It should be noted that the numerical value of h_1 must be approached with caution because of the random numerical smallness in the denominator of (15) for the threshold h_1, but one can hope the qualitative picture of the transitions to remain valid.

We note also that in the presence of dissipation imaginary increments appear in the expressions for the squares of the frequencies of the natural oscillations, but all the conclusions concerning the stability and the expression for the thresholds remain in force.

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1)One more structure with triangular symmetry is possible, but in our case it coincides with the hexagonal structure (7) and is obtained from the latter by shifting the origin (see[9]).

2)Generally speaking, the third-terms make a definite contribution to the fourth-order terms, but this contribution is small in terms of the parameter χ^2.

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Features of the volt-ampere characteristics and oscillations of the electric potential in superconducting channels

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The physical nature of the supercritical resistive current states in narrow superconducting channels and the connection of this phenomenon with relaxation processes of "mixing" of the electron and hole components of the normal-excitation spectrum are discussed. The exact solutions of the kinetic equations in the vicinity of the singular points of the structure of the resistive state are investigated and the effective boundary conditions at these points for the macroscopic equations of the structure are found. The solutions of these equations for large currents of the order of the upper critical current j_{c2} and the volt-ampere characteristics of a long channel are constructed. The role of the principle of minimum entropy production in the formation of the structure of the resistive state is noted. At low currents the static structure is found to be unstable, generally speaking. The physical reasons for the instability are analyzed together with the corresponding manifestations of the nonstationarity in the resistive state.

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It is well known[1] that the peculiar diamagnetic properties of a superconductor are, in a certain sense, a more fundamental characteristic than the infinite conductivity. Significant in this respect is, e.g., the explanation of the nature of the dissipative current states in bulk type-I and type-II superconductors. From a microscopic point in view[2,3] the electric fields that arise in these superconductors on passage of a transport current have, in essence, an induction origin. They are associated with the dynamics of the magnetic fluxes in the superconductor and with the acceleration of the superconducting condensate in the vortex electric fields:

$$\partial \mathbf{p}_s/\partial t = e\mathbf{E}, \quad \text{rot}\, \mathbf{p}_s = -e\mathbf{H} \tag{1}$$

($\mathbf{p}_s$ is the condensate momentum per electron; $\hbar = c = 1$).

A different situation arises in narrow superconducting "channels" connected to a current source. In view of the small transverse dimensions of the samples the dissipative current states observed experimentally in them are not explained by vortex mechanisms[2] or by the structure of the intermediate state,[3] and, thus, a new physical aspect of superconductivity is manifested here—a singularity in the response of the superconductor to a nonequilibrium longitudinal electric field.

As already noted,[4] the question of the nature of the resistive states in narrow channels abuts primarily upon the study of the Cooper instability in the normal current state at below-critical temperatures $T < T_c$. Unlike a condensate-accelerating vortex electric field, which is associated with the change of magnetic flux and (in accordance with Anderson's theorem[5] on violation of time-reversal symmetry) suppresses the superconductivity, a longitudinal field, if it is sufficiently small, should not, generally speaking, impede the formation of superconducting droplets. The condensate can rapidly "adjust itself" to the distribution of the chemical potential of the Cooper pairs $\Phi = e\varphi + \frac{1}{2}\partial\chi/\partial t$ (φ is the electric potential and χ is the phase of the superconducting-order parameter), which modulates the magnitude of the order parameter, whereas the slow relaxation of the normal excitations in the field Φ via diffusion fluxes ensures normal conduction in accordance with Ohm's law[1)].

In previous papers[4,9] we have already considered certain fundamental aspects of the problem of resistive states and have analyzed qualitatively the structure of these states and the volt-ampere characteristics for not-too-large currents. In the present report we give a more detailed discussion of the physical nature of the phenomenon and adduce, together with a treatment of certain important mathematical aspects omitted in[4,9], a number of further results on the structure at large currents and on the characteristic manifestations of the nonstationarity in the resistive state.

The starting point is the kinetic equations for "dirty" superconductors[9,11] (with nonmagnetic impurities)

$$\hat{u}\sigma_z\hat{u} = \sigma_z, \quad \sigma_z\hat{\omega}\hat{u} - \hat{u}\hat{\omega}\sigma_z = i_s D\nabla(\hat{u}\sigma_z\nabla\hat{u}), \quad i_s = \pm i;$$

$$\hat{u}_+\{\hat{\omega},\hat{f}\}\sigma_z - \sigma_z[\hat{\omega},\hat{f}]\hat{u}_- = iD[\nabla(\sigma_z\nabla\hat{f} + \hat{u}_+\sigma_z\nabla\hat{f}u_-) + \hat{u}_+\sigma_z\nabla\hat{u}_+\nabla\hat{f} + \nabla\hat{f}\nabla\hat{u}_-\sigma_z\hat{u}_-], \tag{2}$$

$$[\sigma_z, \hat{f}] = 0; \tag{3}$$

$$\hat{\omega} = \hat{\omega}^{(0)} - \sigma_z e\varphi - \hat{\Delta}, \quad \hat{\Delta} = \begin{pmatrix} 0 & \Delta \\ \Delta^* & 0 \end{pmatrix},$$

$$\hat{\omega}^{(0)}(t_1, t_2) = i\frac{\partial}{\partial t_1}\delta(t_1 - t_2). \tag{4}$$

Here $\hat{u}_\pm(\mathbf{r}; t_1, t_2)$ are the reduced "propagation functions" of the electrons (in the normal state, $\hat{u}(t_1, t_2) \equiv \delta(t_1 - t_2)$, fixed by the "retarded" and "advanced" conditions $\hat{u}_\pm(t_1 - t_2) \sim \theta(\pm(t_1 - t_2))$; $\hat{f}(\mathbf{r}, t_1, t_2)$ is the two-component ($\hat{f} = f + \sigma_z f_z$) "distribution function" of the normal excitations[2)]; σ_z is a Pauli matrix and $D = \frac{1}{3}v_F l$ is the electron diffusion coefficient. In the matrix multiplication in Eqs. (2) and (3) contraction of the functions $\hat{u}_\pm(t_1, t_2)$ and $\hat{f}(t_1, t_2)$ with respect to the time is implied. In view of the unimportance of the intrinsic magnetic field of the current in narrow channels, the vector potential is omitted in Eqs. (2) and (3): $\mathbf{A} = 0$.

Equations (2) and (3) are supplemented by formulas for the macroscopic quantities—for the order parameter $\hat{\Delta}$, current density $\mathbf{j}$ and electron-density change δN:

$$\hat{\Delta}(t) = \frac{|g|\nu_F}{2}\pi(\hat{v}_+\hat{f} + \hat{f}\hat{v}_-)(t,t), \tag{5}$$

$$\mathbf{j}(t) = e\nu_F D\frac{\pi}{2}\mathrm{Tr}[\hat{u}_+\sigma_z\nabla\hat{u}_+\hat{f} + \hat{f}\nabla\hat{u}_-\sigma_z\hat{u}_- + \sigma_z\nabla\hat{f} + \hat{u}_+\sigma_z\nabla\hat{f}\hat{u}_-](t,t). \tag{6}$$

$$\delta N(t) = -\nu_F\frac{\pi}{2}\mathrm{Tr}\,\sigma_z(\hat{u}_+\hat{f} + \hat{f}\hat{u}_-)(t,t) - \nu_F e\varphi(t), \tag{7}$$

where $\hat{v}$ is the nondiagonal (with respect to the spin in the electron-hole isotopic space; cf.[12]) part of the matrix $\hat{u}$.

In the chosen gauge $\mathbf{A} = 0$, the gauge transformations in Eqs. (2)–(7) reduce to the following:

$$(\hat{\Delta}, \hat{u}, \hat{f}) \to \exp(i\sigma_z\alpha)(\hat{\Delta}, \hat{u}, \hat{f})\exp(-i\sigma_z\alpha),$$
$$e\varphi \to e\varphi - \partial\alpha/\partial t \quad (\alpha = \alpha(t), \ \nabla\alpha = 0). \tag{8}$$

Since the resistive states should be observed near T_c, where the Joule heat can be neglected,[3)] the distribution function of the excitations is close to equilibrium, and, since the operator $\hat{\omega}^{(0)}$ in (4) corresponds to the total energy, including the potential of the nonequilibrium electric field, the distribution function can be represented in the form[9]

$$\hat{f} = f^{(0)}(\hat{\omega}^{(0)} - \sigma_z e\varphi) + \hat{f}', \tag{9}$$

where $\hat{f}'$ is the kinetic correction to the equilibrium distribution function[9,11] and

$$f^{(0)}(\varepsilon) = \frac{1}{2}\left(1 + \mathrm{th}\frac{\varepsilon}{2T}\right) = T\sum_{\omega_n}\frac{\exp(-i\omega_n 0)}{\varepsilon - i\omega_n},$$
$$\omega_n = \pi T(2n+1), \quad n = 0, \pm 1, \pm 2, \ldots \tag{10}$$

Considering a stationary current state, we can fix the constant $\partial\chi/\partial T$ ($\nabla\partial\chi/\partial T = 0$), i.e., fix the origin in the electric potential (cf. the transformations (8))

$$e\varphi = \Phi - {}^1/_2\partial\chi/\partial t. \tag{11}$$

Proceeding, after this, to the Fourier transformation of the homogeneous functions of the time in Eqs. (2)–(7), (9):

$$f(t_1 - t_2) = \int\frac{d\varepsilon}{2\pi}f(\varepsilon)\exp[-i\varepsilon(t_1 - t_2)]$$

and, in analogy with[9], carrying out the calculations to the first nonvanishing terms in the parameter $\eta = (1 - T/T_c)^{1/2} \ll 1$, we obtain (taking the one-dimensional character of the problem into account)

$$\left[\frac{T_c - T}{T_c} + \frac{\pi}{8T_c}D\left(\frac{d}{dx}\right)^2 - \frac{7\zeta(3)}{8}\frac{2\Phi^2 + \Delta^2}{(\pi T_c)^2}\right]\Delta = -\int\frac{d\varepsilon}{2}(v_+ - v_-)\Psi\frac{df^{(0)}(\varepsilon)}{d\varepsilon}, \tag{12}$$

$$j = e\nu_F D\left[\frac{\pi}{8iT_c}\left(\Delta^*\frac{d\Delta}{dx} - \Delta\frac{d\Delta^*}{dx}\right) - \int\frac{d\varepsilon}{2}\left(1 + u_+u_- + \frac{v_+\tilde{v}_- + \tilde{v}_+v_-}{2}\right)\frac{d\Psi}{dx}\frac{df^{(0)}(\varepsilon)}{d\varepsilon}\right]. \tag{13}$$

$$\delta N = \nu_F\int\frac{d\varepsilon}{2}(u_+ + u_-)\Psi\frac{df^{(0)}(\varepsilon)}{d\varepsilon} - \nu_F\Phi. \tag{14}$$

Here u, v, and $\tilde{v}$ are the elements of the matrix $\hat{u}$:

$$\hat{u} = \begin{pmatrix} u & v \\ \tilde{v} & u \end{pmatrix}, \quad u_- = u_+^*, \quad \tilde{v}_+ = v_-^*, \quad \tilde{v}_- = v_+^*, \tag{15}$$

and the function Ψ is defined by the formula

$$\hat{f}' = \sigma_z\frac{df^{(0)}(\varepsilon)}{d\varepsilon}(\Phi - \Psi) = \sigma_z\frac{df^{(0)}(\varepsilon)}{d\varepsilon}\psi. \tag{16}$$

We emphasize that, under the assumption of ideal heat transfer, it is precisely the σ_z-component (16) of the correction $\hat{f}'$ to the distribution function that turns out to be important, and the principal role in the kinetic equations (3) is played by the particle-number detailed-balance equation associated with this correction (cf.[9]); for the function Ψ of (16), this equation can be written in the form

$$D\frac{d}{dx}\left(\left(1+u_+u_-+\frac{v_+\tilde{v}_-+\tilde{v}_+v_-}{2}\right)\frac{d\Psi}{dx}\right)+\frac{1}{i}(\Delta^*(v_+-v_-)+\Delta(\tilde{v}_+-\tilde{v}_-))\Psi=0. \quad (17)$$

From Eqs. (12), (13) and (17) follows the conservation of the current (13): $dj/dx=0$. The electroneutrality condition

$$\delta N=0 \quad (18)$$

determines, according to (14), the potential Φ for a given current (13).

For the functions u, v and $\tilde{v}$ (15), we obtain from (2) the equations ($u^2-v\tilde{v}=1$)

$$\begin{aligned}\varepsilon v-\Delta u&=\frac{i_eD}{2}\frac{d}{dx}\left(u\frac{dv}{dx}-v\frac{du}{dx}\right),\\ \varepsilon\tilde{v}-\Delta^* u&=\frac{i_eD}{2}\frac{d}{dx}\left(u\frac{d\tilde{v}}{dx}-\tilde{v}\frac{du}{dx}\right).\end{aligned} \quad (19)$$

We shall consider first a characteristic feature of Eqs. (12)–(14), (17)–(19), as manifested in the problem of a superconducting droplet ($\Delta\to 0$). As in the case of small currents,[9] the key role here is played by the fact that the nonequilibrium correction (16) to the distribution function, despite the electroneutrality requirement (14), (18), is not identically equal to zero, i.e., $\Psi\neq\Phi$, in view of the energy dependence of the function Ψ (cf. (17)). This entails important consequences. In the calculation of the left-hand sides of Eqs. (12), (13) (cf.[9]) with the aid of the equilibrium distribution function (10), owing to the analyticity of the functions $\hat{u}_\pm(\varepsilon)$ in the upper or lower half-planes the characteristic energies were determined by the poles of the function $f^{(0)}(\varepsilon)$ (10): $\varepsilon\sim T_c$. The situation is completely different in the right-hand side of (12) (the "anomalous term" in the terminology of Gor'kov and Eliashberg[8]). It can be seen from Eq. (17) that the function Ψ has singularities on both sides of the real energy axis, and, as a result, the characteristic energies in the integral (12) are determined, according to Eqs. (19), by the quantity $\Delta\to 0$. For given boundary conditions $u(\infty)=1$, $v(\infty)=0$, the gradients in these equations (19) tend to zero ($d/dx\to 0$) as $(\Delta,\varepsilon)\to 0$. Since the size of a droplet, $\xi(T)\sim(\xi_0 l)^{1/2}/\eta$ ($\xi_0\sim v_F/T_c$), remains finite, in Eqs. (19) we can replace

$$\Delta(x)\to\lambda\delta(x),\quad \lambda=\int dx\,\Delta(x).$$

Omitting, after this, the unimportant constant phase factor (transformations (8)), we can put

$$v=\tilde{v}=\operatorname{sh}\beta,\quad u=\operatorname{ch}\beta \quad (20)$$

and obtain from (19) the equation

$$\varepsilon\operatorname{sh}\beta-\lambda\delta(x)\operatorname{ch}\beta=\frac{i_eD}{2}\frac{d^2\beta}{dx^2},\quad \operatorname{ch}\beta(\infty)=1. \quad (21)$$

It is not difficult to integrate Eq. (21). However, for the following it is sufficient to use it to estimate the characteristic energies ε in the integral (12). It can be seen from (21), that, for $\Delta\to 0$, $\lambda\to 0$,

$$\frac{d}{dx}\sim\lambda\to 0,\quad \varepsilon\sim\lambda^2\to 0.$$

Comparing these estimates with the formulas (12), (13), (17) and (20), we find that for any finite values of x the right-hand side of Eq. (12) vanishes more rapidly than the left as $\Delta\to 0$. As a result, the term linear in the potential Φ drops out and the droplet is described by the equation that was used in the paper[4] to find the upper critical current j_{c2}.

The developed structure of the resistive state for currents $j<j_{c2}$ is (in a homogeneous channel) a periodic alternation of "normal" and "superconducting" regions.[4,9,14] Here, as can be seen from (11) and (12), because of the unbounded increase of the electric potential φ it is necessrry to introduce discontinuities Φ_0, determined by the discontinuities of the derivative $\partial\chi/\partial t$ (11) of the phase and cancelling the increment of φ at the limits of the period, into the chemical potential Φ of the Cooper pairs.[4,9] From a physical point of view this is the requirement that the chemical potential of the Cooper pairs be constant on the average, and generalizes the analogous requirement of constancy of Φ when we pass across a Josephson junction.[15] The accumulation of electric charges on the plates of the capacitor, which is what this junction is, produces a discontinuity φ_0 in the electric potential, whence follows (cf. (11)) the Josephson relation for the phase discontinuity: $\partial\chi_0/\partial t=-2e\varphi_0$. In contrast to this, in the resistive state the potential φ and the charges are distributed continuously and (for a periodic structure) the increment in the potential over a period appears in the Josephson relation: $\partial\chi_0/\partial t=2e\bar{E}d$, where $\bar{E}$ is the average field intensity and d is the period.

It should be emphasized that the construction considered[4)] is not connected with any inhomogeneities in the sample, as it is in the case of the Josephson effect,[15] although the presence of inhomogeneities should, of course, affect the periodicity of the structure. This construction is a typical (for self-consistent field theory) construction of a special singular solution, and, in this sense, as already noted,[4,9] it does not differ from, e.g., the construction of the vortex singularities in type-II superconductors.[19]

The "centers" at which $\partial\chi/\partial t$ experiences a discontinuity ("phase-slip centers," in the foreign literature[17]) should naturally be associated with the points at which $\Delta=0$.[4,9] Since the order parameter has a different time dependence $\exp(\pm i\Phi_0 t)$ within the limits of each period, there arises the important question of the exact solutions of the initial equations (2)–(7) in the neighborhood of the singularity: these solutions should determine the boundary conditions to the local equation of the form in[4]. The characteristic size of this neighborhood is of the order of $(\xi_0 l)^{1/2}$. Therefore, the corresponding gradients here are large: $d/dx\sim 1/(\xi_0 l)^{1/2}\gg 1/\xi(T)$, while the other quantities remain small: $\partial/$

∂t, Δ, $\Phi \sim \eta$. From this it follows that, in the neighborhood of the singularity, Eqs. (2), (5) and (9) give, in the leading approximation in the parameter η,

$$\Delta(x,t)=\int Q(x-x')\Delta(x',t)dx', \quad (22)$$

$$Q(x-x')=\frac{|g|v_F}{2}\int d\varepsilon f^{(0)}(\varepsilon)\frac{1}{2}\left[\left(\varepsilon-\frac{iD}{2}\left(\frac{d}{dx}\right)^2\right)^{-1}+\left(\varepsilon+\frac{iD}{2}\left(\frac{d}{dx}\right)^2\right)^{-1}\right]\delta(x-x'). \quad (23)$$

Placing the singularities at the coordinate origin, we have

$$\Delta(x,t)=\theta(x)\Delta_+(x)e^{i\alpha t}+\theta(-x)\Delta_-(x)e^{-i\alpha t}. \quad (24)$$

We shall substitute this relation into Eq. (22) and consider, e.g., the region $x>0$:

$$\Delta_+(x)=\theta(x)\int_0^\infty Q(x-x')\Delta_+(x')dx'+e^{-2i\alpha t}\theta(x)\int_{-\infty}^0 Q(x-x')\Delta_-(x')dx'. \quad (25)$$

Since the kernel Q (23) is localized over short distances $(\xi_0 l)^{1/2}$, the second term in Eq. (25) is small, since, unlike the first term, it is determined by the weak nonlocal "overlap" of values of Δ from different regions. Therefore, we can make use of the averaging principle $(\overline{\exp(-2i\alpha t)}=0)$ and consign the second term to the terms of higher order of smallness. Thus, in the leading approximation $\Delta_+(x)$ is determined from the equation

$$\Delta_+(x)=\theta(x)\int_0^\infty Q(x-x')\Delta_+(x')dx'. \quad (26)$$

The latter equation is equivalent to the equation describing the change of Δ near the boundary with a normal metal. The linear asymptotic forms of the solutions of Eq. (26) determine (cf., e.g., [20]) the boundary condition to the local equations, which, to within small higher-order terms, reduces simply to the requirement $\Delta(0)=0$.

The small oscillating corrections (25) to the "averaged" solutions do not enter into the problem of the present paper. We remark only that these oscillations will, obviously, be manifested in all quantities and will give a weak Josephson effect.[15] Despite the smallness of this effect, it has been possible, according to the communication,[17] to observe it experimentally.

From the above discussions flows one important consequence, concerning the volt-ampere characteristics and the structure of the resistive state in the immediate vicinity of the upper critical current j_{c2}. As was seen in the droplet problem (Eqs. (19)–(21)), as $\Delta\to 0$ the range of nonlocality in the functions u, v and $\tilde{v}$ in the "anomalous" term in (12) becomes large. For finite but small Δ this should lead to complete "overlap" of the periods of the structure in the "anomalous" term, which, generally speaking, is now not small compared with the left-hand side of Eq. (12). It is obvious that in these conditions the treatment of the "centers" of singularity as isolated regions with small radius $(\xi_0 l)^{1/2}$ loses its meaning and the problem becomes essentially nonstationary. However, according to (19), the order parameter Δ, at least in this region of currents, is equal to $(d/dx)^2\sim\eta^2\ll\eta$, and this region itself turns out to be very narrow (see below). Therefore, we can postulate that, because of fluctuations not taken into account in the kinetic scheme of [12], the exact solution of the problem in this region of currents has no special physical meaning.[5)]

In view of these arguments we shall consider the developed structure of the resistive state (for currents $j<j_{c2}$) when all quantities have their "natural" orders, determined by the left-hand side of Eq. (12):

$$\Delta\sim\eta,\quad \Phi\sim\eta,\quad d/dx\sim\eta\quad (\eta=(1-T/T_c)^{1/2}). \quad (27)$$

For these orders, the local approximation is valid in Eqs. (19), so that, e.g., in the leading approximation we have

$$u\approx\varepsilon(\varepsilon^2-|\Delta|^2)^{-1/2},\quad v\approx\Delta(\varepsilon^2-|\Delta|^2)^{-1/2},\quad \tilde{v}\approx\Delta^*(\varepsilon^2-|\Delta|^2)^{-1/2}. \quad (28)$$

As in the treatment of an isolated droplet, this enables us to fix the potential "shift" $\partial\chi/\partial t$ (11) within a period and, by using the transformations (8), to reduce the problem to a stationary problem with effective boundary conditions $\Delta=0$ at the singularity "centers."

Turning now to Eqs. (12)–(14), (17)–(19), we note that, as can be seen from formulas (28) and Eq. (17), for energies $|\varepsilon|<|\Delta|$ inside the gap the "field" $\Psi(\varepsilon,x)$ undergoes rapid exponential damping ($\sim e^{-1/\eta}$) in the interior of the superconducting regions. Outside the gap ($|\varepsilon|>|\Delta|$), according to (28) the difference $v_+-v_-\approx 0$ and it is necessary to take into account the next terms of the expansion in the parameter η in Eqs. (19). A straightforward analysis confirms the consistency of the orders of the quantities (27) in the left- and right-hand sides of Eq. (12). Unfortunately, because of the "anomalous" terms in (12), (13), the asymptotically exact equations that arise in this process are exceedingly complicated. It is therefore worthwhile to model Eqs. (12)–(14), (17) qualitatively by simpler equations, making use of the following considerations.

Since, according to (17), for $v_+-v_-\neq 0$ the "field" Ψ attenuates exponentially, we shall assume that, effectively,

$$(v_+-v_-)\Psi\sim 0, \quad (29)$$

and for the functions u, v and $\tilde{v}$ we shall make use of the leading approximation (28). Then, as is not difficult to see from (12), the resistive state is described by a real parameter Δ and, according to (13), (17) and (29), the function Ψ (within the limits of the period $d=2a$; $\Delta(\pm a)=0$) is equal to

$$\Psi=-\frac{2j}{ev_F D}\int_0^x(1+u_+u_-+v_+v_-)^{-1}dx'. \quad (30)$$

Substituting this expression into Eqs. (14) and (18) and taking (28) into account, we find the potential Φ and the average intensity of the electric field:

$$\bar{E}=\frac{1}{2ea}(\Phi(-a)-\Phi(a))=\frac{j}{2\sigma_n a}\int_{-a}^{a}dx\int d\varepsilon\frac{df^{(0)}}{d\varepsilon}\frac{2}{1+(\varepsilon^2+\Delta^2)/|\varepsilon^2-\Delta^2|},$$
$$\bar{E}=\frac{j}{\sigma_n}\left(1-\frac{2\bar{\Delta}}{3T_c}\right), \tag{31}$$

where σ_n is the conductivity of the normal metal and $\bar{\Delta}$ is the average value of the order parameter.

Thus, in the model approximation (29), the "excess" current for a given voltage has a different nature near j_{c2} from that in the case of "small" currents $j\sim j_c \ll j_{c2}$.[9] In that case it was associated with the addition of the ordinary superconducting current to the normal current (cf. (13)), while here it is associated with the increase of the "normal" conductivity in conditions of superconducting pairing of electrons.

Taking (29) into account in Eq. (12) and confining ourselves, in the potential Φ, to the leading approximation in the parameter η, we obtain

$$\left[\frac{T_c-T}{T_c}+\frac{\pi}{8T_c}D\left(\frac{d}{dx}\right)^2-\frac{7\zeta(3)}{8(\pi T_c)^2}\left(2\left(\frac{ejx}{\sigma_n}\right)^2+\Delta^2\right)\right]\Delta=0. \tag{32}$$

Together with formula (31), Eq. (32) solves qualitatively the problem posed about the structure of the resistive state and the volt-ampere characteristics of a superconducting channel at currents $j\sim j_{c2}$. This equation can be integrated in the limiting case $|(j-j_{c2})|/j_{c2}\ll 1$. We write (32) in dimensionless variables:

$$x\to\left(\frac{\pi D}{8(T_c-T)}\right)^{1/2}x,\quad \Delta\to\pi T_c\left(\frac{8(T_c-T)}{7\zeta(3)T_c}\right)^{1/2}\Delta=\Delta_0\Delta,\quad j\to j_{c2}j, \tag{33}$$

where[4]

$$j_{c2}=\sigma_n E_{c2},\quad E_{c2}=\frac{4(T_c-T)}{e}\left(\frac{2\pi T_c}{7\zeta(3)D}\right)^{1/2}.$$

We then have

$$\left(\left(\frac{d}{dx}\right)^2+1-j^2x^2-\Delta^2\right)\Delta=0,\quad \Delta(\pm a)=0. \tag{34}$$

As $j\to 1$ ($j\to j_{c2}$) the quantity $\Delta\to 0$, and the period increases: $a\to\infty$. We shall denote by $j(a)$ the eigenvalue of the linear problem:

$$\frac{d^2\Delta}{dx^2}+(1-j^2(a)x^2)\Delta=0,\quad \Delta(\pm a)=0. \tag{35}$$

Obviously, $j(a)\to 1$ as $a\to\infty$.

Regarding the difference $j-j(a)$ and Δ^2 as a perturbation in Eq. (34), we find that, in the first approximation, the solution for Δ is proportional to the solution of Eq. (35), and the condition for solubility of the equation of the next approximation has the form

$$\int_{-a}^{a}\Delta^4dx=2j(a)(j(a)-j)\int_{-a}^{a}x^2\Delta^2dx. \tag{36}$$

For large a the solution of Eq. (35) is approximately equal to $A\exp(-x^2/2)$, and, taking (36) into account, we find in the first nonvanishing approximation

$$\Delta\approx 2^{3/4}(j(a)-j)^{1/2}\exp(-x^2/2). \tag{37}$$

Hence follows

$$\bar{\Delta}=(\pi/\sqrt{2})^{1/2}(j(a)-j)^{1/2}/a. \tag{38}$$

Applying standard methods of mathematical physics to Eq. (35), we can find the asymptotic formula for large a:

$$1-j(a)\approx\frac{2\sqrt{2}e}{\pi}ae^{-a^2}. \tag{39}$$

In the solution found ((37), (38)) the magnitude of the period $d=2a$ remains undetermined, and this is also typical for the singular solutions of the self-consistent field theory (cf.[19]). To determine the period it is necessary to make use of additional physical considerations. In view of the nonequilibrium character of the resistive state we must postulate[9] that the stationary regime corresponds to the minimum entropy production.[21] Thus, the period is determined from the condition minimizing the average electric field $\bar{E}$ (31) (i.e., maximizing $\bar{\Delta}$ (38)) for a given current j. Using formulas (38) and (39), it is not difficult to convince oneself that this minimum indeed exists and corresponds to the following values of $\bar{\Delta}$ and of the period of the structure:

$$\bar{\Delta}=\left(\frac{\pi}{\sqrt{2}}\right)^{1/2}\frac{\sqrt{1-j}}{a},\quad 1-j\approx\frac{2\sqrt{2}e}{\pi}a^3e^{-a^2}\quad (a\gg 1). \tag{40}$$

As $j\to 1$ the period increases (on account of the increase of the sizes of the normal regions) fairly slowly: $a\sim(-\ln(1-j))^{1/2}$.

Together with the dimensionality formulas (33), the formulas (31), (40) give a parametric representation of the volt-ampere characteristics near j_{c2}. Using them we can estimate the range of currents in which the stationary picture of the resistive state is inapplicable. Since $\Delta\lesssim\eta^2$ in this region, we have, according to (40),

$$(j_{c2}-j)/j_{c2}\lesssim[(T_c-T)/T_c]^2.$$

With decrease of the current the period of the structure decreases and, having reached a minimum value determined by the size $\xi(T)$ of the droplet, begins to increase again, this time because of the increase of the sizes of the superconducting regions. On further decrease of the current, when j reaches values of the order of the lower critical current $j_c\sim\eta^3$, the period of the structure increases to such an extent that effects associated with the depth of penetration of the electric field from the normal region to the superconducting region become important and the description of the structure of the resistive state becomes qualitatively different.[9] The qualitative difference between "small" currents $j\sim j_c$ and "large" currents $j\sim j_{c2}$ is fairly sharply displayed in the experimental volt-ampere characteristics (cf.[12]) for $T\to T_c$, when $j_{c2}/j_c\sim 1/\eta\gg 1$. Without repeating the derivation from[9], we give the final system of equations for currents $j\sim j_c$ in dimensionless variables[9]:

$$\Delta\to\Delta_0\Delta,\quad \Phi\to\frac{\Delta_0}{\sqrt{2}}\Phi,\quad p_s\to\left(\frac{2(T_c-T)}{\pi D}\right)^{1/2}p_s,$$

$$x\to\frac{T_c}{\Delta_0}\left(\frac{D}{\pi(T_c-T)}\right)^{1/2}x,\quad t\to\frac{T_c^2t}{\pi(T_c-T)\Delta_0^2},\tag{41}$$

$$j\to\frac{ev_F\Delta_0^2}{T_c}\left(\frac{\pi D}{2}(T_c-T)\right)^{1/2}j,\quad \Delta_0=\pi T_c\left(\frac{8(T_c-T)}{7\zeta(3)T_c}\right)^{1/2};$$

here we have

$$\Delta^2+p_s^2+\Phi^2=1,\quad \Delta^2p_s+E=j\quad (E=-\partial\Phi/\partial x,\ \Delta\neq0),\tag{42}$$

$$\int_0^\infty\frac{d\varepsilon}{\mathrm{ch}^2\,\varepsilon}\psi(\varepsilon,x)=0,\quad \left[\frac{\partial}{\partial t}+2\left(\frac{p_s\Delta}{\pi\varepsilon}\right)^2-\left(\frac{\partial}{\partial x}\right)^2\right]\psi=\left[2\left(\frac{p_s\Delta}{\pi\varepsilon}\right)^2-\left(\frac{\partial}{\partial x}\right)^2\right]\Phi.\tag{43}$$

Here Δ denotes the modulus of the order parameter and $p_s=\frac{1}{2}\partial\chi/\partial x$ is the momentum of the condensate. In the kinetic equation (43) for the nonequilibrium correction ψ (16) to the distribution function the small time derivative $\partial/\partial t\sim(\partial/\partial x)^2\sim\eta^4$ has been kept for what follows (cf. the initial equation (3)).

The boundary conditions to Eqs. (42), (43) contain, besides the continuity of the quantities Δ, p_s, E, ψ and $d\psi/dx$, the discontinuity Φ_0 of the potential Φ at the points $\Delta=0$. For a given current (cf. (42)), the magnitude of the discontinuity Φ_0 is uniquely determined by the period d of the structure. As at large currents, the period is fixed by the condition for minimum entropy production and determines the average electric-field intensity $\bar{E}=\Phi_0/d$, i.e., the volt-ampere characteristic $\bar{E}(j)$. In view of the complexity of Eqs. (43), the calculation of $\bar{E}(j)$ in[9] was performed by modeling (43) by a simpler equation. It must be emphasized, however, that for a comparison of the theory with experiment a large volume of information is contained in the dimensionality formulas (41). These formulas not only enable us to estimate the characteristic orders of all the quantities, but also establish, to within certain unknown dimensionless functions (of order unity) of the current ratio j/j_c, the different temperature dependences (in the spirit of laws of corresponding states (see below)).

Certain simplifications are possible in a long channel for currents $j\to j_c$, when the period of the structure becomes large and we can consider the homogeneous equation

$$\frac{d^2\Psi}{dx^2}-2\left(\frac{p_s\Delta}{\pi\varepsilon}\right)^2\Psi=0\tag{44}$$

for an isolated singularity. The asymptotic solutions of Eq. (44) at large distances $d\gg\delta_E$, where Δ, $p_s\to$ const,

$$\Psi\sim\exp(-\mathrm{const}\cdot d/\varepsilon)$$

determine the potential Φ (cf. Eqs. (14), (16), (43)):

$$\Phi(x)=\int_0^\infty\frac{d\varepsilon}{\mathrm{ch}^2\,\varepsilon}\Psi(\varepsilon,x).$$

Thus, the electric field inside a superconducting region is, in order of magnitude,

$$E(d)\sim\exp(-\mathrm{const}\cdot\sqrt{d}).\tag{45}$$

According to the second Eq. (42), to ensure the minimum entropy production the field (45) must be made equal to the difference $j-j_c$, since the critical current $j_c=2/3\sqrt{3}$ is the maximum value of the superconducting current $j_s=\Delta^2p_s$. Since, on the other hand, as $j\to j_c$, $\Phi_0\to$ const and $d=\Phi_0/\bar{E}$, the asymptotic volt-ampere characteristic of a long channel for $j\to j_c$ follows from (45):

$$\frac{j-j_c}{j_c}\sim\exp\left[-\mathrm{const}\cdot\left(\frac{\sigma_nj_c}{\bar{E}}\right)^{1/2}\right].\tag{46}$$

We shall discuss the physical meaning of Eqs. (43) in more detail. As already noted, the key role in the phenomena under consideration is played by the circumstance that, unlike in a normal metal, in which the electroneutrality condition makes the kinetic correction ψ to the distribution function vanish identically, in a superconductor this condition is fulfilled only integrally (the first Eq. (43)) and $\psi\neq0$. According to (14), (16), ψ determines the charge density in ordinary space and in energy space, and Eq. (43) is the detailed-balance equation for the charge in the superconductor. It follows from Eq. (43) that the electric field is damped over a fairly large distance $\delta_E\sim(\xi_0l)^{1/2}(1-T/T_c)^{-1}$ into the superconducting regions, and that the electric charges $\sim\mathrm{div}\mathbf{E}/4\pi$ compensating the field are distributed over these same distances. The essential point is that ψ (16) defines the correction (to the distribution function) that is nonsymmetric (the σ_z-component!) in the electron-hole "space,"[12] and, thus, the kinetic equation (43) essentially describes, as it should, the "mixing" of the electron and hole components of the excitation spectrum. Here the quantity $2(p_s\Delta/\pi\varepsilon)^2$ in this equation plays the role of the characteristic "mixing frequency."

From this point of view it becomes clear that the inclusion of any factor that violates the time-reversal symmetry in the sense of Anderson's theorem should have a substantial effect on the processes considered. The time-reversal operation "interchanges" the electron and hole states, and the specific correlations between these states constitute the nature of superconductivity.[22,23] The appearance of the quantity $p_s^2(t\to-t,\ p_s\to-p_s)$ in the "frequency" of mixing in Eq. (43) is already comprehensible from these considerations. Indeed in the case of a condensate at rest ($p_s=0$), owing to the full symmetry between excitations of the electron and hole types, scattering by ordinary impurities does not lead to "mixing" of the excitations.[6)] A more radical way of "mixing" electrons and holes is by scattering by paramagnetic impurities. In the "dirty" limit $\tau_sT_{c0}\ll1$ (τ_s is the time between spin-flips of an electron and T_{c0} is the temperature of the "clean" superconductor), the energy gap in the spectrum of the superconductor disappears[25] and the dynamics of the condensate ceases to have any influence at all on the energy distribution of the excitations. This circumstance also affects the electroneutrality equations (43). According to the work of Gor'kov and Éliashberg,[8] in this case the constant τ_s appears in Eq. (43) in place of the energy-dependent factor Dp_s^2/ε^2. As a result, in such a superconductor it follows from the electroneutrality condition, as in a normal metal, that $\psi=0$ and this ensures the validity of the nonstationary Ginzburg–Landau equation.

Equations (42), (43) possess an interesting feature. The static structure described by them for the resistive state at sufficiently small currents can turn out to be unstable. As already indicated,[9] the reason for this instability is that, generally speaking, the thermodynamically unstable "falling" parts of the so-called "pair-breaking" curves, i.e., the curves of the dependence of the superconducting current $j_s = \Delta^2 p_s$ on the condensate momentum p_s (cf. formula (42)), are used in forming the structure. The mechanism of the development of the instability is determined by the detailed-balance equation (43) for the charge and consists in the following. On a decreasing part of the pair-breaking curve the order parameter Δ increases with increasing superconducting current j_s. In view of the conservation of the total current (cf. (42)), this is accompanied by a decrease of the electric field. But this tendency agrees exactly with the electroneutrality equations (43), according to which, the greater is Δ the more rapidly are the electric fields attenuated, and, thus, "positive feedback" for small fluctuations of Δ is ensured.

In the case of a long channel, owing to the uniformity of the conditions in space and time, motion of the structure as a whole, with constant velocity u, can arise as a result of the instability, i.e., in (43) we can replace $\partial/\partial t \to -u\partial/\partial x$. The magnitude u of the velocity is fixed by the requirement that the processes of redistribution of charge in the moving structure be consistent, i.e., it should be determined as the nontrivial eigenvalue $u \neq 0$ of the system of equations (42), (43). In this case, in view of the symmetry $u \to -u$, $x \to -x$, $\Phi \to -\Phi$, both signs of the velocity are possible.[7)]

Irrespective of its concrete form, the motion of the structure should be manifested experimentally in small oscillations of the electric potential over a finite length of the sample. Evidently, it is by just this mechanism that the generation of monochromatic electric oscillations in the resistive state of narrow superconducting films, observed by Dmitriev, Churilov and Beskorsyĭ[26] (cf. also[14,27]), can be explained. From the general dimensionality formulas (41) there follows the law of corresponding states for the frequency of the oscillations:

$$\nu/T_c(1-T/T_c)^2 = g(j/j_c(T)), \qquad (47)$$

which is in fair agreement with the experimental results (cf.[13]). It must be emphasized that, owing to the extra temperature factor $(1 - T/T_c)^2$ in (47), the frequencies of these oscillations are found to be anomalously small ($\sim 10^8\ \mathrm{sec}^{-1}$ at $T \approx 0.99T_c$, in agreement with the observations of[14,26] compared with the frequencies of the aforementioned Josephson oscillations ($\sim 10^{10}\ \mathrm{sec}^{-1}$).[17]

For real samples of finite length (superconducting "bridges"), Eqs. (42), (43) must be supplemented by boundary conditions at the junction between the normal and superconducting metal, which would take into account the character of the penetration of the electric field into the superconductor. The effect of these conditions is particularly important on the initial part of the volt-ampere characteristic, where voltage across the sample first appears. The idea of a periodic structure of the resistive state is inapplicable in this region. However, for a sufficient degree of "supercriticality" $j - j_c > 0$, "centers" of singularity should inevitably appear inside the sample, and the positions of these, together with the magnitude of the discontinuity Φ_0, will be determined by the minimum of the entropy production. The observations by Meyer[28] of small voltage discontinuities in the volt-ampere characteristics of whiskers are interesting in this respect. Although the electric potential φ remains continuous in the mathematical sense, the "penetration" of a new "center" of singularity into the sample is accompanied by a rapid change of φ, and this should register as a voltage "discontinuity" within the limits of the experimental accuracy.

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1)This situation is not described by the so-called nonstationary Ginzburg-Landau equation,[6,7] which gives a negative result[7] in the droplet problem. This equation (cf.[8,9]) is valid only for dirty superconductors with paramagnetic impurities, which explains the agreement of the results of[7,10] (in[4] this was explained, not entirely correctly, as a consequence of the the treatment of the vortex fields in[7,10]). From the point of view of the problem of resistive states, such superconductors are closest to a normal metal, and do not, as it were, "distinguish" longitudinal and vortex fields.

2)In[9,11] this function was denoted by $\hat{\varphi}$.

3)Yu. N. Ovchinnikov has noted that, owing to the long energy-relaxation times of the electrons, this condition can turn out to be rather severe. In practice, temperatures $T \gtrsim 0.99T_c$ are found to be sufficient.[13]

4)Discontinuities of Φ in the resistive state were also introduced in the above-cited work of Fink[6] (cf. also[16]). Unfortunately, Fink uses the nonstationary Ginzburg-Landau equation, and this practically reduces his results to nought. From a physical point of view, the treatment of resistive states proposed in the recent paper of Skocpol, Beasley and Tinkham[17] is close to that expounded here (especially in its understanding of the problem of the depth of penetration of the electric field from the normal region into the superconducting region[18]). However, the specific model proposed in[17] for the phenomenological description of resistive states does not find confirmation in the microscopic theory.

5)In this, the resistive state is essentially different from the mixed state in type-II superconductors near the critical field H_{c2}, in which one can trace the gradual "development" of the vortex singularities, starting from the field H_{c2}. This difference is due to the nonequilibrium nature of the resistive state.

6)This can be seen particularly clearly in "clean" superconductors, for which it is possible to construct the Boltzmann integral for collisions of quasi-particles with impurities.[24]

7)The "forces" moving the structure have a thermodynamic origin and are not connected directly with the electromagnetic interactions. This explains the symmetry $u \to -u$. Since the instability of the initial symmetric structure can develop in both directions with equal probability, in the framework of the approximation used the final direction of motion is determined by chance causes.

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Electron-phonon interaction spectrum in copper

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The nonlinear current-voltage characteristics of microscopic copper point junctions are investigated at low temperatures (≤ 4.2°K). The electron-phonon interaction function $g(\omega)=\alpha^2(\omega)F(\omega)$ is reconstructed from the voltage dependence of the second derivative of the current-voltage characteristics. It is found that $g(\omega)$ differs appreciably from the phonon state density $F(\omega)$, owing to the strong mean square matrix element dependence of the electron-phonon interaction energy α^2. New effects are observed at low energies corresponding to large mean free paths. These include oscillations of the second derivative and a minimum of conductivity at $V=0$. These effects are apparently due to quantum size effects and to nonequilibrium occupation of the electron states near the Fermi level.

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1. INTRODUCTION

One of us[1] has proposed a new method of investigating electron-phonon interactions (EPI) in normal metals. It has turned out that at $T\approx 0$ the second derivatives of the current-voltage characteristics of point junctions are directly proportional to a function of the EPI

$$g(\omega)=\alpha^2(\omega)F(\omega), \quad (1)$$

equal to the product of the square of the matrix element of the EPI, averaged over the Fermi surface, and to the density of the phonon states.[2] This was experimentally demonstrated for point junctions with dimensions on the order of several dozens angstroms made of metals such as Pb, Sn,[1] and In,[3] for which the function $g(\omega)$ is known from tunnel measurements in the superconducting state.[4]

For many metals, however, an investigation of nonlinear effects in point junctions is the only way of determining the function $g(\omega)$ over the entire energy interval. This pertains primarily to noble metals with weak EPI, such as Cu, Ag, and Au. The present paper is devoted to an experimental investigation of the spectrum of the EPI in copper. The function $g(\omega)$ obtained by us differs

