(3) with $q=5$, $p=1$, inasmuch as the term with the third derivative vanishes for this direction.[11] Another example of interest for applications is the propagation of capillary-gravitational waves in shallow water, for which the realization of solitons with oscillations and the formation of bound states are also possible (cf.[9]).

The authors are grateful to E. N. Pelinovskiĭ for useful comments.

[1)]The possibility of such an interpretation of the interaction of solitons has already been noted previously.[4]

[2)]In the following we consider solitons each of which is determined by only one phase, of the type $x-vt$; this excludes "envelope solitons" (e.g., Langmuir solutions) from consideration, although, in principle, the given approach appears to be perfectly possible for these also.

[3)]This possiblity has now been confirmed by means of a numerical investigation of Eq. (3).

[4)]Weakly bound states in which the solitons are coupled by oscillations further from their maxima have also been observed experimentally. In this case the characteristic period of the oscillations is substantially increased, the changes in the amplitudes and velocities turn out to be considerably smaller, and the oscillograms are less revealing.

[1]A. C. Scott, F. Y. F. Chu, and D. W. McLaughlin, Proc. IEEE 61, 1443 (1973).
[2]R. K. Dodd, R. K. Bullough, and S. Duckworth, J. Phys. A 8, L64 (1975).
[3]A. E. Kudryavtsev, Pis'ma Zh. Eksp. Teor. Fiz. 22, 178 (1975) [JETP Lett. 22, 82 (1975)].
[4]L. A. Ostrovskiĭ, V. V. Papko, and E. N. Pelinovskiĭ, Izv. Vuzov, Radiofizika 15, 580 (1972).
[5]N. J. Zabusky, Nonlinear Partial Differential Equations, Academic Press, N.Y., 1967.
[6]K. A. Gorshkov, L. A. Ostrovsky, and E. N. Pelinovsky, Proc. IEEE 62, 1B, 11 (1974).
[7]V. E. Zakharov, Zh. Eksp. Teor. Fiz. 60, 993 (1971) [Sov. Phys. JETP 33, 538 (1971)].
[8]P. D. Lax, SIAM Rev. 11, 7 (1969) (Russ. transl. in "Matematika" ("Mir", M.) 13, No. 5, 128 (1969)).
[9]T. Kawahara, J. Phys. Soc. Japan 33, 260 (1972).
[10]G. M. Zaslavskiĭ, Zh. Eksp. Teor. Fiz. 62, 2129 (1972) [Sov. Phys. JETP 35, 1113 (1972)].
[11]T. Kakutani and H. Ono, J. Phys. Soc. Japan 26, 1305 (1969).

Translated by P. J. Shepherd

Stationary model of the "corona" of spherical laser targets

Yu. V. Afanas'ev, E. G. Gamaliĭ, O. N. Krokhin, and V. B. Rozanov

P. N. Lebedev Institute, USSR Academy of Sciences
(Submitted February 16, 1976)
Zh. Eksp. Teor. Fiz. 71, 594–602 (August 1976)

An analytic stationary model of the "corona" of spherical laser targets is considered with account taken of the major physical processes, viz. hydrodynamic processes, laser radiation absorption in the vicinity of the critical point, and electron thermal conductivity. Exressions for the "corona" parameters are derived as functions of the laser pulse parameters (radiation flux and frequency) and of the target (radius and thermophysical properties).

PACS numbers: 79.20.Ds

1. As follows from a number of studies,[1,2] all the known schemes for laser initiation of thermonuclear reactions consist of three physical stages: evaporation, compression, and thermonuclear combustion. These stages are governed by different physical processes and exert different influences on the set of final parameters characterizing the laser-induced thermonuclear fusion process as a whole.

The initial stage of the interaction of the laser radiation with the target material consists of evaporation and heating of a definite fraction of the medium, i.e., formation of a "corona," which is a hot plasma of relatively low density that expands in a direction opposite to the incident radiation. During this stage, a pressure pulse is generated at the boundary between the corona and the dense cold material and accelerates the unevaporated part of the target towards the center. The principal parameter characterizing the evaporation stage is the hydrodynamic efficiency,[3] i.e., the ratio of the energy of the unevaporated part of the target to the total laser-emission energy. The magnitude of this ratio determines both the energy balance in the system, i.e., the temperature of the central region of the target, and the maximum value of the thermonuclear-fuel mass that can be compressed by the radiation to a high density at a given energy.[1] Moreover, the degree of the compression of the target material also depends on the shape and amplitude of the pressure pulse.

The indicated quantities—the hydrodynamic efficiency and the pressure amplitude—depend essentially on the physical state of the corona, a state determined by the parameters of the laser pulse (flux density, duration, radiation frequency) and of the target (radius and thermophysical constants of the evaporated layer).

It should be noted that the process of formation and expansion of the corona as well as the compression process can occur simultaneously[2] and can be separated by a stage of inertial motion of the accelerated matter towards the center.[1] In all cases, however, the velocity of the material in the corona is much higher than the velocity of the compressed material, so that the hydrodynamics of the corona can be regarded independently of the compression process.

We describe in this paper an analytic model of the corona produced under conditions when spherical laser targets are symmetrically irradiated, with account taken of the decisive physical processes.

2. We consider the main physical factors that determine the state of the corona. It follows from both the experimental data[4] and numerical calculations[1,2] that, in all the cases of interest in which laser radiation acts on a spherical target, the plasma manages to expand during the time of action of the pulse to a distance exceeding the initial target radius. This means that the problem should be treated in a spherical geometry.

As the next step it is natural to introduce some mechanism whereby the laser radiation is absorbed. The mechanism hitherto considered in theoretical investigations of the interaction of laser radiation with matter (see, e.g., [5,6]) was principally inverse bremsstrahlung, an approach valid at relatively low flux densities $q \sim 10^{10}-10^{11}$ W/cm^2, when the plasma produced has a relatively low temperature. An investigation of the mechanisms whereby laser radiation is absorbed by a high-temperature plasma[7] shows that the radiation is most effectively absorbed in the plasma region near the critical density $\rho_{cr} = \omega_0^2 M_i m / 4\pi z e^2$, where ω_0 is the cyclic frequency of the radiation, M_i and m are the masses of the ion and of the electron, and z is the ionization multiplicity. The most justified assumption is consequently the condition that the absorption of the incident flux be δ-like at the point $\rho = \rho_{cr}$. Besides the radiation-absorption mechanism, a decisive process in the corona is the heat transfer via the electronic thermal-conductivity mechanism. Thus, it is assumed in the problem under consideration that the plasma moving towards the radiation is fully transparent at $\rho < \rho_{cr}$, whereas at the point $\rho = \rho_{cr}$ the light flux is transformed into electron thermal-conductivity fluxes that travel both into the region with $\rho > \rho_{cr}$ and into the region $\rho < \rho_{cr}$. In this formulation, obviously, it is easy to include in the analysis the reflection of the radiation from the plasma at the point $\rho = \rho_{cr}$.

The equations describing the stationary flow of the plasma take in this case the form

$$\rho v R^2 = \rho^* v^* R^{*2};$$

$$\frac{d}{dR}(p+\rho v^2) = -\frac{2\rho v^2}{R}, \quad p+\rho v^2 = p^* + \rho^* v^{*2} + \int_R^{R^*} dR \frac{2\rho v^2}{R}, \tag{1}$$

$$\rho v R^2\left(\varepsilon + \frac{v^2}{2} + \frac{p}{\rho}\right) - R^2 \varkappa_0 T^{5/2} \frac{dT}{dR} = \begin{cases} Q_0, & R > R_{cr} \\ 0, & R < R_{cr} \end{cases},$$

where Q_0 is the radiation flux in a unit solid angle, ρ, v, and $p = \rho z T/M_i$ are the density, velocity, and pressure of the material, $\varepsilon = 3p/2\rho$ is the specific internal energy, T is the electron temperature, and $\varkappa_0 T^{5/2}$ is the coefficient of the electron thermal conductivity. The quantities

$$\rho^* = \rho(R^*), \quad v^* = v(R^*), \quad p^* = p(R^*)$$

are referred for the time being to an arbitrary but fixed cross section. It is natural to consider the problem (1) under the assumption that the initial radius R_0 remains constant and is assumed to be a known parameter. The latter is valid if the state of the corona "follows" the change (decrease) of the radius of the unevaporated part of the target. Formally, the condition $R_0 = \text{const}$ leads to the condition $\rho_0 = \infty$ (ρ_0 is the density of the unevaporated part). The boundary conditions then take the form

$$v(R_0) = T(R_0) = 0. \tag{2}$$

Furthermore, it is physically obvious that at $R \to \infty$ and $v \to v_\infty = \text{const}$ we have $T^{5/2} dT/dR \to 0$ and, as follows now from (1), we have p, ρ, and $T \to 0$. As a result we have an addition to (2)

$$v(\infty) = v_\infty, \quad T(\infty) = 0. \tag{3}$$

Furthermore,

$$\rho(R_{cr}) = \rho_{cr}. \tag{4}$$

Thus, the posed problem (1) with conditions (2)–(4) contains five specified parameters: $\varkappa_0$, Q_0, R_0, ρ_{cr}, and M_i/z. It is required to determine the coordinate R_{cr} of the critical point, the velocity v_∞ of the matter as $R \to \infty$, and the scales of the hydrodynamic quantities ρ^*, v^*, and T^* together with the coordinate R^*.

As shown by analysis, it is convenient, in analogy with Nemchinov's demonstration,[6] to choose as the reference coordinate R^* the Jouguet point, i.e., the point at which the velocity is equal to the local speed of sound. In this case, owing to the non-adiabatic character of the motion, the reference is to the isothermal speed of sound $c_T = (zT/M_i)^{1/2}$. As a result we have $zT^* = M_i v^{*2}$ at $R = R^*$, i.e., the sought scales of velocity and temperature are related.

Introducing furthermore the dimensionless functions $\theta = T/T_*$ and $\eta = v^2/v^{*2}$ and the variable $x = R^*/R$, we ultimately rewrite the problem (1) in the form

$$\frac{\rho}{\rho^*} = \frac{x^2}{\eta^{1/2}}, \quad x\frac{d\eta}{dx}\left(1 - \frac{\theta}{\eta}\right) + 2x\frac{d\theta}{dx} + 4\theta = 0,$$

$$\beta\theta^{5/2}\frac{d\theta}{dx} = \frac{1}{2}(\varphi - \eta) - \frac{5}{2}\theta, \quad \beta = \frac{\varkappa_0 T^{*7/2}}{\rho^* v^{*3} R^*}; \tag{1'}$$

$$\varphi = \begin{cases} 0, & x > x_{cr} \\ \varphi_0 = 2Q_0/\rho^* v^{*3} R^{*2} & x < x_{cr} \end{cases}.$$

The conditions (2)–(4) become

$$\theta(x_0) = \eta(x_0) = 0, \quad \theta(0) = 0, \quad \eta(0) = \varphi_0 = v_\infty^2/v^{*2},$$

$$\rho_{cr} = \rho^* x_{cr}^2/\eta_{cr}^{1/2}, \quad x_0 = R^*/R_0. \tag{2'}$$

In addition, we now have

$$\theta(1)=\eta(1)=1. \quad (5)$$

When the problem is formulated in terms of dimensionless variables, the unknown parameters are β, φ_0, x_{cr}, and x_0. If these quantities are obtained by solving Eqs. (1) with conditions (2) and (5), then the relations

$$\beta=\frac{\varkappa_0 T^{*7/2}}{\rho^* v^{*3} R^*}, \quad \varphi_0=\frac{2Q_0}{\rho^* v^{*3} R^{*2}}, \quad zT^*=M_i v^{*2},$$
$$x_0=R^*/R_0, \quad \rho_{cr}=\rho^* x_{cr}^2/\eta_{cr}^{1/2} \quad (6)$$

can be used to determine R_{cr}, R^*, v^*, T^*, ρ^*, and v_∞.

3. We consider now the general scheme for solving the problem (1) and (2). As will be shown below, there exists a characteristic dimensionless parameter

$$\gamma_0=\frac{\varkappa_0^{2/7} Q_0}{\rho_{cr}^{9/7} R_0^{11/7}}\left(\frac{M_i}{z}\right)^{11/7},$$

on which the qualitative picture of the state of the corona depends. At $\gamma_0>10^2$ we have $x_{cr}<1$ ($R_{cr}>R^*>R_0$), i.e., the coordinate of the critical point is strongly displaced towards the incident radiation in comparison with the position of the Jouguet point. In this case Eqs. (1′) in the region $x>x_{cr}$ ($R<R_{cr}$) are universal with a singular point $x=1$. Indeed, from the condition that there be no infinite acceleration at the Jouguet point ($x=1$) we obtain with the aid of the second and third equations of the system (1)

$$d\theta/dx=-2, \quad d\eta/dx=-(\sqrt{13}+1) \quad \text{at} \quad x=1, \quad \beta=1.5. \quad (7)$$

Next, integrating the equations in the region $x\in[1,x_0]$ (going out of the singular point $x=1$, $\theta(1)=\eta(1)=1$ with the aid of (7)) we obtain the quantity ($\theta(x_0)=\eta(x_0)=0$). As shown by a simple numerical calculation, $x_0=1.2$, i.e., $R^*=1.2R_0$. It is possible to integrate similarly the universal equations at $x<1$ (but $x>x_{cr}$). In the region $x<x_{cr}$ the problem contains, at $\gamma_0>10^2$ only one parameter φ_0, which can be determined together with the value of x_{cr} from the condition that the functions $\theta(x)$ and $\eta(x)$ be continuous at the point $x=x_{cr}$, i.e., as a result of joining the solutions in the regions $x>x_{cr}$ and $x<x_{cr}$.

Thus, in the region $\gamma_0>10^2$ the problem (1′), (2′) turns out to be closed within the framework of the considered formulation, with a solution corresponding to a continuous transition through the Jouguet point and the critical point. There exists in this case a universal connection between the coordinate of the Jouguet point and the target radius $R^*=1.2R_0$, while the critical point is supersonic. It should also be noted that $\gamma_0>10^2$ the derivatives $d\rho/dR$, dv/dR, and dT/dR experience a finite discontinuity, this being due to the need for introducing a discontinuity in the electronic thermal conductivity flux by virtue of the δ-like character of the energy released at $\rho=\rho_{cr}$. As a result, the maximum temperature is reached in our problem at the critical point. With decreasing γ_0, however, the point with $\rho=\rho_{cr}$ approaches the Jouguet point, and at a certain finite value $\gamma_0=\gamma_0^*\sim 10^2$ the two points coincide, i.e., $x_{cr}=1$. As shown by analysis, at $\gamma_0<\gamma_0^*$ one can no longer construct a solution that is continuous at the point $x=x_{cr}=1$ with a finite discontinuity of the derivatives under the condition of a jumplike change in the dimensionless flux φ from φ_0 to zero at $x=x_{cr}=1$. We indicate that formally, as $\gamma_0\to\gamma_0^*$, the derivative of the velocity tends to infinity at the point $x=x_{cr}=1$. In this case it becomes necessary to construct a new solution, different from the solution corresponding to $\gamma_0>\gamma_0^*\sim 10^2$. As will be shown below, a physically correct solution in the region $\gamma_0<\gamma_0^*$ can be constructed by introducing the distributed energy release in the vicinity of the critical point. Within the framework of the foregoing formulation of the problem, this reduces to replacement of the relations $\varphi=\varphi_0$, $x<x_{cr}$; $\varphi=0$, $x>x_{cr}$ by

$$\varphi=\begin{cases}\varphi_0, & x\leqslant x_{cr}-0=1-0\\ 0, & x\geqslant x_{cr}+0=1+0\end{cases} \quad (8)$$

The solution in the region $\gamma_0<\gamma_0^*$, with allowance for (8), then corresponds to the conditions $x_{cr}\equiv 1$ ($R_{cr}\equiv R^*$) and $x_0\to x_{cr}\equiv 1$ as $\gamma_0\to 0$, i.e., the universal connection between the target radius and the coordinate $R^*=1.2R_0$ of the Jouguet point no longer holds. Accordingly, the parameter $\beta\neq 1.5$ and tends to zero as $\gamma_0\to 0$.

4. We present now a formal solution of the problem (1′), (2′).

A. We consider first the case $x_{cr}<1$, i.e., $R_{cr}>R^*$, which corresponds to large values of the parameter γ_0. In the region $x_0\geqslant x\geqslant x_{cr}$ ($R_0\leqslant R\leqslant R_{cr}$) Eqs. (1′) are universal ($\beta=1.5$) and their approximate solution is

$$\eta=1-6.4\ln x=1-6.4\ln(R^*/R);$$
$$\theta=[1+15.4(1-x)+6.4x\ln x]^{2/7}$$
$$=\left[1+15.4\left(1-\frac{R^*}{R}\right)+6.4\frac{R^*}{R}\ln\frac{R^*}{R}\right]^{2/7}. \quad (9)$$

Expressions (9) satisfy the conditions $\theta(1)=\eta(1)=1$, $\theta(x_0)=\eta(x_0)=0$, or $x_0=1.2$.

The solution of (1′) at $x<x_{cr}$ ($R>R_{cr}$) can be represented in the form

$$\theta=Ax^{4/5}, \quad \eta=\varphi_0-5Ax^{4/5}, \quad (10)$$

where A is an arbitrary constant. Relations (10) were obtained under the condition that at $x\to 0$ ($R\to\infty$) the flux of the electronic thermal conductivity is a quantity of higher order of smallness than the hydrodynamic part of the flux. This physical condition makes it possible to go out of the singular point $x=0$ of the equations.

It should be noted that as $x\to 0$ there exists formally also the solution $\theta=x^{2/5}$, $\eta=\varphi_0-x^{2/5}$, which corresponds to an equal order of smallness of the thermal and hydrodynamic fluxes. The use of this solution, however, makes the problem overdetermined.

Using next the condition for the continuity of the functions at the point $x=x_{cr}=R^*/R_{cr}=1.2R_0/R_{cr}$, the first equation of the system (1′), and the relations $\varphi_0=2Q_0/\rho^* v^{*3}R^{*2}$, $\frac{3}{2}=\varkappa_0 T^{*7/2}/\rho^* v^{*3}R^*$, we obtain a system of algebraic equations for the critical radius and for the plasma parameters at the Jouguet point and at the critical point:

$$1-6.4\ln\frac{1.2R_0}{R_{cr}}=\varphi_0-5A\left(\frac{1.2R_0}{R_{cr}}\right)^{4/3},$$

$$\left[1+15.4\left(1-\frac{1.2R_0}{R_{cr}}\right)+6.4\frac{1.2R_0}{R_{cr}}\ln\frac{1.2R_0}{R_{cr}}\right]^{2/7}=A\left(\frac{1.2R_0}{R_{cr}}\right)^{4/3}, \tag{11}$$

$$\varphi_0=\frac{2Q_0}{\rho^* v^{*3}(1.2R_0)^2},\quad \frac{3}{2}=\frac{\varkappa_0 T^{*7/2}}{\rho^* v^{*3}(1.2R_0)},\quad zT^*=M_i v^{*2},$$

$$\rho_{cr}=\rho^*\left(\frac{1.2R_0}{R_{cr}}\right)^2\left(1-6.4\ln\frac{1.2R_0}{R_{cr}}\right)^{-1/2}.$$

The system (11) constitutes six equations for the determination of six parameters R_{cr}, A, φ_0, ρ^*, v^*, and T^* as functions of the quantities Q_0, $\varkappa_0$, R_0, ρ_{cr} and M_i/z. The approximate solution of (11) yields

$$R_{cr}=0.5R_0\gamma_0^{2/7}(1+6.4\ln(\gamma_0^{2/7}/4))^{-1/4}\approx 0.3R_0\gamma_0^{2/7},$$

$$v^{*2}=0.56\gamma_0^{-8/21}\left(\frac{Q_0}{\rho_{cr}R_0^2}\right)^{2/3},\quad v_{cr}^2=v^{*2}\left(1+6.4\ln\frac{\gamma_0^{2/7}}{4}\right),$$

$$T^*=\frac{M_i v^{*2}}{z},\quad T_{cr}=2\frac{M_i v^{*2}}{z}, \tag{12}$$

$$v_\infty^2=v^{*2}[11+6.4\ln(\gamma_0^{2/7}/4)],\quad \rho^*=1.7\cdot 10^{-1}\rho_{cr}\gamma_0^{4/7},$$

$$A=0.64\gamma_0^{8/21}(1+6.4\ln(\gamma_0^{2/7}/4))^{-1/3}.$$

The spatial distribution of the quantities in the region $R_0\leqslant R\leqslant R_{cr}$ takes accordingly the form

$$v^2=v^{*2}\left(1-6.4\ln\frac{1.2R_0}{R}\right),$$

$$T=T^*\left(1+15.4\left(1-\frac{1.2R_0}{R}\right)+\frac{7.7R_0}{R}\ln\frac{1.2R_0}{R}\right)^{2/7}, \tag{13}$$

$$\rho=\rho^*\left(\frac{1.2R_0}{R}\right)^2\left(1-6.4\ln\frac{1.2R_0}{R}\right)^{-1/2}.$$

In the region $R>R_{cr}$ we have

$$v^2=v^{*2}\left[11+6.4\ln\frac{\gamma_0^{2/7}}{4}-\frac{3.2\gamma_0^{8/21}(1.2R_0/R)^{4/3}}{(1+6.4\ln(\gamma_0^{2/7}/4))^{1/3}}\right],$$

$$T=0.64T^*\gamma_0^{8/21}\left(\frac{1.2R_0}{R}\right)^{4/3}\left(1+6.4\ln\frac{\gamma_0^{2/7}}{4}\right)^{-1/3}, \tag{14}$$

$$\rho=\rho^*\left(\frac{1.2R_0}{R}\right)^2\left[11+6.4\ln\frac{\gamma_0^{2/7}}{4}-\frac{3.2\gamma_0^{8/21}(1.2R_0/R)^{4/3}}{(1+6.4\ln(\gamma_0^{2/7}/4))^{1/3}}\right].$$

Formulas (12)–(14) are valid at $R_{cr}/R^*\approx 0.25\gamma_0^{2/7}>1$, which leads to the condition $\gamma_0>\gamma_0^*\approx 10^2$.

Thus, formulas (12)–(14) solve our problem at

$$\gamma_0=[Q_0\varkappa_0^{8/4}(M_i/z)^{21/4}](R_0^{11/4}\rho_{cr}^{7/4})^{-1}>\gamma_0^*\approx 10^2,$$

which corresponds physically to the action of large light fluxes with low radiation frequency on a target with relatively small dimensions.

B. Inasmuch as the coordinate of the critical point decreases with decreasing parameter γ_0, the radii of the critical point and of the Jouguet point coincide at a certain value $\gamma_0\approx\gamma_0^*\sim 10^2$. In this case, as already indicated, a singularity arises in Eqs. (1′) at the point $x=x_{cr}=1$, and is removed by introducing a distributed absorption in the vicinity of $x=1$.

Assuming $x_{cr}\equiv 1$ and stipulating, as before, that the transition through the Jouguet point $x=x_{cr}=1$ be continuous, we obtain with the aid of the second and third equations of the system (1′), with (8) taken into account, the relation

$$\beta={}^3/_2-{}^1/_4\varphi(1),\quad 0\leqslant\varphi(1)\leqslant\varphi(0). \tag{15}$$

From the relations

$$\beta=\varkappa_0 T_{cr}^{7/2}/\rho_{cr}v_{cr}^3 R_{cr},\quad \varphi_0=2Q_0/\rho_{cr}v_{cr}^3 R_{cr}^2$$

it is easy to verify that at $\gamma_0<\gamma_0^*$ we have $\beta\sim\gamma_0^{4/3}$, since $\rho^*=\rho_{cr}$, $v^*=v_{cr}$, $T^*=T_{cr}$, and, as will be shown below, $R_{cr}\to R_0$ as $\gamma_0\to 0$.

In the region $1+0\leqslant x\leqslant x_0$ ($R_{cr}-0\leqslant R\leqslant R_0$) the solution of (1′) can be obtained with the aid of the planar approximation

$$\theta\approx\left[1-\frac{15}{2\beta}(x-1)\right]^{2/5},\quad \theta=2\eta^{1/2}-\eta. \tag{16}$$

Hence $x_0=1+2\beta/15$ or, taking (15) into account,

$$x_0=1.2-{}^1/_{30}\varphi(1). \tag{17}$$

It follows from (15) and (17) that the limiting value of $\varphi(1)$ as $\beta\to 0$ ($\gamma_0\to 0$) is $\varphi(1)=6$. Consequently, according to (8), $\varphi_0\to 6$ as $\beta\to 0$. As a result, at small values of γ_0, with the aid of the relations

$$\varphi_0=6=2Q_0/\rho_{cr}v_{cr}^3 R_{cr}^2 \text{ and } \beta=\varkappa_0 T_{cr}^{7/2}/\rho_{cr}v_{cr}^3 R_{cr},$$

of formulas (15) and (17), we obtain

$$\beta=[{}^1/_3\gamma_0]^{4/3},\quad \rho^*=\rho_{cr},\quad R_{cr}=R^*=(1+{}^2/_{15}\beta)R_0,$$

$$v^{*2}=v_{cr}^2=\left(\frac{Q_0}{3\rho_{cr}R_0^2}\right)^{2/3}\left(1+\frac{2}{15}\beta\right)^{-4/3},\quad T^*=T_{cr}=\frac{M_i v_{cr}^2}{z}. \tag{18}$$

The approximate solution of (1′) in the region $x<x_{cr}=1$ ($R>R_{cr}\equiv R^*$) then takes the form

$$\theta\approx x^{4/3},\quad \eta\approx 6-5x^{4/3},\quad \rho=\rho_{cr}x^2(6-5x^{4/3})^{-1/2}. \tag{19}$$

Formulas (18) and (19) solve our problem at small values of γ_0. Thus, as $\gamma_0\to 0$ the critical density "lies" on the target, i.e., $R_{cr}\approx R_0$. If $\gamma_0\to 0$ because $R_0\to\infty$, but in such a way that $Q_0/R_0^2\to q_0=\text{const}$, then a hydrodynamic discontinuity takes place on the target boundary, namely:

$$v^2=\begin{cases}(q_0/3\rho_{cr})^{2/3} & \text{at } R=R_{cr}+0,\\ 0 & \text{at } R=R_{cr}-0.\end{cases}$$

The result means, in principle, that in an almost plane geometry $R_0\to\infty$, $Q_0/R_0^2\to q_0$ there can exist a stationary layer which is heated by electronic thermal conductivity, with the conditions $\rho=\rho_{cr}$, $v^2=(zT/M_i)$ on the outer boundary. Adjacent to the stationary layer should be a nonstationary "tail," e.g., an isothermal rarefaction wave.

This circumstance was not taken into account in[9], where it was assumed that the acoustic point has a density $\rho=\rho_0$ (where ρ_0 is the initial target density). As a result, the coefficient of the transformation of the incident-radiation energy into kinetic energy of the material, calculated in[9], was strongly underestimated.

5. We present now some significant consequences of our results. As shown in[3], the hydrodynamic efficiency for laser targets is determined by the heat flux Q^* introduced into the discontinuity. At $\gamma_0 > \gamma_0^*$, according to the model considered in[3], the quantity Q^* is equal to the heat flux in the section R^*. Using the relation (12), we easily obtain

$$Q^* \approx 0.31 Q_0, \qquad (20)$$

and the transfer coefficient η is equal, consequently, to

$$\eta = \eta_{max}(Q^*/Q_0) = 0.31\eta_{max}. \qquad (21)$$

According to (18) in this case, the analogous quantities are equal, at $\gamma_0 < \gamma_0^*$

$$Q^* = Q_0[1 - {}^8/_{15}({}^1/_3\gamma_0)^{4/3}], \qquad (22)$$
$$\eta = \eta_{max}[1 - {}^8/_{15}({}^1/_3\gamma_0)^{4/3}]. \qquad (23)$$

Thus, the coefficient of the transformation of the radiation energy into kinetic energy of the target increases on going to larger target radii and to higher-frequency radiation.

The second important consequence is connected with the position of the critical radius as a function of the parameters of the problem. As shown above, the critical density is far from the target at $\gamma_0 \to \gamma_0^*$ and approaches the latter as $\gamma_0 \to 0$. The latter means that in the case, for example, of low-frequency radiation the radiation energy input to the plasma subtends over several radii, thus facilitating the focusing conditions at small initial target radii. For the investigation of some nonlinear processes,[7] a very important role is played by the state of the plasma in the region of the critical density, and particularly by the quantity $\alpha = (\rho v^2/p)$. According to (12) $\alpha = 0.5(1 + 6.4 \ln(\gamma_0^{2/7}/4))$ at $\gamma_0 > \gamma_0^*$ and $\alpha = 1$ at $\gamma_0 < \gamma_0^*$. The temperature at the critical point is determined by the following relations, according to (12) and (18):

$$T_{cr} \sim Q_0^{1/6} \varkappa_0^{-1/4} R_0^{-5/12} \rho_{cr}^{-1/12}, \quad \gamma_0 > \gamma_0^*;$$
$$T_{cr} \sim Q_0^{2/3} \rho_{cr}^{-2/3} R_0^{-4/3} (M_i/z), \quad \gamma_0 < \gamma_0^*. \qquad (24)$$

As follows from (24), the temperature at the critical point at $\gamma_0 > \gamma_0^*$ is practically independent of the critical density.

A similar situation obtains also for the pressure p_0 applied to the target:

$$p_0 \sim \varkappa_0^{1/8} Q_0^{1/2} \rho_{cr}^{1/24} R_0^{-7/4}, \quad \gamma_0 > \gamma_0^*;$$
$$p_0 \sim Q_0^{2/3} \rho_{cr}^{1/3} R_0^{-4/3}, \quad \gamma_0 < \gamma_0^*. \qquad (25)$$

We indicate one more circumstance that follows from the analysis of our problem. It can be shown that in the case when distributed absorption with a finite radiation mean free path l_0 in the vicinity of the critical point is introduced, the maximum of the temperature does not coincide with the critical density and shifts towards lower density, by a distance $\sim l_0$. This means that a noticeable contribution to the radiation of the corona itself in the x-ray band can be made by a region lying ahead of the critical density.

In our problem it was assumed that the main contribution to the pressure is made by the electronic component, i.e., $zT_e > T_i$. For a number of applications of our results, however, interest attaches to the profile of the ion temperature $T_i(r)$. Within the formulation of this problem, the function $T_i(r)$ can be obtained from the equation

$$v\frac{dT_i}{dR} = \frac{T_e(R) - T_i}{\tau_{ei}(T_e, \rho)}, \qquad (26)$$

where $\tau_{ei} = aT_e^{3/2}\rho^{-1}$ is the time of the electron-ion relaxation, and $a = (\frac{3}{8}\sqrt{2\pi})M_i^2(m^{1/2}e^4\Lambda z^3)^{-1}$. The solution of (26) takes in this case the form

$$T_i(R) = \exp\left\{-\int_{R_0}^{R}\frac{dR}{v\tau_{ei}}\right\}\int_{R_0}^{R}dR'\exp\left\{\int_{R_0}^{R'}\frac{dR}{v\tau_{ei}}\right\}\frac{T_e(R')}{v\tau_{ei}}. \qquad (27)$$

Estimates show that in the experimental situations of practical interest[4] we have $\gamma_0 \leqslant \gamma_0^*$. Using next the relations (18) and (14), we obtain near $R = R_{cr}$ with the aid of (27)

$$T_i(R) \approx T_{cr}\left[1 - \exp\left(-\frac{R_0^2 - R_0 R}{R v_{cr}\tau_{cr}}\right)\right]$$
$$\tau_{cr} = \tau_{ei}(\rho_{cr}, T_{cr}). \qquad (28)$$

We consider next the limits of applicability of theory developed in this paper. Obviously, the results will describe correctly the parameters of the corona if the time τ_{ss} in which the steady state is established is much shorter than the characteristic time of variation τ_{var} of the external parameters, such as the flux Q_0. The time τ_{ss} is of the order of the time required to heat the corona by thermal conductivity, i.e., $\tau_{ss} \sim (R_{cr} - R_0)^2 \times (\varkappa_0 T_{cr}^{5/2}\rho_{cr} z/M_i)^{-1}$. The condition $\tau_{ss} \ll \tau_{var}$, both at $\gamma_0 > \gamma_0^*$ and at $\gamma_0 < \gamma_0^*$, is equivalent to the relation

$$q_0 = Q_0/R_0^2 \gg \rho^*(R_0/\tau_{var})^3. \qquad (29)$$

For problems in which the duration τ_{var} and the dimension R_0 of the target are matched, (i.e., the compression of the target takes place at the end of the pulse), the inequality (29) in a sufficiently slowly varying flux Q_0 has a simple physical meaning: the velocity of the dense layers of the target is much smaller than the velocity at which the corona expands.

[1]Yu. V. Afanas'ev, N. G. Basov, P. P. Volosevich, E. G. Gamaliĭ, S. P. Krudyumov, O. N. Korkhin, E. I. Levanov, V. B. Rozanov, A. A. Samarskiĭ, and A. N. Tikhonov, Pis'ma Zh. Eksp. Teor. Fiz. 21, 150 (1975) [JETP Lett. 21, 68 (1975)].

[2]J. Nuckolls, L. Wood, G. Zimmerman, and A. Thiessen, Nature (Lond.) 239, 139 (1972).

[3]Yu. V. Afanas'ev, E. G. Gamaliĭ, O. N. Krokhin, and V. B. Rozanov, Fiz. Metal. Metalloved. 39, 453 (1975).

[4]N. G. Basov, A. A. Kologrivov, O. N. Korkhin, A. A. Rupasov, G. V. Sklizkov, and A. S. Shikanov, Pis'ma Zh. Eksp. Teor. Fiz. 23, 474 (1976) [JETP Lett. 23, 428 (1976)].

[5]Yu. V. Afanas'ev and O. N. Korkhin, Kvantovaya radiofizika

(Quantum Radiophysics), Trudy, Phys. Inst. Acad. Sci. USSR, 52, Nauka, 1970.
[6]I. V. Nemchinov, Fiz. Metal. Metalloved. 31, 300 (1967).
[7]V. V. Pustovalov and V. P. Silin, Zh. Eksp. Teor. Fiz. 59, 2215 (1970) [Sov. Phys. JETP 32, 1198 (1971)]; V. P. Silin, Parametricheskoe vozdeĭstvie izlucheniya bol'shoĭ moshchnosti na plazmu (Parametric Action of High Power Radiation on a Plasma), Nauka, 1973.
[8]Yu. V. Afans'ev, N. G. Basov, P. P. Volosevich, E. G. Gamaliĭ, O. N. Korkhin, S. P. Kurdyumov, E. I. Levanov, V. B. Rozanov, and A. A. Samarskiĭ, Preprint No. 66 Phys. Inst. Acad. Sci. 1972.
[9]K. A. Brueckner, P. M. Campbell, and R. A. Grandey, Nuclear Fusion 15, 471 (1975).

Translated by J. G. Adashko

Propagation of a microwave discharge in heavy atomic gases

V. M. Batenin, I. I. Klimovskiĭ, and V. R. Khamraev

Institute of High Temperatures, USSR Academy of Sciences
(Submitted February 26, 1976)
Zh. Eksp. Teor. Fiz. 71, 603–612 (August 1976)

Results are presented of an experimental investigation of the parameters of a moving microwave discharge in argon, viz., the velocity, geometry, temperature, and electron density. It is found that at pressures exceeding a certain critical value diffusion of resonance radiation plays the main role in the discharge motion. At low pressures, electron diffusion exerts an additional effect on the discharge velocity. Satisfactory agreement between the experimental results and the theory of a microwave discharge set in motion by resonance-radiation diffusion {[V. I. Myshenkov and Yu. P. Raĭzer, Zh. Eksp. Teor. Fiz. 51, 1822 (1972) [Sov. Phys. JETP 24, 969 (1972)]} can be obtained by taking into account the dependence of the excited-atom ionization constant and the fraction of the microwave energy consumed by excitation of the resonance levels on E/P. The motion of the ionization front in argon is accompanied by contraction of the discharge and the formation of shock waves. Addition of molecular hydrogen or nitrogen gas reduces the discharge velocity considerably, the quenching of the argon resonance levels by the molecular impurities playing a major role in the velocity reduction.

PACS numbers: 52.80.Pi

Earlier investigations[1,2] of the mechanism whereby a microwave discharge propagates in air or nitrogen have demonstrated the exceptional usefulness of the idea of the analogy between the propagation of a discharge and the process of slow combustion; this analogy is based on the decisive role played by the thermal conductivity of the gas.[3] However, the very first experiments on the motion of microwave discharges in inert gases at high pressure[1,4,5] have led to the conclusion that their speed, 10^4–10^6 cm/sec, can apparently not be connected with atomic thermal conductivity.

On the other hand, the use of the energy equation demonstrates that at such velocities of the discharge front, the gas behind the front remains practically unheated, i.e., we are dealing with a non-equilibrium-ionization wave, which is not connected with the motion of the gas as a whole.

From an analysis of the published data it is seen that to describe the propagation of microwave discharges in inert gases one can invoke the following mechanism[6–8]: microwave breakdown on the discharge front, diffusion of the resonant radiation, and diffusion of the charged particles.

A theoretical analysis of microwave breakdown[6] would call, in the course of the solution of the problem, for far-reaching assumptions, principal among which are constancy of the electron temperature during the course of the development of the ionization by the direct electron impact, a Maxwellian type of distribution function of the electrons, and the use of the geometrical optics approximation. It has turned out that the result of the solution depends strongly on the form of the initial distribution of the electron density in the plasma cluster, while typical values of the discharge velocities agree in order of magnitude with those observed.

Calculations of the process of the motion of the ionization wave as a result of diffusion of the resonant radiation[7] were made under the assumption that this process determines the density of the excited atoms, which are then ionized by direct electron impact. Recombination and diffusion of the electrons proceed slowly and are insignificant in the energy-release zone. The discharge velocities are large and close to those typical of microwave breakdown.

Finally, the influence of the diffusion of the charged particles was analyzed by Bulkin, Ponomarev, and Solntsev,[6] who studied the motion of an ionization front in long tubes at pressures ~0.1 mm Hg. They have assumed that during the second stage of the discharge development the ionization wave constitutes an electron-density wave whose motion is due to diffusion. The electron losses are also determined by their diffusion to the

