Nonadiabatic effects in scattering from deuterons

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It is shown under very general assumptions that at small momentum transfers Δ ($\Delta \lesssim 1/R$, where R is the deuteron radius) considerable cancellation takes place between the following nonadiabatic effects: 1) effects associated with the motion of the nucleons (recoil in the elementary scattering events) cancel against the contribution from processes in which the incident particle is successively scattered from different nucleons, which are rescattered from one another in the meantime; and 2) effects due to the change in amplitude for the elementary event on moving off the energy shell cancel against the contribution from processes in which the incident particle is successively scattered by a single nucleon, which interacts with the other deuteron nucleon in the meantime. This situation obtains both at high energies, and at medium and low energies, and helps to explain the success and wide range of applicability of the widely used theories of multiple scattering based on the idea of fixed (rigidly connected) nucleons. Such cancellation also shows how dangerous it can be to attempt "to improve partially" simple theories.

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1. INTRODUCTION

Widely used theoretical schemes for calculating the scattering of high- and low-energy particles from nuclei (see, e.g., [1,2]) are based on the idea of fixed ("rigidly connected") nucleons.[3] The multiple-scattering amplitude is calculated for fixed positions of the nucleons, and only at the very end is it averaged over the coordinates of the nucleons with the aid of the squared modulus of the nuclear wave function. Moreover, nonadiabatic effects associated with recoil of the nucleons in the elementary scattering events, excitation of the nucleus to intermediate states, and possible rescatterings of the nuclear nucleons from one another are not taken into account. Further, in these schemes one also neglects the fact that, owing to the internal motion and binding energy of the nucleons, the amplitudes for the elementary scatterings of the projectile from the nuclear nucleons, which contribute to the amplitude for scattering from the nucleus, are off the energy shell.

Despite the fact that some of the nonadiabatic corrections are not small,[4,5] these schemes work well even beyond the region in which they would seem to be applicable. The following questions therefore arise: Is this accidental? How well do we understand the mechanism of elastic scattering of particles from nuclei? How much confidence can we have in nuclear-structure information derived with the aid of the Glauber approximation, for example, or using the Kisslinger-Erickson optical potential?

The first indications that this situation is not accidental, but that the nonadiabatic effects partially cancel one another, is apparently to be found in[6,7]. There, however, only high energies were discussed and the assumption that the interaction between nucleons is described by a local potential was essential. Later, numerical calculations for two centers at an energy of about 1 GeV[8] and at low energies[9] (separable potentials for the πN and NN interactions were used in[9]) again revealed considerable cancellation among the nonadiabatic effects. Some arguments in favor of such cancellation at high energies, but within the limitations of a nonrelativistic formalism, are given in[10].

In the present study we find a unified method for investigating nonadiabatic effects in scattering from the simplest nuclear target, the deuteron, which is valid for any form of the NN interaction and is the same for all energies. This method is based on the use of Feynman diagrams (see[11]), so it is relativistically covariant, as it must be for medium and high energies. It is shown in general form that for small momentum transfers Δ ($\Delta \lesssim 1/R$, where R is the deuteron radius) there is considerable cancellation of the nonadiabatic effects associated with recoil and rescattering of the nucleons from one another and with the fact that the amplitudes for the elementary processes are off the energy shell. As the momentum transfer increases, the cancellation gradually ceases, the relevant parameter being $(\Delta R)^2/8$. This cancellation is also of methodological interest, for it indicates the danger of the "partial" improvement of simple theoretical schemes based on a lucid physical idea. (It turns out that an "unimproved" theory based on the fixed-nucleon approximation is more accurate than a theory in which some but not all of the adiabatic effects are taken into account.)

For the sake of argument, let us suppose that the particle incident on the deuteron is a pion. Each of the nonadiabatic corrections to the double-scattering amplitude, e.g., that due to nucleon recoil, is of the order of $1/mR$ at medium energies[4,6] and of the order of $(\mu/m)^{1/2}$ at low energies[5] (μ and m are the pion and nucleon masses). As will be shown, the degree of cancelling, i.e., the ratio of the sum of all the corrections to each one of them, is of the order of 10–15% at high energies and of the order of $(\mu/m)^{1/2} \sim 40\%$ at low energies. Numerically, therefore, the cancelling at low energies is not especially good, and the nonadiabatic effects can make an appreciable contribution of the order of μ/m. As numerical calculations show, the degree of cancelling at medium energies is ~15–20%.

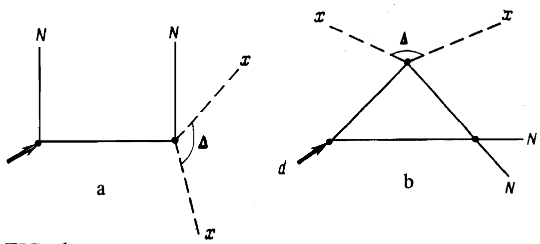


FIG. 1.

2. INTEGRAL RELATION BETWEEN THE *NN* SCATTERING AMPLITUDE AND THE DEUTERON VERTEX FUNCTION

The treatment to follow will be based to a considerable extent on an integral relation between the off-shell *NN* scattering amplitude and the deuteron vertex function, which is essentially a consequence of the Hilbert identity for the resolvent of the Lippmann–Schwinger equation (see[12] and Sec. 4 below). Now we shall derive this relation in two ways. The first way, although indirect, shows not only that no assumptions concerning the form of the nucleon–nucleon potential are required for the validity of the relation, but also that it is apparently valid even outside the limitations of a potential approach.

Let us consider the breakup of the deuteron under the action of a particle whose interaction with nucleons can be regarded as weak. Then only diagrams in which the incident particle is scattered only once need be considered, i.e., the amplitude will be given by the sum of expressions corresponding to the diagrams of Fig. 1:

$$M_{1a}+M_{1b}=f_{XN}(\Delta)\left(\varphi_d(\mathbf{p}_s)+\frac{1}{2\pi^2}\int d^3p\,\frac{\varphi_d(\mathbf{p})f_{NN}(\mathbf{p}+\Delta/2,\mathbf{p}_s-\Delta/2,E^*)}{(\mathbf{p}+\Delta/2)^2-(\mathbf{p}_s-\Delta/2)^2-i\eta}\right). \tag{1}$$

Here $\varphi_d(\mathbf{p}_s)$ is the deuteron wave function in the momentum representation, f_{XN} is the amplitude for scattering of the incident particle by a nucleon, Δ is the momentum transfer, f_{NN} is the nucleon–nucleon scattering amplitude normalized so that the square of its modulus on the energy shell is equal to the differential cross section, $\mathbf{p}_s$ is the spectator-nucleon momentum, and E^* is the energy of the relative motion of the nucleons being rescattered: $mE^*=(\mathbf{p}_s-\Delta/2)^2$.

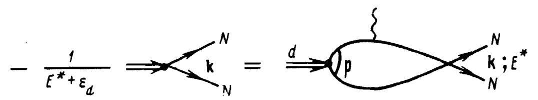
The breakup amplitude should vanish in the limit $\Delta\to 0$ on account of the orthogonality of the states of the two-nucleon continuum to the bound state (the deuteron state). From this it follows that

$$\frac{M_d(\mathbf{k})}{\varepsilon_d+E^*}=\frac{1}{2\pi^2 m}\int d^3p\,\frac{\varphi_d(\mathbf{p})f_{NN}(\mathbf{p},\mathbf{k},E^*)}{E^*-p^2/m+i\eta}. \tag{2}$$

Here we have introduced $\mathbf{k}$ in place of $\mathbf{p}_s$. The deuteron vertex function $M_d(\mathbf{k})$ is defined as follows:

$$M_d(\mathbf{k})=(k^2/m+\varepsilon_d)\varphi_d(\mathbf{k}),$$

where ε_d is the deuteron binding energy. Equation (2) can be written graphically in the form



(2′)

in which the wavy line represents a "spurion," which carries energy but not momentum. This graphical equation shows that a block containing a deuteron vertex and the *NN* scattering amplitude and forming part of a complex Feynman diagram can frequently be simply replaced (except for a factor) by a deuteron vertex. Sometimes it is convenient to replace Eq. (2) by the following equation, which is obtained by a simple transformation:

$$M_d(\mathbf{k})=\frac{1}{2\pi^2 m}\int d^3p\,M_d(\mathbf{p})\left[\frac{1}{E^*-p^2/m+i\eta}+\frac{1}{p^2/m+\varepsilon_d}\right]f_{NN}(\mathbf{p},\mathbf{k},E^*). \tag{3}$$

Equations (2) and (3) can be derived in the manner sketched above only for the case in which the amplitude f_{NN} is "half on the energy shell," i.e., $mE^*=k^2\neq p^2$. Using the Lippmann–Schwinger equation, it is easy to obtain these same equations for a fully off-shell amplitude f_{NN} when E^* and k^2 are not related to one another. In fact, taking the Lippmann–Schwinger equation in the form

$$f_{NN}(\mathbf{q},\mathbf{k},E^*)=-2\pi^2 mV(\mathbf{q},\mathbf{k})+\int d^3p\,\frac{V(\mathbf{q},\mathbf{p})f_{NN}(\mathbf{p},\mathbf{k},E^*)}{E^*-p^2/m+i\eta}, \tag{4}$$

in which $V(\mathbf{q},\mathbf{k})$ is the (in general nonlocal) potential in the momentum representation, multiplying both sides by $\varphi_d(\mathbf{q})$, integrating over $\mathbf{q}$, and using the relation

$$M_d(\mathbf{p})=-\int V(\mathbf{p},\mathbf{p}')\varphi_d(\mathbf{p}')\,d^3p', \tag{5}$$

we immediately obtain Eq. (3) for arbitrary $\mathbf{k}$ and E^*.

3. NUCLEON RECOIL AND RESCATTERING EFFECTS

Let us consider the double scattering of a pion of total energy ε and momentum $\mathbf{k}$ corresponding to the diagram of Fig. 2a, and show that the nonadiabatic corrections to the amplitude for this process at low momentum transfers Δ reduce mainly to the contribution from the diagram of Fig. 2b. The amplitude M_{2a} corresponding to the diagram of Fig. 2a, is of the form

$$M_{2a}=-\frac{1}{(2\pi)^6}\int d^3q\,d^3p\,\frac{\tilde f_1(\mathbf{q})\tilde f_2(\Delta-\mathbf{q})\varphi_d(\Delta/2-\mathbf{p})\varphi_d(\mathbf{p}-\mathbf{q})}{[\varepsilon-p^2/2m-(\mathbf{p}-\mathbf{q})^2/2m-\varepsilon_d]^2-(\mathbf{k}-\mathbf{q})^2-\mu^2+i\eta}. \tag{6}$$

Here $\tilde f_i$ is the amplitude for scattering of the incident pion by the i-th nucleon, normalized so that

$$\left.\frac{d\sigma}{d\Omega}\right|_{\rm cms}=\left(\frac{\varepsilon_1\varepsilon_2}{2\pi(\varepsilon_1+\varepsilon_2)}\right)^2|\tilde f_i|^2,$$

where ε_1 and ε_2 are the total energies of the colliding pion and nucleon in their own c.m. system. In the fixed-scatterers approximation, the denominator of Eq.

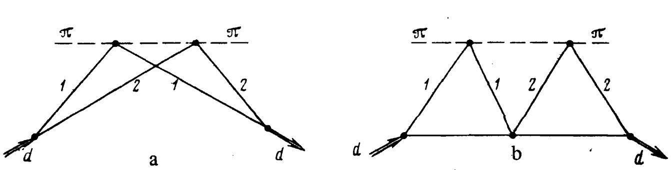


FIG. 2.

(6) would not contain the terms $p^2/2m$ and $(\mathbf{p}-\mathbf{q})^2/2m$. Thus, these terms lead to the nonadiabatic corrections for nucleon recoil.

For what follows we shall find it convenient to separate M_{2a} into two parts by an identity transformation:

$$M_{2a}=\frac{1}{2(2\pi)^6}\left\{\int d^3q\,d^3p\frac{f_1(\mathbf{q})f_2(\Delta-\mathbf{q})\varphi_d(\mathbf{p}-\mathbf{q})\varphi_d(\Delta/2-\mathbf{p})}{[(\mathbf{k}-\mathbf{q})^2+\mu^2]^{1/2}}\right.$$
$$\times\left(\frac{(\mathbf{p}-\mathbf{q}/2)^2}{m}+\frac{q^2}{4m}+\varepsilon_d-\varepsilon+[(\mathbf{k}-\mathbf{q})^2+\mu^2]^{1/2}-i\eta\right)^{-1}$$
$$-\int d^3q\,d^3p\frac{f_1(\mathbf{q})f_2(\Delta-\mathbf{q})\varphi_d(\mathbf{p}-\mathbf{q})\varphi_d(\Delta/2-\mathbf{p})}{[(\mathbf{k}-\mathbf{q})^2+\mu^2]^{1/2}}$$
$$\left.\times\left(\frac{(\mathbf{p}-\mathbf{q}/2)^2}{m}+\frac{q^2}{4m}+\varepsilon_d-\varepsilon-[(\mathbf{k}-\mathbf{q})^2+\mu^2]^{1/2}+i\eta\right)^{-1}\right\}. \quad (7)$$

We shall show that only the nonadiabatic corrections to the first term are important. We recall that the characteristic momenta of the nucleons in the deuteron are of the order of $1/R$, where R is the deuteron radius, so that the recoil energy will be of the order of $1/mR^2$. Now let us consider the high-, low-, and medium-energy cases separately.

1) $kR\gg 1$. The expression in brackets in the denominator of the first term of (7) is of the order of $|q_\varepsilon|\sim 1/R$, and for the second term, of the order of $2k$. Thus, the second term is smaller than the first by a factor of $1/kR$ and can be neglected. The nonadiabatic corrections to this term are even more negligible.

2) $kR\ll 1$. Here

$$(\varepsilon-\mu)/(\varepsilon+\mu)\sim k^2/4\mu^2\ll 1/4\mu^2R^2\ll 1$$

and again we can neglect the second term in (7).

3) $kR\sim 1$. Although the quantities $(\varepsilon-((\mathbf{k}-\mathbf{q})^2+\mu^2)^{1/2})^{-1}$ and $(\varepsilon+((\mathbf{k}-\mathbf{q})^2+\mu^2)^{1/2})^{-1}$ are of the same order, they occur squared in the ratio of the nonadiabatic corrections to the first and second terms, so that for this ratio we shall have, as might be expected,

$$\left(\frac{\varepsilon-\mu}{\varepsilon+\mu}\right)^2\leqslant\left(\frac{k^2}{k^2+4\mu^2}\right)^2\approx\frac{1}{25}.$$

This result is confirmed by numerical calculations, which give a result of the order of 10^{-2}.

Thus, we may neglect the nonadiabatic corrections except those to the first term of Eq. (7), which we shall denote by $M_{2a}^{(1)}$. Their magnitude ΔM is obtained by subtracting the same expression but without the terms $(\mathbf{p}-\mathbf{q}/2)^2/m$, ε_d, and $q^2/4m$ in denominator, from $M_{2a}^{(1)}$:

$$\Delta M=\frac{1}{2(2\pi)^6}\int d^3q\,d^3p\frac{f_1(\mathbf{q})f_2(\Delta-\mathbf{q})\varphi_d(\mathbf{p}-\mathbf{q})\varphi_d(\Delta/2-\mathbf{p})}{[(\mathbf{k}-\mathbf{q})^2+\mu^2]^{1/2}}$$
$$\times\left(\left\{\frac{(\mathbf{p}-\mathbf{q}/2)^2}{m}+\frac{q^2}{4m}+\varepsilon_d-\varepsilon+[(\mathbf{k}-\mathbf{q})^2+\mu^2]^{1/2}-i\eta\right\}^{-1}\right.$$
$$\left.+\{\varepsilon-[(\mathbf{k}-\mathbf{q})^2+\mu^2]^{1/2}+i\eta\}^{-1}\right). \quad (8)$$

This relation is valid at all energies.

Now let us consider the diagram of Fig. 2b, in which the incident particle is scattered first from one of the deuteron nucleons and then from the other, the two nucleons being rescattered from one another in the meantime. The corresponding amplitude has the form

$$M_{2b}=\frac{1}{(2\pi)^9m}\int d^3p_1\,d^3p_2\,d^3q\frac{f_1(\mathbf{q})f_2(\Delta-\mathbf{q})}{[(\mathbf{k}-\mathbf{q})^2+\mu^2]^{1/2}}$$
$$\times\frac{\varphi_d(\mathbf{p}_1)\varphi_d(\Delta/2-\mathbf{p}_2)f_{NN}(\mathbf{p}_1+\mathbf{q}/2,\mathbf{p}_2-\mathbf{q}/2,E^*)}{[(\mathbf{p}_1+\mathbf{q}/2)^2/m-E^*-i\eta][(\mathbf{p}_2-\mathbf{q}/2)^2/m-E^*-i\eta]}, \quad (9)$$

$$E^*=\varepsilon-\varepsilon_d-q^2/4m-[(\mathbf{k}-\mathbf{q})^2+\mu^2]^{1/2}. \quad (10)$$

We note that the denominator of the integrand in Eq. (9) contains two factors representing the deviation from the energy shell in the NN scattering amplitude for the initial and final states.

Let us transform (9) using Eq. (2). We can obtain the following expression with an accuracy of ~10% (see the Appendix):

$$M_{2b}=-\frac{1}{2(2\pi)^6}\int d^3q\,d^3p\frac{f_1(\mathbf{q})f_2(\Delta-\mathbf{q})\varphi_d(\Delta/2-\mathbf{p})\varphi_d(\mathbf{p}-\mathbf{q}/2)}{[(\mathbf{k}-\mathbf{q})^2+\mu^2]^{1/2}}$$
$$\times\left[\frac{1}{(\mathbf{p}-\mathbf{q}/2)^2/m-E^*-i\eta}+\frac{1}{\varepsilon_d+E^*}\right]. \quad (11)$$

On comparing ΔM from (8) with M_{2b} from (11), we easily see that these expressions are opposite in sign and very close in magnitude.

These expressions differ in two places. First, the argument of deuteron wave function is $\mathbf{p}-\mathbf{q}$ in (8) and $\mathbf{p}-\mathbf{q}/2$ in (11). For $\Delta=0$, this difference can be neglected with good accuracy on account of the additional angular integration (see the Appendix). Then the first terms in Eq. (8) and (11) cancel one another accurately. As regards the second terms, the denominator of one of them contains an "extra" term $q^2/4m$, which prevents them from cancelling one another rigorously. Let us examine the corrections due to the presence of this term in more detail.

At low energies we have

$$\varepsilon_d+E^*=\varepsilon-[(\mathbf{k}-\mathbf{q})^2+\mu^2]^{1/2}-\frac{q^2}{4m}\approx\frac{\mathbf{kq}}{2\mu}-\left(1+\frac{\mu}{2m}\right)\frac{q^2}{2\mu}.$$

Thus, the term $q^2/4m$ leads to a correction of the order of μ/m, but this is a correction to a "large" quantity which is approximately equal to the double-scattering amplitude (see the diagram of Fig. 2a). As was mentioned before, at low energies the nonadiabatic correction is a quantity of the order of $(\mu/m)^{1/2}$ times the double-scattering amplitude. From this it is evident that while the presence of the "extra" term $q^2/4m$ prevents ΔM and M_{2b} from cancelling one another exactly, they still cancel with an accuracy of at least $(\mu/m)^{1/2}$, i.e.,

$$|\Delta M+M_{2b}|/|\Delta M|\sim(\mu/m)^{1/2}. \quad (12)$$

At high and medium energies the term $q^2/4m$ must be compared with the terms in the denominator of the first term of (8) that determine the nonadiabatic correction. Their ratio is

$$\frac{q^2}{4m}\Big/\left(\frac{(\mathbf{p}-\mathbf{q}/2)^2}{m}+\frac{q^2}{4m}+\varepsilon_d\right)\sim\frac{1}{10}.$$

Thus, at these energies the terms come fairly close to cancelling one another and, in view of all that was said above, in the limit $\Delta\to 0$ we have

$$|\Delta M+M_{2b}|/|\Delta M|\leqslant 0.1-0.2. \tag{13}$$

If the momentum transfer Δ is large, some of the approximations made in the Appendix are no longer valid, since the integration over the direction of **p** becomes complicated and, on the other hand, the characteristic momentum **q** increases and becomes equal to $\Delta/2$. All this spoils the cancelling of ΔM against M_{2b}, the relevant parameter being $(\Delta R)^2/8$. The nonadiabatic effects may be expected to appear in full force at $(\Delta R)^2/8\sim 1$, i.e., at $\Delta\sim 200$ MeV/c.

4. EFFECTS ASSOCIATED WITH THE OFF-SHELL CHARACTER OF THE ELEMENTARY SCATTERING AMPLITUDE

Amplitudes for scattering of the incident particle (pion) from off-shell deuteron nucleons occur in the multiple-scattering series, and in particular, in the single-scattering term (see the diagram of Fig. 3a). Of course, to estimate the effect of the deviation of these amplitudes from the energy shell we must use some dynamical scheme for the πN interaction. In this section we shall assume that the potential approach is adequate and shall use nonrelativistic kinematics. Strictly speaking, the results will therefore be valid only for low and medium energies. Generalization to a relativistically covariant scheme (such as the quasi-potential approach), however, presents no particular difficulty.

We shall use an expression for the difference between scattering operators for the same momenta but different energies that can be derived from the Hilbert identity[12]:

$$g_u^{-1}-g_v^{-1}=v-u, \tag{14}$$

where $g_u=(H-u-i\eta)^{-1}$ is the resolvent (Green's function) for the Lippmann–Schwinger equation with the Hamiltonian $H=H_0+V$. We also introduce the free-particle Green's function $g_u^{(0)}=(H_0-u-i\eta)^{-1}$. From (14) and the relations

$$t_E=V-Vg_EV,\quad g_EV=g_0t_E \tag{15}$$

for the scattering operator, it follows that

$$t_u-t_v=t_u(g_v^{(0)}-g_u^{(0)})t_v, \tag{16}$$

or in expanded form,

$$t(\mathbf{q}_1,\mathbf{q}_2,u)-t(\mathbf{q}_1,\mathbf{q}_2,v)=\int d^3p\, t(\mathbf{q}_1,\mathbf{p},u)\times\left[\frac{1}{\mathbf{p}^2/2m-v-i\eta}-\frac{1}{\mathbf{p}^2/2m-u-i\eta}\right]t(\mathbf{p},\mathbf{q}_2,v). \tag{16a}$$

By inserting the appropriate values of u and v in (16a) we can find the difference between the on- and off-shell scattering operators.

Let us consider the triangle diagram of Fig. 3a (for

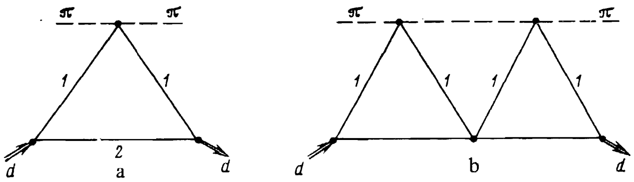


FIG. 3.

simplicity we put $\Delta=0$ at once) and denote the change in the amplitude corresponding to it resulting from the deviation of the πN scattering amplitude from the energy shell by $\Delta M'$. In other words, $\Delta M'$ is the difference between the true amplitude M_{3a} and the amplitude corresponding to the diagram of Fig. 3a, but with the πN scattering operator taken for the energy v corresponding to the interaction of the pion with a free nucleon of momentum p. Using (16a), we obtain

$$\Delta M'=-\left(1+\frac{\mu}{m}\right)\int d^3p\,d^3q\,\varphi_d^2(\mathbf{p})\,t(\mathbf{q}_1,\mathbf{q},v)\,t(\mathbf{q},\mathbf{q}_2,u)\times\left[\frac{1}{q^2/2m_0-v-i\eta}-\frac{1}{q^2/2m_0-u-i\eta}\right], \tag{17}$$

where

$$m_0=\frac{\mu m}{\mu+m},\quad u=-\frac{(\mathbf{p}+\mathbf{k})^2}{2(\mu+m)}+\left(\varepsilon_\pi-\frac{p^2}{2m}-\varepsilon_d\right),$$

$$v=-\frac{(\mathbf{p}+\mathbf{k})^2}{2(\mu+m)}+\varepsilon_\pi+\frac{p^2}{2m},\quad \varepsilon_\pi=\frac{k^2}{2\mu}.$$

For $\Delta=0$,

$$\mathbf{q}_1=\mathbf{q}_2=\frac{\mu\mathbf{p}-m\mathbf{k}}{\mu+m}\approx-\mathbf{k}.$$

The approximate equality $\mathbf{q}_1=\mathbf{q}_2\approx-\mathbf{k}$ is valid only for $k\gtrsim 1/R$; it is not valid for small k, but that does not matter since then the πN scattering amplitude can be regarded as an S-wave amplitude depending only on the energy variable.

The off-shell effect turns out to be small[1)] and, neglecting higher-order corrections, we can replace $t(\mathbf{q},\mathbf{q}_2,u)$ in the integrand in (17) by the "half-on-shell" amplitude $t(\mathbf{q},\mathbf{q}_2,v)$. Then (17) assumes a form related to that of formula (8) for the nucleon-recoil effect, and this suggests that $\Delta M'$ may be cancelled by the contribution from the diagram of Fig. 3b.

Let us compare $\Delta M'$ as given by (17) with the amplitude for the process corresponding to the diagram of Fig. 3b, in which the incident particle is scattered twice by the same deuteron nucleon while the two deuteron nucleons rescatter from each other in the meantime:

$$M_{3b}=-\int d^3p\,d^3q\,\varphi_d\left(\mathbf{p}-\frac{\mathbf{q}}{2}\right)\varphi_d\left(\mathbf{p}-\frac{\Delta}{2}\right)t(\mathbf{k},\mathbf{k}-\mathbf{q},\varepsilon_\pi)\times t(\mathbf{k}-\mathbf{q},\mathbf{k},\varepsilon_\pi)\left[\frac{1}{(\mathbf{p}-\mathbf{q}/2)^2/m-E^*-i\eta}+\frac{1}{\varepsilon_d+E^*}\right]; \tag{18}$$

E^* is given by (10). We proceed further just as in the preceding section. For $\Delta=0$ we can, with good accuracy (see the Appendix), replace $\varphi_d(\mathbf{p}+\mathbf{q}/2)$ in (18) by $\varphi_d(\mathbf{p})$ (having first made the change in variables $\mathbf{p}-\mathbf{q}/2$

$\to \mathbf{p}$); then we obtain

$$\Delta M' + M_{3b} = \int d^3p\, d^3q\, \varphi_d^2(\mathbf{p})\, t(\mathbf{k}, \mathbf{q}, \varepsilon_\pi)\, t(\mathbf{q}, \mathbf{k}, \varepsilon_\pi)$$
$$\times \left\{ \left(1 + \frac{\mu}{m}\right) \left[\frac{q^2}{2\mu}\left(1 + \frac{\mu}{m}\right) + \frac{p^2}{m} + \varepsilon_d - \frac{k^2}{2\mu}\left(1 - \frac{\mu}{m}\right) - i\eta \right]^{-1} \right.$$
$$- \left[\frac{q^2}{2\mu}\left(1 + \frac{\mu}{2m}\right) + \frac{p^2}{m} + \varepsilon_d - \frac{k^2}{2\mu}\left(1 - \frac{\mu}{2m}\right) - i\eta \right]^{-1}$$
$$- \left(1 + \frac{\mu}{m}\right) \left[\frac{q^2}{2\mu}\left(1 + \frac{\mu}{m}\right) - \frac{k^2}{2\mu}\left(1 - \frac{\mu}{m}\right) - i\eta \right]^{-1}$$
$$\left. + \left[\frac{q^2}{2\mu}\left(1 + \frac{\mu}{2m}\right) - \frac{k^2}{2\mu}\left(1 - \frac{\mu}{m}\right) - i\eta \right]^{-1} \right\}. \qquad (19)$$

Here we have replaced the πN scattering amplitudes by their on-shell values from the very beginning; this corresponds to neglecting the higher order corrections (see the argument given above).

The further study of Eq. (19) is fully analogous to the study carried through in the preceding section. An "extra" term $q^2/4m$ arises, as a result of which the accuracy to which ΔM and M_{3b} cancel one another is of the order of $(\mu/m)^{1/2}$ at low energies and of the order of 10–15% or better at medium energies. For $\Delta \neq 0$ the cancellation becomes rapidly less complete with increasing Δ, the relevant parameter being $(\Delta R)^2/8$.

We note that the variant used in[7] in which the πN scattering amplitude is treated as a function of the squared four-momentum transfer t alone, i.e., is determined by a sum of exchange diagrams, seems entirely reasonable at high energies. At high energies $t \approx -q_\perp^2$, where $q_\perp$ is the component of the three-momentum transfer in the plane perpendicular to the direction of the incident particle. In this approximation the effect due to the deviation of the πN scattering amplitude from the energy shell does not arise. But then, as can be shown,[7] the contribution from the diagram of Fig. 3b vanishes and the over-all picture remains the same as before.

5. CONCLUSION

In discussing single and double scattering of pions from deuterons we have succeeded in tracing the relations between a number of effects that seem at first glance to have nothing to do with one another. It turned out that the nonadiabatic effects due to nucleon recoil and rescattering, and the effects due to the deviation from the energy shell of the amplitude for the elementary scattering event, are of the same order of magnitude and cancel one another out to a considerable extent. At low momentum transfers ($\Delta \lesssim 1/R$) the ratio of the uncancelled part of the corrections to the double-scattering term (Fig. 2a) amounts in order of magnitude to less than $(0.1-0.2)/mR$ at high and medium energies and to μ/m at low energies. These parameters also determine the accuracy of the nuclear information that can be derived with the aid of theoretical schemes based on the fixed-nucleon approximation. The cancellation becomes less complete with increasing Δ and disappears entirely at $(\Delta R)^2/8 \sim 1$, i.e., at $\Delta > 200$ MeV/c.

We note that the presence of a high-momentum component in the deuteron wave function, which we did not take explicitly into account, cannot affect the estimates obtained for the case of scattering with comparatively low momentum transfers (the effect due to the high-momentum component does not exceed a few percent).

It should be emphasized that most of the results were essentially obtained without using dynamical models. Only in calculating the off-shell effects was it necessary to resort to some definite model, and there the potential approach was used.

The possibility of generalizing the results to the case of heavier nuclei is of great interest. It is not difficult to see that the results on single and double scattering carry over in the most direct way, since the initial relationships (the Hilbert identity and the vanishing of the nuclear-disintegration amplitude in the limit $\Delta \to 0$) are also valid for complex nuclei. It may be assumed that higher-multiplicity scatterings will also make no qualitative changes in the picture; here, however, an explicit treatment encounters considerable technical difficulties and has not yet been carried through.

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APPENDIX

We make the change of variable $\mathbf{p}_1 + \mathbf{q}/2 = \mathbf{p}$ in formula (9). Then

$$M_{2b} = \frac{1}{(2\pi)^8 m} \int d^3p\, d^3q\, d^3p_2 \frac{\tilde{f}_1(\mathbf{q})\, \tilde{f}_2(\Delta - \mathbf{q})}{[(\mathbf{k}-\mathbf{q})^2 + \mu^2]^{2}}$$
$$\times \frac{f_{NN}(\mathbf{p}, \mathbf{p}_2 - \mathbf{q}/2, E^*)\, \varphi_d(\mathbf{p} - \mathbf{q}/2)\, \varphi_d(\mathbf{p}_2 - \Delta/2)}{(\mathbf{p}^2/m - E^* - i\eta)\,[(\mathbf{p}_2 - \mathbf{q}/2)^2 - E^* - i\eta]}. \qquad (A.1)$$

Now we expand $\varphi_d(\mathbf{p} - \mathbf{q}/2)$ in a power series in $\mathbf{q}/2$:

$$\varphi_d(\mathbf{p} - \mathbf{q}/2) = \varphi_d(\mathbf{p}) - \frac{q}{2} \nabla_{\mathbf{p}} \varphi_d \Big|_{\mathbf{q}=0} + \frac{q_i q_j}{8} \frac{\partial^2 \varphi_d}{\partial p_i\, \partial p_j} \Big|_{\mathbf{q}=0} + \ldots \qquad (A.2)$$

On substituting the first term into (A.1) we obtain formula (11). The second term vanishes on integrating over the angle between $\mathbf{p}$ and $\mathbf{q}$, in view of the fact that the nucleon–nucleon scattering amplitude is an S-wave amplitude at low energies. Bearing in mind that $\Delta_p \varphi_d \sim R^2 \varphi_d$, it will be seen that the third term in (A.2) leads to a correction of the order of $(qR)^2/8$. Since $qR \lesssim 1$, this correction amounts to about 10%.

For $\Delta = 0$, a similar transformation can be carried through with the same accuracy in the sum of expressions (8) and (11), passing from $\varphi_d(\mathbf{p} - \mathbf{q})$ to $\varphi_d(\mathbf{p} - \mathbf{q}/2)$. The condition $\Delta = 0$ is important, since otherwise the angular integration becomes complicated and the term analogous to the second term of (A.2) does not vanish.

Experience in working with a local NN potential[7] suggests that the combined accuracy of the two approximations described above will actually be much better than 10%, i.e., that the errors made in the two stages of the calculation will cancel each other out to a considerable extent. However, a rigorous proof of this is lacking.

[1] At low energies, the corresponding parameter, which can be obtained from (16) with $k=0$, is $\Delta f_{\pi N}/f_{\pi N} \sim (\mu/m)^{1/2} a_{\pi N}/R$.

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A theory of direct four-fermion interactions

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A solution has been obtained for the "parquet" equations for the vertex $\Gamma(p_1, p_2; p_3, p_4)$ of the direct four-fermion interaction in a space of dimension $d=2+\epsilon$. For the existence of such a solution it is necessary that the interaction have a symmetry of the type of $SU(2)$-invariance, and that the coupling constant G be positive. For high energies $Gp^2 \gg 1$ this solution is scale-invariant and corresponds to a stable fixed point of the Gell-Mann–Low equations. It is shown that a similar solution approximately satisfies the system of equations in four-dimensional space $d=4$, where all the integrals in the equations turn out to be convergent. With the help of this solution the contribution of the so-called "non-parquet" terms is estimated, terms which have not been taken into account in the equations. It is shown that these terms are numerically small. The solution can be used as a zeroth approximation of an iterative method of solution of the exact equations.

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1. INTRODUCTION

The direct point interaction of fermions

$$H=G_0(\bar{\nu}O_\alpha e)(\bar{e}O_\alpha \nu) \tag{1}$$

for $O_\alpha=\gamma_\alpha(1+\gamma_5)/2$ (the $V-A$ variant) describes well in the Born approximation all weak interaction processes at low energies. However, since the cross section for this interaction increases with energy, $\sigma \sim G^2E^2$, and only the S-wave participates in scattering, at an energy $E\sim 10^3$ GeV the growth of the cross sections runs into contradiction with unitarity, and it becomes necessary to take into account terms of higher order in the coupling constant.[1]

In order to determine such higher-order contributions one cannot make use of perturbation theory, since the interaction (1) is not renormalizable in the usual sense. For this reason the renormalizable Weinberg–Salam scheme[2] for the weak interactions has acquired popularity in recent years. Unfortunately, this scheme requires the introduction of a series of new particles and is not quite simple. Also, the strong interaction scheme which is based on the intermediate nonabelian gauge vector fields is not simple. All other types of renormalizable interactions (e.g., the Yukawa $\pi N\bar{N}$ interaction, or meson self-interactions of the type $\lambda\varphi^4$, as well as the electromagnetic $\gamma\bar{e}e$ interaction) lead to the well known problem of "vanishing charge,"[3] i.e., the vanishing of the physical coupling constant in these theories in the limit of a point interaction, i.e., in the local limit. This manifests itself also in the fact that the effective coupling constant $g^2(p^2)$, which characterizes the interaction at a momentum p^2, increases with p^2, in distinction from the asymptotically free gauge theories, where it decreases. Theories are possible where $g^2(p^2)\to g_1^2=\text{const}$ for $p^2\to\infty$, the so-called theories with a "fixed point." This is the kind of possibility that will be explored for the four-fermion interaction in this paper.

The weak interaction has been investigated in a number of papers by means of dispersion relations. This