}\gg\bar{\nu}\gg(qd)^{-1}, \qquad (3.11)$$

the principal contribution to the function $G(\omega,\nu,\Omega)$ is made by the central section. The contribution of the remaining sections with extremal diameters does not have a resonance character. Therefore, only the terms from the central sections remain for the absorption coefficient and the sound velocity dispersion coefficient in the sums (3.9) and (3.10).

At very high frequencies and large magnetic fields,

$$v/s\ll\Omega/\nu\ll(v/s)(\omega/\nu)^{1/2},\quad\text{or}\quad (qd)^{-1}\gg\bar{\nu}\gg(qd)^{-2}, \qquad (3.12)$$

when the asymptotic form of the function G is given by (3.7), the increment to the frequency is equal to

$$\Phi_{\Lambda i}=\Phi_{\Lambda i}^{(1)}+\Phi_{\Lambda i}^{(2)}=\frac{\Phi_0}{2}\left\{C_i^{(1)}\left[1+\sin\left(qd-\frac{\pi\omega}{\Omega}\right)\right]I\right\}_{p_x=0}. \qquad (3.13)$$

(This formula was obtained earlier for the absorption coefficient.[9]) The period of change of the sine significantly exceeds the interval over H in which this formula is applicable. Outside this interval, the results for the absorption and dispersion coefficients do not differ from the case (3.11).

We now proceed to the analysis of the role of the electromagnetic field which accompanies the sound propagation in the metal. In the range of high frequencies (1.2), the field increment Φ_{ei} becomes as important as the deformation increment if cyclotron waves are excited.

The spectrum and the damping of the longitudinal cyclotron wave are determined by the equation $\sigma_{zz}(\omega, k, H) = 0$, as is well known.[10] This wave is weakly damped in the range of frequencies and wave vectors k in which $(kd)^2\bar{\nu} < 1$. In order that the sound spectra and the longitudinal cyclotron wave spectra overlap in this region, it is necessary to have pure samples and to use high acoustic frequencies: $\nu/\omega < (s/v)^2$. At the present time, this condition is difficult to achieve. Therefore, we shall not take into account the coupling of the sound with the longitudinal electric fields.

The spectrum of transverse cyclotron waves is found from the dispersion equation

$$\mathrm{Re}\,\det\left\{k^2\delta_{\alpha\beta} - \frac{4\pi i\omega}{c^2}\sigma_{\alpha\beta}\right\} = 0, \quad \alpha, \beta = x, y. \tag{3.14}$$

Their attenuation is small in the range of frequencies and magnetic fields at which

$$|\mathrm{Re}\,\sigma_{\alpha\beta}| \ll |\mathrm{Im}\,\sigma_{\alpha\beta}|. \tag{3.15}$$

The conductivity tensor is represented in the form of the sum $\sigma_{\alpha\beta} = \sigma^{(1)}_{\alpha\beta} + \sigma^{(2)}_{\alpha\beta}$. The first term $\sigma^{(1)}_{\alpha\beta}$ contains the cyclotron resonance, while the tensor $\sigma^{(2)}_{\alpha\beta}$ describes the geometric oscillations of the conductivity. The contribution to the tensor $\sigma^{(1)}_{\alpha\beta}$ from electrons with extremal masses, near ACR, is equal to

$$\sigma^{(1)}_{\alpha\beta} = \sigma_0[\alpha f_{\alpha\beta} I(\Delta, \bar{\nu})]_{p_x = p_l},$$
$$\sigma_0 = \frac{4\pi e^2 m_0 p v}{h^3 k}, \quad f_{\alpha\beta} = \sum_{m=1,2} \frac{m}{m_0} \frac{n_\alpha(\tau_m) n_\beta(\tau_m)}{|n_z'(\tau_m)|}. \tag{3.16}$$

The asymptotic values of the tensor $\sigma^{(2)}_{\alpha\beta}$ depend on the relations of the parameters $\bar{\nu}^{-1}$ and qd, and are determined by the equations (3.6) and (3.7).

It follows from the dispersion equation (3.14) that the necessary condition for the existence of waves near the cyclotron resonance is a positive value of the parameter b. The shortwave portion of the spectrum is formed by the resonant electrons with maximum cyclotron mass (minimum $\Omega(p_x)$). The inequality (3.15) is satisfied for the conductivity due to these electrons if $\bar{\nu} < |\Delta_l| \ll 1$ and $\Delta_l < 0$. The dissipative part of the conductivity can generally be increased at the expense of the other electrons, with non-maximal mass, the cyclotron frequency of which satisfies the condition $|\omega - n\Omega| \ll \omega$. This leads to the result that the spectrum of the weakly damped waves is located within the frequency intervals

$$(n-1)\Omega_{max} < \omega < n\Omega_{min} \tag{3.17}$$

($\Omega_{\max}$ and $\Omega_{\min}$ are the maximum and minimum values of the cyclotron frequency). Outside the interval (3.17), the damping of the cyclotron wave is greater. Here the field contribution to the absorption and to the sound-velocity dispersion turns out to be small in comparison with the deformation contribution. The frequency interval (3.17) exists, at least in the case $n = 1$.

The dispersion equation (3.14), generally speaking, admits of two solutions—for two transverse waves. If the absolute maximum of the mass is achieved on the central section, only one weakly damped—extraordinary—wave can exist in the metal ($\mathbf{E} \perp \mathbf{H}$). In the case in which electrons from the vicinity of the turning point have the greatest mass, the ordinary wave ($\mathbf{E} \parallel \mathbf{H}$) is undamped.

The field increment to the frequency (2.7) contains the resonance denominator $\det\{q^2\delta_{\alpha\beta} - 4\pi i\omega\sigma_{\alpha\beta}/c^2\}$. It follows from the relations (3.14) and (3.15) that the function $\Phi_{\varepsilon i}$ increases strongly near the crossing of the spectra of the acoustic and cyclotron waves. Owing to the oscillations of the conductivity $\sigma^{(2)}_{\alpha\beta}$, there is a large number of points of intersection of the dispersion curves. The behavior of the functions $\Gamma_i(H)$ and $D_{\varepsilon i}(H)$ then depends on the ratio of the absolute value of the oscillating conductivity $|\sigma^{(2)}_{\alpha\beta}|$ to the dissipative part $\sigma^{(1)}_{\alpha\beta}$. If

$$|\sigma^{(2)}_{\alpha\beta}| < \mathrm{Re}\,\sigma^{(1)}_{\alpha\beta}, \tag{3.18}$$

then the conductivity oscillations lead to small increments—to geometric oscillations of $\Gamma^{(2)}_{\varepsilon i}(H)$ and $D^{(2)}_{\varepsilon i}(H)$. In the opposite limiting case,

$$|\sigma^{(2)}_{\alpha\beta}| > \mathrm{Re}\,\sigma^{(1)}_{\alpha\beta} \tag{3.19}$$

the resonance denominator in the region of spectrum crossing oscillates with the period of the geometric resonance, changing from a value of the order of $4\pi\omega\,\mathrm{Re}\sigma^{(1)}_{\alpha\beta}/c^2$ to the value $4\pi\omega|\sigma^{(2)}_{\alpha\beta}|/c^2$. The plots of $\Gamma_{\varepsilon i}(H)$ and $D_{\varepsilon i}(H)$ represent a series of resonance peaks with the period of the geometric oscillations.

The condition (3.18) is satisfied in the frequency range of (3.8). The coefficients of absorption and sound velocity dispersion in this case can be written in the form $\Gamma_{\varepsilon i} = \Gamma^{(1)}_{\varepsilon i} + \Gamma^{(2)}_{\varepsilon i}$ and $D_{\varepsilon i} = D^{(1)}_{\varepsilon i} + D^{(2)}_{\varepsilon i}$, where

$$\Gamma^{(1)}_{\varepsilon i} = \frac{\Phi_0}{\omega} \sum_{r=1,2} \frac{\zeta_r c_r \,\mathrm{Re}\, I}{(y^3 - |\zeta_r \,\mathrm{Im}\, I|)^2 + (\zeta_r \,\mathrm{Re}\, I)^2}, \tag{3.20}$$

$$D^{(1)}_{\varepsilon i} = -\frac{\Phi_0}{\omega} \sum_{r=1,2} \frac{c_r (y^3 - |\zeta_r \,\mathrm{Im}\, I|)}{(y^3 - |\zeta_r \,\mathrm{Im}\, I|)^2 + (\zeta_r \,\mathrm{Re}\, I)^2}. \tag{3.21}$$

Here we have introduced the notation $y^3 = q^2/k_0^3$, $k_0^3 = 16\alpha\pi^2 e^2 \omega m_0 p v / c^2 h^3$,

$$c_r = \left\{\frac{|\zeta_r \xi_i - \eta_i|}{\pi(\zeta_1 - \zeta_2)}\right\}_{p_x = p_l}, \quad \xi_i = \Psi_{ix}^2 + \Psi_{iy}^2,$$
$$\eta_i = \Psi_{ix}^2 f_{yy} + \Psi_{iy}^2 f_{xx} - 2\Psi_{ix}\Psi_{iy} f_{xy},$$
$$\Psi_{i\alpha} = \frac{1}{2\pi} \int_0^1 \frac{m}{m_0} dx \oint_0^{2\pi} d\tau \frac{g_{iz} n_\alpha}{n_z},$$
$$\zeta_{1,2} = \tfrac{1}{2}[f_{xx} + f_{yy} + ((f_{xx} - f_{yy})^2 + 4f_{xy}^2)^{1/2}]_{p_x = p_l}.$$

The quantities $\zeta_{1,2}$ represent the principal values of the tensor $f_{\alpha\beta}$.

The absorption coefficient $\Gamma_{\varepsilon i}(H)$ reaches a maximum at values of the magnetic field satisfying the condition of coupling of the sound with the cyclotron wave:

$$\Gamma^{(1)}_{\varepsilon i\,max}=\frac{\Phi_0}{\omega}\frac{c_r}{\zeta_r \operatorname{Re} I(\Delta_{lr})}, \quad |\operatorname{Im} I(\Delta_{lr})|=\frac{q^3}{k_0^3\zeta_r}. \tag{3.22}$$

The dispersion $D^{(1)}_{\varepsilon i}(H)$ has a maximum and minimum at values $|\Delta_l|=-\Delta_{lr}\mp\bar{\nu}$:

$$|D^{(1)}_{\varepsilon i}|_{ext}=\frac{\Phi_0}{\omega}\frac{c_r}{2\zeta_r \operatorname{Re} I(\Delta_{lr})}. \tag{3.23}$$

The dispersion $D^{(1)}_{\varepsilon i}$ vanishes at the maximum of the absorption coefficient, since the resonance lines $\Gamma_{\varepsilon i}(\Delta_l)$ and $D_{\varepsilon i}(\Delta_l)$ have a Lorentzian shape. Their widths are determined by the parameter $\bar{\nu}$. The extremal values of $\Gamma^{(1)}_{\varepsilon i}$ and $D^{(1)}_{\varepsilon i}$ are comparable with the resonance values of deformed quantities $\Gamma^{(1)}_{\Lambda i}$ and $D^{(1)}_{\Lambda i}$:

$$\Gamma^{(1)}_{i\varepsilon\,ext}/\Gamma^{(1)}_{\Lambda i\,ext}\sim|D^{(1)}_{\varepsilon i}|_{ext}/|D^{(1)}_{\Lambda i}|_{ext}\sim|\Delta_{lr}|^{1/2}\bar{\nu}^{-1/2}.$$

The extrema of $\Phi_{\varepsilon i}$ are shifted relative to the position of the ACR by a value $|\Delta_{lr}|\gg\bar{\nu}$ toward higher magnetic fields ($\Omega_r>\omega/n$). With increase in the number of the resonance n, the interval Δ_{lr} between the extrema of field and deformation origin falls off like n^{-2}. The role of the field correction decreases, since $\Phi_{\Lambda i}\sim n^{-1}$, while $\Phi_{\varepsilon i}\sim n^{-2}$.

The functions $\Gamma^{(2)}_{\varepsilon i}$ and $D^{(2)}_{\varepsilon i}$ describe geometric oscillations. Their amplitudes increase sharply near the crossing of the spectra. We write out the formulas for $\Gamma^{(2)}_{\varepsilon i}$ and $D^{(2)}_{\varepsilon i}$ near the maxima of the amplitudes:

$$\Gamma^{(2)}_{\varepsilon i}\approx\frac{3\sqrt{3}}{2}\Gamma^{(1)}_{\varepsilon i\,max}\sum_\mu B_\mu\frac{\operatorname{Im}\operatorname{sh}^{-1}\pi\gamma}{(q|d''|)^{1/2}\operatorname{Re} I}\sin(qd+\delta), \tag{3.24}$$

$$D^{(2)}_{\varepsilon i}\approx D^{(1)}_{\varepsilon i\,max}\sum_\mu B_\mu\frac{\operatorname{Im}\operatorname{sh}^{-1}\pi\gamma}{(q|d''|)^{1/2}\operatorname{Re} I}\sin(qd+\delta), \tag{3.25}$$

B_μ is a number of the order of unity. The maximal values of $\Gamma^{(2)}_i$ and $D^{(2)}_{\varepsilon i}$ are comparable with the extremal values of $\Gamma^{(2)}_{\Lambda i}$ and $D^{(2)}_{\Lambda i}$: $|\Phi^{(2)}_{\varepsilon i}|_{max}/|\Phi^{(2)}_{\Lambda i}|_{max}\sim\Delta^2_{lr}/\bar{\nu}$. Consequently, the oscillations of the geometric resonance in the correction to the frequency $\Phi^{(2)}_i$ are described by the sum $\Phi^{(2)}_{\Lambda i}+\Phi^{(2)}_{\varepsilon i}$, the amplitude of which increases sharply in the neighborhood of ACR and in the region of excitation of cyclotron waves.

In the region of higher frequencies (3.11), both conditions (3.18) and (3.19) can be satisfied. We shall prove this by the example of a cyclotron wave corresponding to the central section of the Fermi surface. The case in which the spectrum of the wave is formed by electrons of a non-central section does not differ in principle from this, but its analysis is more cumbersome. The asymptotic values of the absorption and dispersion coefficients are equal to

$$\Gamma_{\varepsilon i}=\frac{\Phi_0}{\omega}\frac{[\Psi_{iy}^2-\varphi_{iy}^2A_n^2\cos^2(qd+\delta)]\zeta\operatorname{Re} I}{[y^3-\zeta|\operatorname{Im} I|+\zeta A_n\sin(qd+\delta)]^2+(\zeta\operatorname{Re} I)^2}, \tag{3.26}$$

$$D_{\varepsilon i}=-\frac{\Phi_0}{\omega}\frac{[\Psi_{iy}^2-\varphi_{iy}^2A_n^2\cos^2(qd+\delta)][y^3-\zeta|\operatorname{Im} I|+\zeta A_n\sin(qd+\delta)]}{[y^3-\zeta|\operatorname{Im} I|+\zeta A_n\sin(qd+\delta)]^2+(\zeta\operatorname{Re} I)^2}$$

$$A_n=\frac{(-1)^n(2\pi)^{1/2}}{\pi n|\Delta_l|(q|d''|)^{1/2}}, \quad \varphi_{iy}=2\left[\frac{m}{m_0}\frac{q_{iz}(\tau_1)n_y(\tau_2)}{|n_z'(\tau_1)|}\right]_{p_x=0}. \tag{3.27}$$

The condition (3.18) means that the amplitude of the oscillations $A_n<\operatorname{Re}I$, i.e., $|\Delta_l|<\bar{\nu}^2qd$. If the coupling of the sound with the cyclotron wave takes place in this range of frequencies and magnetic fields, then the expansion of the expressions (3.26) and (3.27) in the small parameter $A_n/|\operatorname{Re}|$ leads to the formulas (3.20)–(3.25).

At

$$|\Delta_l|>\bar{\nu}^2qd \tag{3.28}$$

(the condition (3.19)), it is not possible to represent the expressions (3.26) and (3.27) in the form of a sum of "monotonic" and oscillating components. The range in the magnetic field in which $\Gamma_{\varepsilon i}$ and $D_{\varepsilon i}$ undergo significant changes becomes broader and is determined by the parameter $(|\Delta_{li}|/(qd)^{1/2}>\bar{\nu}$. The absorption coefficient $\Gamma_{\varepsilon i}$ as a function of H in this range represents a series of peaks with amplitudes of the order of $\Phi_0/\omega\operatorname{Re}I(\Delta_{li})$. The locations of the maxima correspond to values of the roots of the dispersion equation (3.14). The dispersion $D_{\varepsilon i}(H)$ is described by a complicated alternating function. Vanishing of $D_{\varepsilon i}(H)$ at zero takes place at points of maximum absorption $\Gamma_{\varepsilon i}$. Its extremal values ($D_{\varepsilon\,max}\sim|D_{\varepsilon\,min}|$) are characterized by the quantity $\Phi_0/2\omega\operatorname{Re}I(\Delta_{l1})$. The separation between neighboring extremal functions $\Gamma_{\varepsilon i}$ and $D_{\varepsilon i}$ is proportional to the parameter $(qd)^{-1}$. The amplitude of the oscillations decreases with separation from the region (3.19) and the possibility arises of the separation of the monotonic component $\Phi^{(1)}_{\varepsilon i}$ and the geometric component $\Phi^{(2)}_{\varepsilon i}$.

In the region of frequencies and magnetic fields (3.12), the overlap of the spectra can occur in the range $\bar{\nu}\ll|\Delta_l|\ll(qd)^{-1}$. The field contribution to the frequency has in this case the form

$$\Phi_{\varepsilon i}=i\Phi_0\left\{\frac{\varphi_{iy}^2\cos^2(qd-\pi\omega/\Omega)I^2}{y^3-if_{yy}[1-\sin(qd-\pi\omega/\Omega)]I}\right\}_{p_x=0}. \tag{3.29}$$

The absorption and dispersion coefficients as functions of Δ_l change strongly over an interval of width $\bar{\nu}$, reaching the values $\Gamma_{\varepsilon i\,ext}\sim|D_{\varepsilon i\,ext}|\sim\Phi_0|\Delta_{l1}|^{1/2}/\nu$ at resonance.

Weakly damped cyclotron waves also exist far from the cyclotron resonances. This is not difficult to see from the expression for the asymptote of the conductivity tensor $\sigma^{(1)}_{\alpha\beta}$:

$$\sigma^{(1)}_{\alpha\beta}=\sigma_0[\bar{\nu}a_{\alpha\beta}+ib_{\alpha\beta}], \quad |\Delta_l|\leqslant1,$$
$$a_{\alpha\beta}=\pi\int_0^1\frac{dx\,f_{\alpha\beta}}{\Omega\sin^2(\pi\omega/\Omega)}, \quad b_{\alpha\beta}=\int_0^1 dx\,f_{\alpha\beta}\operatorname{ctg}(\pi\omega/\Omega), \tag{3.30}$$

$a_{\alpha,\beta}$ and $b_{\alpha,\beta}$ are monotonic functions of the frequencies ω and Ω, the specific form of which is determined by the form of the Fermi surface. The spectra of the acoustic and cyclotron waves can overlap far from ACR if the sound frequency is equal in order of magnitude to

$$\omega\sim k_0 s\sim\omega_0 s^{3/2}/cv^{1/2},$$

where ω_0 is the plasma frequency. The formulas for the absorption and dispersion coefficients are obtained from the expressions (3.20), (3.21), (3.24)–(3.27) by replacement of the quantities $f_{\alpha\beta}\,\mathrm{Im}I$ and $f_{\alpha\beta}\,\mathrm{Re}I$ by $b_{\alpha\beta}$ and $\bar{\nu}a_{\alpha\beta}$, respectively. The extremal values of $\Gamma_{\varepsilon i}$ and $D_{\varepsilon i}$ are greater than in the case of coupling of waves near the ACR, and exceed the resonance values of the deformation quantities $\Gamma_{\Lambda i}$ and $D_{\Lambda i}$ by a factor of $\bar{\nu}^{-1/2}$.

Thus, in the region of frequencies (1.1) and (1.2), the solenoidal electric fields can lead to a number of interesting features in the behavior of the absorption and sound velocity dispersion coefficients.

4. NONRESONANCE REGION

In the region of strong magnetic fields, when

$$\omega,\ \nu<\Omega<qv, \tag{4.1}$$

the field increment $\Phi_{\varepsilon i}$ to the frequency is always small in comparison with the deformation increment $\Phi_{\Lambda i}$. The coefficient of deformation absorption $\Gamma_{\Lambda i}$ under the conditions (4.1) has been well studied, and we give the result for the sound velocity dispersion in a metal with an arbitrary Fermi surface:

$$D_i=\frac{\Phi_0}{\pi}\frac{\Omega_0}{\nu^2+\omega^2}\left\{\sum\int_0^1 dx\,C_i^{(1)}(x)+(2\pi)^{1/2}\sum_\mu\left[C_i^{(2)}(x)\frac{\sin(qd+\delta)}{(q|d''|)^{1/2}}\right]_{p_x=p_\mu}\right\}$$

$$\Omega_0=eH/m_0c. \tag{4.2}$$

The quantity D_i differs from the absorption coefficient Γ_i by the additional factor ω/ν.

The argument of the oscillating factor $(qd+\delta)$ depends not only on the magnetic field but also on its orientation relative to the crystal axes. Therefore, geometric oscillations of the sound velocity, as well as those of the absorption coefficient, can occur in the case of rotation of the vector **H** in a plane perpendicular to the wave vector **q**. Such oscillations were observed in Ga.[5]

In the relations $\omega\sim\nu\sim\Omega$ existing in the experiment of Shepelev, Ledenev and Filimonov,[5] the contribution of the field increment $\Phi_{\varepsilon i}$ to the geometric oscillations is comparable with the deformation increment. We shall not write out the formulas describing the absorption coefficient $\Gamma_i=\Gamma_{\Lambda i}+\Gamma_{\varepsilon i}$ and the dispersion coefficient $D_i=D_{\Lambda i}+D_{\varepsilon i}$ because of their cumbersome nature.

5. SPHERICAL FERMI SURFACE

The case of a metal with quadratic isotropic dispersion law for the electrons requires special consideration. This is connected with the fact that some terms of the deformation potential tensor vanish at points of stationary phase τ_m. Moreover, the cyclotron mass does not depend on p_x. The electrons that are singled out lie on the central section.

Transverse cyclotron waves exist in the range of magnetic fields at which[10]

$$\Delta=(\omega-n\Omega)/\omega<0,\quad |\Delta|>\bar{\nu}. \tag{5.1}$$

Coupling of acoustic and cyclotron waves takes place if the acoustic frequency satisfies the conditions

$$\bar{\nu}^{-1}>y^3>1. \tag{5.2}$$

The contribution to the sound frequency Φ_i depends considerably on the polarization of the acoustic oscillations. For a longitudinal wave, the quantity Φ_z is determined purely by the deformation mechanism of interaction of the electrons with the lattice:

$$\Phi_z=\Phi_{\Lambda z}=\frac{\Phi_0}{18}\left\{\pi\,\mathrm{cth}\,\pi\gamma+\left(\frac{2\pi}{qd}\right)^{1/2}\frac{\sin(qd-\pi/4)}{\mathrm{sh}\,\pi\gamma}\right\}. \tag{5.3}$$

The field increment $\Phi_{\varepsilon z}$ is small:

$$\Gamma_{\varepsilon z}^{(1)}/\Gamma_{\Lambda z}^{(1)}\sim D_{\varepsilon z}^{(1)}/D_{\Lambda z}^{(1)}\sim(qd)^{-1},\ \Gamma_{\varepsilon z}^{(2)}/\Gamma_{\Lambda z}^{(2)}\sim D_{\varepsilon z}^{(2)}/D_{\Lambda z}^{(2)}\sim(qd)^{-1/2}.$$

Equation (5.3) for the sound absorption coefficient was obtained previously.[9] The ACR lines of the absorption and dispersion coefficients are characterized by the same width (of the order of $\bar{\nu}$) as in the case of an arbitrary dispersion law. However, the resonance values of $\Gamma_{\Lambda z\,\mathrm{ext}}^{(1)}$ and $D_{\Lambda z\,\mathrm{ext}}^{(1)}$ are $\bar{\nu}^{-1/2}$ times greater, as a consequence of the fact that all the electrons of the Fermi surface take part in the resonance. Near the ACR, the relative amplitude of the geometric oscillations is small: $\Gamma_{\Lambda z}^{(2)}/\Gamma_{\Lambda z}^{(1)}\sim D_{\Lambda z}^{(2)}/D_{\Lambda z}^{(1)}\sim(qd)^{-1/2}$.

The absorption coefficient and the dispersion of the transverse sound velocity are due to the electric fields. The deformation increment $\Phi_{\Lambda\alpha}$ is smaller by a factor of at least qd than the quantity $\Phi_{\Lambda z}$ for longitudinal sound. For sound polarized along the magnetic field, geometric oscillations do not develop in the considered approximation:

$$\Phi_x=\Phi_{\varepsilon x}=-{}^4/{}_9 i\Phi_0[y^3-i\pi\,\mathrm{cth}\,\pi\gamma]^{-1}. \tag{5.4}$$

It is not difficult to see that Γ_x reaches its maximum value at the point Δ_i of crossing of the spectra of acoustic and cyclotron waves: $y^3=\pi\,\mathrm{Im}\,\mathrm{coth}\pi\gamma$. The dispersion at this point changes sign. The locations of its extrema, determined by the relations

$$y^3=\pi[\mathrm{Im}\,\mathrm{cth}\,\pi\gamma\pm\mathrm{Re}\,\mathrm{cth}\,\pi\gamma],$$

are shifted along Δ by a value of the order of $\bar{\nu}$ relative to its zero. The extremal values of $\Gamma_{x\,\mathrm{ext}}$ and $D_{x\,\mathrm{ext}}$ are the same in order of magnitude: $\Gamma_{x\,\mathrm{ext}}\sim|D_{x\,\mathrm{ext}}|\sim\Phi_0\Delta_1^2\nu^{-1}$. They are comparable with the resonance values of the absorption coefficient and the longitudinal sound dispersion if coupling of the waves occurs in the range of magnetic fields not too close to ACR.

If the transverse sound is polarized perpendicular to the magnetic field, the contribution to the frequency is described by the formula:

$$\Phi_y=\Phi_{\varepsilon y}=-{}^4/{}_9 i\Phi_0[y^3-i\pi\,\mathrm{cth}\,\pi\gamma+2i(2\pi)^{1/2}[(qd)^{1/2}\,\mathrm{sh}\,\pi\gamma]^{-1}\sin(qd-\pi/4)]^{-1}.$$

The oscillations of the conductivity lead to geometric oscillations of $\Phi_y^{(2)}$ with small amplitude if the solution of the dispersion equation for the cyclotron wave falls in the region $\bar{\nu} < |\Delta_1| < \bar{\nu}(qd)^{1/2}$. The formula for the resonant, nonoscillating term $\Phi_y^{(1)}$ coincides with the expression (5.4).

Under conditions when the coupling takes place in a region that is further from the ACR, $1 > |\Delta_1| > \bar{\nu}(qd)^{1/2}$, the curves $\Gamma_y(H)$ and $D_y(H)$ have the same character as in the case (3.28) of an arbitrary dispersion law for the electrons. The extremal values of D_y and Γ_y are equal: $\Gamma_{y\,\mathrm{ext}} \sim D_{y\,\mathrm{ext}} \sim \Phi_0 \Delta_1^2/\nu$.

1) In this case, as is well known, [9] the resonance should not be very "sharp," in order that the condition $qR(p_x) \approx qR_{\max}(\nu/\Omega)^{1/2} \gg 1$ be satisfied.

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Translated by R. T. Beyer

Transition of second order in the field in a two-dimensional Heisenberg ferromagnet

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(Submitted May 13, 1976)
Zh. Eksp. Teor. Fiz. **71**, 2402–2405 (December 1976)

It is shown that a second-order phase transition occurs near the point $h_c = \lambda$ (λ is the anisotropy constant) in a two-dimensional Heisenberg ferromagnet with anisotropy of the easy plane type. The magnetic susceptibility is infinite below the transition point in a weak field parallel to the plane. The field dependences of the magnetic moment $n_z(h_z)$ are determined above and below the transition point.

PACS numbers: 75.30.Gw, 75.30.Jy, 75.10.Gj

We consider a planar ferromagnetic Heisenberg magnet with easy-axis anisotropy. We shall show that in such a system, at sufficiently low temperatures, a second-order phase transition takes place at a definite value of the magnetic field perpendicular to the plane.

We consider first the case $T = 0$. The Hamiltonian is given by

$$H = \int \left(\frac{\lambda n_z^2}{2} - h_z n_z \right) \frac{d^2x}{a^2}. \tag{1}$$

Here $\lambda > 0$ is the anisotropy constant, h_z is the magnetic field perpendicular to the plane of the magnetic (the XY plane), and a is the lattice constant. It is easily seen that the minimum energy corresponds to the value (Fig. 1)

$$n_z = \begin{cases} h_z/\lambda, & h_z \leqslant \lambda, \\ 1, & h_z > \lambda. \end{cases}$$

The phase transition manifests itself in a change of the magnetic susceptibility χ_{XY} relative to an infinitesimally weak field h_X or h_Y parallel to the plane. The susceptibility is infinite below the phase-transition point ($h_z < \lambda$), since the spontaneous moment has a component parallel to the plane. The susceptibility $\chi_{XY} = 1/(h_z - \lambda)$ above the transition point is finite.

We show now that at sufficiently low $T \neq 0$ the transition takes place as before. The Hamiltonian now contains exchange terms

$$H = \int \left[\frac{1}{2} J (\partial_\mu \mathbf{n})^2 + \frac{\lambda}{2a^2} n_z^2 - \frac{h_z n_z}{a^2} \right] d^2x, \tag{2}$$

where $J > 0$ is the exchange constant. Consider the case $h_z \ll \lambda$. We shall show that $\chi_{XY} = \infty$. We change over to the variables φ and α; then

$$\mathbf{n} = ((1-g^2\varphi^2)^{1/2} \cos g\alpha,\ (1-g^2\varphi^2)^{1/2} \sin g\alpha,\ g\varphi).$$

Here $g^2 = T/J \ll 1$. The Hamiltonian takes the form

$$\frac{H}{T} = \int \left[\frac{1}{2} \frac{(\nabla\varphi)^2}{1-g^2\varphi^2} + \frac{1}{2}(1-g^2\varphi^2)(\nabla\alpha)^2 + \frac{1}{2} m^2\varphi^2 - \left(\frac{h\varphi}{g}\right) \right] d^2x,$$

where $m^2 = \lambda/J$, $h = h_z/J$, and the lattice constant is set equal to unity.

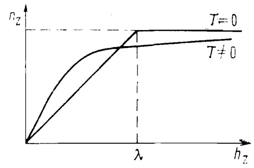

FIG. 1.