Spin relaxation of electrons due to scattering by holes

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We evaluate the electron spin relaxation time connected with the exchange and annihilation interaction between electrons and free and bound holes. We show that this mechanism can play a determining role under conditions of optical orientation of electrons in semiconductors. We construct a theory of the electron spin relaxation when holes are strongly scattered by impurities and taking into account the fast relaxation of the hole spin. We show that the rate of electron spin relaxation due to holes is proportional to the time of interaction with the holes, i.e., the time during which the distance between them is less than the electron wavelength. Under conditions when this time equals the time of diffusion of holes through the interaction region, strong scattering of holes leads to a decrease in the electron spin relaxation time. On the other hand, under conditions when the hole spin relaxation time becomes less than the interaction time, strong hole spin relaxation leads to an increase in the electron spin relaxation time due to an efficient averaging of the hole spin.

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1. INTRODUCTION

In experiments on the optical orientation of electrons in a number of crystals of the A2B5 and A3B5 groups one normally uses samples with a high density of equilibrium holes, thanks to which it is possible to observe a rather strong luminescence (see the surveys[1, 2]). Under those conditions the main mechanism for the electron spin relaxation may be their scattering by holes involving spin flip. The way the electron spin relaxation time depends on the hole density, in particular, the appreciable decrease of the electron spin relaxation time in p-type samples as compared to n-type samples[3] which has been observed in a number of papers indicates the important role played by this mechanism.

We show below that the scattering of electrons by holes involving spin flip may be caused both by exchange and by annihilation interactions which lead also to a longitudinal-transverse splitting of the exciton levels. Both these interactions have a contact character[4].

We shall consider the scattering of electrons by holes involving spin flip in the Born approximation and we shall assume that the wavefunction of the electron-hole pair may differ from a plane wave due to the Coulomb interaction.

The Hamiltonian for the interaction between an electron and a hole which involves spin-flip can be written in the form

\[ \mathcal{H} = \alpha \delta D \delta (r) \delta k' \]

where \( \mathbf{r} = \mathbf{r}_e - \mathbf{r}_h \) is the difference in the coordinates of the electron and the hole, \( \alpha = \alpha_e / e m \) is the exciton Bohr radius (everywhere in this paper \( h = 1 \)), \( m_e = m_h = m_0 \), \( \kappa_0 \) is the static dielectric constant, \( \mathbf{k} = \mathbf{k} + \mathbf{p} \) is the total momentum of the electron-hole pair, \( \mathbf{k} \) the electron momentum, and \( \mathbf{p} \) the hole momentum. The operator \( \delta \) depends on the electron and hole spin operators and for the different cases has the form:

Exchange interaction

a) in the case of simple bands with axial symmetry \[ D = \Delta \sigma_x \]

where \( \sigma_x \) and \( \sigma_y \) are the electron and hole Pauli matrices, \( \sigma_x = 1/2 (\mathbf{1} + x y) \). For A2B5 crystals for holes in the \( \Gamma_5 \) band \( \Delta = 0 \) and for cubic symmetry \( \Delta = \Delta = \Delta \).

In the case of the \( \Gamma_5 \) band for cubic crystals

\[ D = \Delta (\mathbf{l}) \]

where \( \mathbf{l} \) is the hole angular momentum operator \( \mathbf{l} = 3/2 \).

(1)

(2)

We note that according to (1) the constants \( \Delta_{ij}, \Delta_{ij} \), and \( \Delta \) determine the exchange splitting of the exciton ground state.

Annihilation interaction

According to \( m \) in that case

\[ \gamma_{e} \delta D_{e}(k) = \frac{4e\kappa}{m_0 \omega_e} \frac{(P_e(k)P_e(k))}{K^2} \]

where the matrix elements of the momentum operator \( P \) are taken between the initial state (vacuum) and a state with one electron-hole pair, \( \omega_e \) is the dielectric constant at the exciton excitation frequency, \( E_e \) the width of the forbidden band, and \( m_0 \) is the mass of a free electron.

We note that the macroscopic long-range interaction (4) leads to a spin flip of electrons and holes only thanks to the spin-orbit splitting of the valence band (or the conduction band) as only in that case do the momentum matrix elements depend on the spin states of the pair. It is just this fact which is connected with the possibility for optical orientation of the electrons. Together with the considered spin relaxation mechanisms connected with the exchange of spin between an electron and a hole, there is also another mechanism for electron spin relaxation which is connected with the spin-orbit interaction in the Coulomb field of a hole.\[6\] This mechanism equally also determines the spin relaxation involving scattering by impurities[7] where the contribution from the holes connected with the spin-orbit mechanism is less than the contribution connected with the scattering by charged impurities. Estimates given in Sec. 4 show that in uncompensated samples the contribution from the exchange mechanism considered by us exceeds appreciably the contribution from the spin-orbit mechanism connected with the scattering by impurities.

2. EVALUATION OF THE ELECTRON SPIN RELAXATION TIME

The electron spin relaxation drift time \( \tau_{sr}(k) \), which we denote by \( 2 \tau_{sr}(k) \), is given in the Born approximation by the formula

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The effective mass, which is usually the case in semiconductors, is 
where \( \varepsilon_{\text{f}} \) and \( \varepsilon_{\text{p}} \) are the electron and momentum, \( \varepsilon_{\text{p}} \) and \( \varepsilon_{\text{p}} \) the hole energy and momentum, \( f_{\text{p}} = \varepsilon_{\text{p}} \) the hole distribution function, and \( \varphi(r) \) the wave function of the relative motion of the electron and the hole.

We shall in what follows consider the case when the hole effective mass is much larger than the electron effective mass, which is usually the case in semiconductors in which optical orientation is studied. It is then clear that the momentum transfer and the electron momentum are small for \( \varepsilon_{\text{e}} \leq \varepsilon_{\text{p}} \) compared to the hole momentum and \( K \approx \varepsilon_{\text{p}} \). This means that \( D(p, q + q) \) depends only on the direction of the vector \( p \) both for the annihilation interaction and for the exchange interaction in the hole band.

However, in this case in the case of Fermi hole degeneracy the hole velocity \( v_{\text{h}} \) may be either less or larger than the electron velocity \( v_{\text{e}} \).

We consider in what follows two limiting cases:

a) \( v_{\text{h}} \ll v_{\text{e}} \) and b) \( v_{\text{h}} \gg v_{\text{e}} \). In those cases the velocity of the relative motion is determined either by the electron velocity (a) or by the hole velocity (b). The quantity \( \langle \varphi(0) \rangle^2 \) then depends only on the absolute magnitude of the momentum or the relative motion.

It is well known that the value of \( \langle \varphi(0) \rangle^2 \) for the Coulomb potential is determined by the Sommerfeld factor. For a screened potential this quantity depends strongly on the screening radius \( l_{\text{d}} \), when \( l_{\text{d}} \ll a \) we have \( \langle \varphi(0) \rangle^2 = 1 \). We consider various limiting cases.

1) If the holes are not degenerate, using the fact that the energy transfer is small compared to \( \varepsilon_{\text{p}} \) when \( m_{\text{e}} \ll m_{\text{h}} \), we find from (5)

\[
\frac{1}{2\Delta_{\text{e}}} = \frac{1}{\tau_{\text{e}}} \frac{v_{\text{e}}}{v_{\text{h}}} |\varphi(0)|^4,
\]

where

\[
\frac{1}{\tau_{\text{e}}} = \frac{n_{\text{e}}}{4N_{\text{e}}} \frac{D_{\text{e}}^2}{E_{\text{e}}} = \frac{1}{2} \left( \frac{N_{\text{e}}^2}{E_{\text{e}}} \right) \frac{D_{\text{e}}^2}{E_{\text{e}}},
\]

(7)

Here

\[ E_{\text{e}} = 1/2m_{\text{e}}v_{\text{e}}, \quad E_{\text{h}} = 1/2m_{\text{h}}v_{\text{h}}, \quad v_{\text{e}} = 1/m_{\text{e}}, \]

(8)

N is the hole density.

2) If the holes are strongly degenerate so that the Fermi energy \( \varepsilon_{\text{F}} \gg \varepsilon_{\text{p}} \), it is convenient to introduce in (5) the energy transfer as the variable:

\[
\omega = \varepsilon_{\text{e}} - \varepsilon_{\text{e}} - \varepsilon_{\text{h}} - \varepsilon_{\text{h}} = q^2 + q^2 E_2,\]

where \( q^2 = \cos \theta \). After that, replacing everywhere except in the \( \delta \) function \( p \) by \( p \) and bearing in mind that when \( \varepsilon_{\text{F}} > T \)

\[
\int d\varepsilon_{\text{h}} f_{\text{h}}(\varepsilon_{\text{h}})[1 - f_{\text{h}}(\varepsilon_{\text{h}} + \omega)] = \omega/(1 - e^{-\omega/T}),
\]

we get

\[
\frac{1}{2\Delta_{\text{e}}} = \sum_{n=1}^{N} \int d\omega \frac{\omega}{1 - e^{-\omega/T}} \int d\omega V_{\text{n}}(p) \frac{1}{4n},
\]

706 Sov. Phys.-JETP, Vol. 42, No. 4

For a strongly degenerate hole gas we have then \( \langle \varphi(0) \rangle^2 = 1 \). If the electron velocity is much larger than the hole velocity, i.e., \( \varepsilon_{\text{pe}}/\varepsilon_{\text{pm}} \ll 1 \), we find that for \( m_{\text{e}} \ll m_{\text{h}} T^2 \ll 1 \) the energy transfer \( \omega \ll T \), \( \varepsilon_{\text{p}} \).

We then get (see (60))

\[
\frac{1}{2\Delta_{\text{e}}} = \frac{1}{\tau_{\text{e}}} \frac{v_{\text{e}}}{v_{\text{h}}} \frac{1}{T},
\]

(10)

If the hole velocity is much larger than the electron velocity, i.e., \( \varepsilon_{\text{pe}}/\varepsilon_{\text{pm}} \gg 1 \), we have \( \omega \approx T/v_{\text{F}} \) and \( v_{\text{h}} \ll 1 \). If \( D_{\text{h}} \) depends on the direction of \( p \), \( 1/2\tau_{\text{se}} \) will in this case depend on the direction of \( k \) relative to the electron spin direction or the crystal axis direction. The quantity \( (1/2\tau_{\text{se}}) \) which is averaged over all directions of the electron motion is as before determined by \( D_{\text{h}}^2 \) from (7). In the case of fast holes we get for \( 1/2\tau_{\text{se}} \) (see (61))

\[
\langle \frac{1}{2\tau_{\text{e}}} \rangle = \frac{1}{v_{\text{e}} v_{\text{h}}} (T)^{1/2} \frac{1}{m_{\text{e}}}. \frac{1}{T},
\]

(11)

It follows from (11) that the electron spin relaxation time is determined in that case by the larger of the quantities \( T \) or \( \varepsilon_{\text{p}} \): we have \( 1/2\tau_{\text{se}} \approx \langle \tau^2 \rangle / \varepsilon^2 \) when \( \varepsilon_{\text{e}} < T \) and \( 1/2\tau_{\text{se}} \approx \langle \varepsilon \rangle / \langle \varepsilon \rangle^2 \) when \( \varepsilon_{\text{p}} \gg T \).

Electron spin relaxation when the holes are strongly scattered

The formulae given above are valid only under the conditions where the electron-hole interaction time \( \tau_{\text{int}} \) is much smaller than the hole scattering time \( \tau \) and the hole spin flip time \( \tau_{\text{sh}} \). The interaction time \( \tau_{\text{int}} = \lambda/\nu \), where \( \lambda \) is the size of the interaction region, equal to the inverse of the momentum transfer (for a short-range potential) while \( \nu \) is the relative velocity. When \( m_{\text{e}} \ll m_{\text{h}} \) and \( \varepsilon_{\text{e}} \ll \varepsilon_{\text{p}} \) we have \( \lambda \sim 1/k \) and when \( \nu_{\text{e}} > \nu_{\text{h}} \) when the interaction time is determined by the electron velocity and \( \tau_{\text{int}} = \tau_{\text{sh}} \). The condition \( \tau_{\text{int}} \ll \tau \) reduces to \( \tau_{\text{sh}} > 1 \). When \( \nu_{\text{e}} < \nu_{\text{h}} \) the time \( \tau_{\text{int}} \) is determined by the hole velocity and equal to \( \tau_{\text{int}} = 1/kv_{\text{F}} \) and the condition \( \tau_{\text{int}} \ll \tau \) reduces to \( k_{\text{F}} > 1 \) where \( k_{\text{F}} \) is the hole mean free path. The range of values of \( \varepsilon_{\text{F}} \) and \( m_{\text{e}} \) the condition \( \tau_{\text{int}} > \tau_{\text{sh}} \) is satisfied, i.e., \( \varepsilon_{\text{F}} < 1 \) or \( k_{\text{F}} > 1 \) and where Eqs. (10) and (11) are valid are indicated in Fig. 1 by the letters a and b, respectively. The solid line corresponding to the condition \( \nu_{\text{e}} = \nu_{\text{h}} \), separates the regions of slow and fast holes. When \( \tau_{\text{int}} \gg \tau_{\text{sh}} \), i.e., when \( \varepsilon_{\text{F}} > 1 \) and \( k_{\text{F}} > 1 \), a hole manages to be scattered repeatedly during the interaction time and its motion in the region \( \lambda \sim 1/k \) is thus diffusive in nature and it traverses this region in a time \( \tau_{\text{int}} = 1/Dk_{\text{F}} \), where \( D = 1/3v_{\text{F}} \tau \) is the hole diffusion coefficient. The boundary between the slow and fast hole regions in the quantum region, i.e., when \( \varepsilon_{\text{F}} \) \( k_{\text{F}} \) \( 1 \) is thus a dot-dash line.

When \( \tau_{\text{int}} \ll \tau_{\text{sh}} \) the hole spin retains its direction during the interaction time. When \( \tau_{\text{int}} \gg \tau_{\text{sh}} \), i.e., when the conditions \( \varepsilon_{\text{F}} \ll 1 \) and \( Dk_{\text{F}} \ll 1/\tau_{\text{sh}} \) are
simultaneously satisfied, the hole spin manages to change its direction during the interaction many times and as a result the probability that an electron is scattered with spin flip is decreased.

In Sec. 5 below we evaluate the electron spin relaxation time when the holes are strongly scattered by impurities, when the interaction time $t_{\text{int}}$ is equal to $T_{\text{sh}}$ and the effective interaction time is given by the formula (see (63))

$$T_{\text{int}} = \frac{1}{g} \sum_{m} |D_{m,n}|^2 \Delta_{m,n}^2,$$

where $g$ is the degree of degeneracy of the acceptor center ground state.

### 3. EVALUATION OF THE EFFECTIVE SPIN-SPIN INTERACTION CONSTANT

We now evaluate $D_{\text{exch}}$ for different cases, expressing it in terms of the exchange or the longitudinal-transverse annihilation splitting of the excitons.

In hexagonal crystals for a $\Gamma_7 \times \Gamma_9$ electron-hole pair it follows from (2) that

$$D_{\text{exch}} = 2\Delta_{\text{exch}} = \frac{1}{6} \Delta_{\text{exch}}^2,$$  \hspace{1cm} (15)$$

where $\Delta_{\text{exch}}$ is the exchange splitting between the $\Gamma_7$ and $\Gamma_9$ exciton states.

For a $\Gamma_7 \times \Gamma_9$ pair $\Delta_{\text{exch}} = 0$ and $D_{\text{exch}} = 0$, i.e., the exchange mechanism does not lead to scattering with spin flip.

Equation (15) is valid also for nondegenerate bands in a cubic crystal for which $\Delta_{\text{exch}}$ is the singlet-triplet splitting.

In a degenerate $\Gamma_7$ valence band of a cubic crystal the hole wave functions transform in the spherical approximation, i.e., when $D = \sqrt{3} B$, like the spherical harmonics $Y_{m}^{l}$ with $m = \pm 1/2$ or $m = \pm 3/2$ with the quantization axis parallel to p. (When the constants $A$, $B$, and $D$ have the same sign, the states with $m = \pm 3/2$ correspond to the heavy holes.)

In the spherical approximation the averaging over the direction of p is equivalent to averaging over the direction of J. Bearing in mind that the $J_l$ under rotations transform as the components of a vector we get for holes with $m = \pm 3/2$ and $m = \pm 1/2$.

$$D_{\text{exch}} = \frac{3}{32} \Delta_{\text{exch}}^2,$$  \hspace{1cm} (16)$$

where $\Delta_{\text{exch}}$ is the exchange splitting between the exciton states with $J = 1$ and $J = 2$.

We now consider the annihilation interaction. For nondegenerate bands the matrix elements $P_{mn}$ are independent of the direction of p and after averaging we get: for a $\Gamma_7 \times \Gamma_9$ in hexagonal crystals

$$D_{\text{ann}} = \frac{1}{45} \Delta E_1^2,$$  \hspace{1cm} (17)$$

for a $\Gamma_7 \times \Gamma_7$ electron-hole pair

$$D_{\text{ann}} = \frac{1}{45} \Delta E_1^2 + \frac{1}{30} \Delta E_2 \Delta E_1 + \frac{1}{46} \Delta E_3,$$  \hspace{1cm} (18)$$

Here $\Delta E_1$, $\Delta E_2$, and $\Delta E_3$ are the longitudinal-transverse splitting of the corresponding excitons with $K = z$ and $K = \pm z$, where $z$ is a principal axis of the crystal. $\Delta E_3 = 0$ for a $\Gamma_7 \times \Gamma_7$ exciton.) In the quasi-cubic approximation
when the crystal splitting is much smaller than the spin-orbit one, we have for the upper Γ8 band (the one closest to the Γ6 band) $\Delta E^{(1)} = 4\Delta E^{(0)} = 4/3\Delta E^{(0)}$, where $\Delta E^{(0)}$ is the corresponding splitting for the $\Gamma_7 \times \Gamma_8$ exciton, and it is clear from (17) and (18) that in that case $D_{S\text{ann}}$ is the same for the neighboring $\Gamma_7$ and $\Gamma_8$ bands. For the $\Gamma_7$ band which due to the spin-orbit interaction is split off we have in this approximation $\Delta E_I = 2/3\Delta E^{(0)}$ and according to (18)

$$D_{S\text{ann}} = \frac{1}{4} \Delta E_{\text{ann}}^{(0)}.$$  

where $\Delta E_{\text{ann}}$ is the difference in energy of the longitudinal and the transverse excitons. Equation (19) is also valid for nondegenerate bands in cubic crystals.

The calculation shows that for degenerate bands in a cubic crystal, in contrast to the exchange interaction, the annihilation interaction is appreciably different for heavy and light holes with $m = \pm 3/2$ and $m = \pm 1/2$.

$$D_{S\text{ann}} = \frac{1}{4} \Delta E_{\text{ann}}^{(0)} R.$$  

Here $\Delta E_{\text{ann}}^{(0)}$ is the difference in energy of the longitudinal and the transverse $\Gamma_7 \times \Gamma_8$ excitons. For light holes ($m = \pm 1/2$) in the spherical approximation $R = 1$, and for heavy holes $R = 0$ in the spherical approximation.

If we use the exact hole functions to evaluate $D_{S\text{ann}}$ we can show that for heavy holes

$$R < 10^{-4} (D/2B-1).$$  

The annihilation interaction between electrons and heavy holes contributes therefore practically nothing to the scattering of electrons involving spin flip.

For holes which are bound to an acceptor the constants $D_{\text{ex}}^{(0)}$ are the same in the case of a nondegenerate band for the exchange interaction as for free holes.

For a degenerate valence band when the acceptor wave function is given by a single smooth $s$-type function,

$$D_{\text{ex}}^{(0)} = \frac{1}{4} \Delta \Gamma \sum_{\text{spin}} |(l_{\text{ex}})|^2 = \frac{5}{3} \Delta^{s} - \frac{5}{3} \Delta^{e} \Delta_{\text{ex}}.$$  

In the case of annihilation radiation and when the acceptor wave function is spherically symmetric we get for $D_{S\text{ann}}$

$$D_{S\text{ann}} = \left(\frac{5}{3} m^{2} E_{B}^{(s)} + a_{B}^{2} \right) \frac{1}{4} \sum_{\text{spin}} \langle P_{\text{ex}} | P_{\text{ann}} \rangle |^{2}.$$  

We note that the contribution of the annihilation interaction to the exchange constants of a bound exciton is determined by the same matrix (7) which determines in (22) the contribution from this interaction to $D_{\text{ex}}^{(0)}$.

As according to (15) $D_{S} \sim \Delta \Gamma$ and as it follows from general considerations for a $\Gamma_7 \times \Gamma_8$ exciton that $\Delta \Gamma = 0$, in scattering by bound holes neither the exchange nor the annihilation interaction contribute to $D_{S\text{ann}}$, i.e., the electron spin does not change in elastic scattering of an electron by bound $\Gamma_8$ holes.

For a pair from the $\Gamma_7 \times \Gamma_8$ bands we have from (22) in a hexagonal crystal

$$D_{S\text{ann}} = \frac{5}{12} \Delta E_{\text{ann}}^{(0)},$$  

for nondegenerate bands in a cubic crystal

$$D_{S\text{ann}} = \frac{5}{12} \Delta E_{\text{ann}}^{(0)},$$  

and for a degenerate $\Gamma_8$ valence band

$$D_{S\text{ann}} = \frac{5}{24} \Delta E_{\text{ann}}^{(0)}.$$  

We note that in those cases where the annihilation and exchange interactions give approximately the same contribution to $D_{S}^{(0)}$ we must take into account the cross terms, i.e., add the transition matrix elements and afterwards average the whole thing. For instance, for the scattering by bound holes for a pair from the $\Gamma_7 \times \Gamma_8$ bands we have thus in agreement with (34a) from [9]

$$D_{\text{ann}} = \frac{1}{4} (\Delta_{\text{exch}} + \Delta E_{\text{ann}})^{(0)}.$$  

4. DISCUSSION OF THE RESULTS AND NUMERICAL CALCULATIONS

It is clear from the formulae given above that owing to changes in the nature of the scattering the hole density and temperature dependence of the spin relaxation rate depends on the degree of degeneracy of the holes and their spin relaxation time and turns out to be different for free and for bound holes.

It is well known that at not too high acceptor densities, when the screening radius $l_{d} = (\pi a_{B}^{2}/4\pi)^{1/2}$ is larger than the acceptor Bohr radius $a_{B}^{2} E_{B}$, the holes get bound to impurities when the temperature is lowered and at high temperatures the hole gas is nondegenerate. In that case, taking into account scattering by free and by bound holes we get from (6) and (14)

$$D_{\text{ann}} = \frac{1}{4 \tau_{a}} \left[ 1 + \frac{N}{N_{a}} |\langle \psi(0)|i\rangle - 1| \right],$$  

where $\kappa = \sqrt{E_{B}/E_{D}}$ and $N/N_{a}$ is the degree of ionization of the acceptors. Here $\tau_{a}$ is determined according to (14) by the acceptor density. At low temperatures $1/\tau_{a} \propto \tau_{0}^{1/2}$, but already at low degrees of ionization of the acceptors it increases faster as at low temperatures

$$|\psi(0)|i = \frac{2a_{B}}{\sqrt{1-e^{-2a_{B}^{2}}}} \gg 1.$$  

At an appreciable ionization of the impurities, the decrease in $|\psi(0)|i$ with increasing temperature compensates for the increase in $\kappa$. It is clear from (27) that $1/\tau_{a} \propto N_{a}$ both for a low and for a high degree of ionization of the impurities, but in the intermediate region the increase with density is slower.

At high impurity densities, when $l_{d} \leq a_{B}$, the impurities are always ionized and for not too high temperatures the holes are degenerate.

For a given hole density and temperature the applicability of the obtained formulae for $1/\tau_{a}$ is, as we noted earlier, determined by the point with the coordinate $v^{2} k^{2}$ in Fig. 1.

For a fixed density this point moves with increasing temperature ($v^{2} k^{2} \sim T$) along a straight line through the origin with slope $\epsilon = v^{2} k^{2}/h^{2} = 1/6m_{e}D$. If $\epsilon < 1$, the point intersects successively with increasing temperature the regions e, d, c, and a where Eqs. (13), (12), (11), and (10) apply, respectively. The temperature dependence of $1/\tau_{a}= T^{1/2} (e), T^{3/2} (d), T^{2} (c)$, and $T^{1/2} (a)$.

If, as in the $\Gamma_{8}$ band, $\tau_{a} = \tau$, there is practically no region d. If $\epsilon > 1$, the point goes directly from the region e to the regions b and a, in which Eq. (13) is valid.

For fixed temperature and $\epsilon = \text{constant}$ this point moves along a horizontal line. When $\epsilon T > 1$ it moves
from the region a where \(1/\tau_{s\text{Se}} \propto N_3^{1/3} \) into the region c, where \(1/\tau_{s\text{Se}} \) is independent of \(N\). When \(\tau_T < 1\) the point moves from the region b or e, where \(1/\tau_{s\text{Se}} \propto N_3^{1/3} \), into the region \(d\) where according to (12) \(1/\tau_{s\text{Se}} \) decreases with increasing \(N\), and after that enters the region c, i.e., in that case the \(N\)-dependence of \(1/\tau_{s\text{Se}} \) may be nonmonotonic.

As an example we have evaluated the value of \(1/\tau_{s\text{Se}} \) for GaAs which is often used in optical orientation experiments. The basic parameters of this crystal are given in the Table. The critical density corresponding to the condition \(I = \alpha^2 A \propto 1 \times 10^{14} \text{ cm}^{-2} \) for GaAs. We made the calculations for two densities, \(N_1 = 10^{17} \text{ cm}^{-3} \) and \(N_2 = 4 \times 10^{19} \text{ cm}^{-3} \).

For the density \(N_1 = 10^{17} \text{ cm}^{-3} \) the temperature dependence of \(1/\tau_{s\text{Se}} \) was calculated from (27). For \(N = N_2 = 4 \times 10^{19} \text{ cm}^{-3} \) \(1/\tau_{s\text{Se}} \) was given by Eq. (13) in the temperature range 0 to 20 K, by Eq. (11) from 20 to \(100 \text{ K}\), and by Eq. (10) for \(100 \text{ K} \leq T \leq 350 \text{ K}\).

In the calculations we took the hole spin relaxation time in the degenerate band to be the same as the transport time \(\tau\) which for \(N = N_1\) equals \(\tau = 2.4 (E_F)^{1/2}\). The condition \(E_F > 1\) is satisfied when \(E_F > 10 \text{ K}\). In Fig. 2 we have plotted the ratio \(\tau/\tau_{s\text{Se}}\), where \(\tau_{s\text{Se}}\) is the value of \(\tau\) for \(N = 10^{17} \text{ cm}^{-3}\). According to (7) and (16) for GaAs

\[
\frac{1}{\tau_{s\text{Se}}} = 8 \times 10^{-6} (\Delta_{\text{exch}}^{\text{GaAs}}/10^{14} \text{ eV})^4,
\]

where \(\Delta_{\text{exch}}\) is the exchange splitting of the exciton, i.e., the difference in energy of the states with \(J = 1\) and \(J = 2\). (As we mentioned earlier, the annihilation interaction does not contribute significantly in this case to \(\Delta_{\text{exch}}\).)

There are no reliable data on the magnitude of \(\Delta_{\text{exch}}\) in GaAs. If we take in accordance with \(\Delta_{\text{exch}}^{\text{GaAs}} = 5 \times 10^{-3} \text{ eV}\), we get \(1/\tau_{s\text{Se}} > 3 \times 10^8 \text{ s}^{-1}\). According to Fig. 2 the quantity \(1/\tau_{s\text{Se}}\) then exceeds \(3 \times 10^{10} \text{ s}^{-1}\) for \(N = 4 \times 10^{19} \text{ cm}^{-3}\), while for \(N = 10^{17} \text{ cm}^{-3}\) we have \(1/\tau_{s\text{Se}} = 3 \times 10^8 \text{ s}^{-1}\), i.e., in both cases the mechanism considered leads to short electron spin relaxation times which may be comparable with or even less than the life time. Estimates show that the spin relaxation time \(\tau_{s\text{Se}}\), which is connected with the scattering by impurities and caused by the spin-orbit interaction, turns out to be screened Coulomb potential to be in GaAs about two orders of magnitude larger than the spin relaxation time \(\tau_{s\text{Se}}\) connected with the exchange interaction and becomes less than \(\tau_{s\text{Se}}^{\text{exch}}\) for \(\Delta_{\text{exch}}^{\text{GaAs}} < 3 \times 10^{15} \text{ eV}\). In semiconductors with a nondegenerate valence band the spin relaxation time connected with the annihilation interaction is always less than \(\tau_{s\text{Se}}^{\text{exch}}\) when \(m_e < m_h\).

5. EVALUATION OF THE ELECTRON SPIN RELAXATION TIME WHEN HOLES ARE STRONGLY SCATTERED BY IMPURITIES

In this section we derive a kinetic equation to describe the electron spin relaxation due to holes and give a derivation of Eqs. (12) and (13). We shall assume that \(\tau_F \gg 1\) and \(\tau_{e\text{ck}} \gg 1\), where \(\tau_{e\text{ck}}\) is the electron relaxation time due to impurities. As \(m_e < m_h\), the quantity \(\tau_T\) can be small here.

We use Keldysh's technique. We consider the case of simple spherical bands with a hole–impurity interaction Hamiltonian of the form

\[
\mathcal{H}_{\text{imp}} = i \mathcal{U}(t) + i \mathcal{V}(t) (\mathbf{S}_i),
\]

where \(\mathbf{S}_i\) is the impurity spin operator and \(\mathcal{U}(t)\) the hole spin operator. We shall assume the holes to be strongly degenerate.

We use (9) to introduce the electron and hole Green function matrices \(G_{\alpha\beta}^{<}(\mathbf{k}, x_j)\) and \(G^{>}(\mathbf{k}, x_j, x_k)\). We shall assume that only the electrons are polarized as far as the spin is concerned and that \(G_{\beta\beta}^{>}(\mathbf{k}, x_j) = 0\), if \(\beta \neq \beta'\) while \(G_{\alpha\alpha}^{<} = \delta_{\alpha\alpha} G_{\alpha\alpha}^{>}\). According to (9) each of the components \(G_{\beta\beta}^{>}(\mathbf{k}, x_j)\) is a matrix

\[
G_{\beta\beta}^{>}(\mathbf{k}, x_j) = \left( G_{\alpha\beta}^{>}(x_j, z_l) G_{\beta\alpha}^{>}(x_j, z_l) \right) .
\]

The matrix \(G_{\beta\beta}^{>}(\mathbf{k}, x_j, x_k)\) has a similar form. In zeroth order in the interaction the electron Green functions \(G_{\alpha\beta}^{>}(\mathbf{k})\) are of the form

\[
G^{>}_{\alpha\beta}(\mathbf{k}) = \left( \begin{array}{c} G_{\alpha\beta}(\mathbf{k}) \\ G_{\beta\alpha}(\mathbf{k}) \end{array} \right) .
\]

Here the \(\tau_i\) are Pauli matrices acting in the space of the Green functions (29). In (31) we have changed to the averaged time \(\tau_T = (t_1 - t_2)/2\) and the coordinate differences \(t = t_1 - t_2\), \(\mathbf{r} = r_1 - r_2\) and Fourier transformed with respect to the time and coordinate differences. A graphical expression for \(G^{>}_{\alpha\beta}(\mathbf{k})\) is given in Fig. 3.

According to (12) it is sufficient to write down the real part of the Dyson equation for \(G_{\beta\beta}^{>}(\mathbf{k})\) and integrate it over \(\mathbf{e}_0\) to obtain the kinetic equation for the electrons:

\[
\frac{dF}{dt} = \int \frac{d\mathbf{e}_0}{2\pi} G_{\beta\beta}^{>}(\mathbf{k}) G_{\beta\beta}^{<}(\mathbf{k}) \mathcal{U}(t) G_{\beta\beta}^{>}(\mathbf{k}).
\]

To evaluate \(S^{>}_{\alpha\beta}(\mathbf{k})\) it is necessary to find the explicit form of the function \(\mathcal{U}(\mathbf{p})\) taking into account the scattering by impurities. The Green functions \(G^{<}(\mathbf{p})\) and \(G^{>}(\mathbf{p})\) are connected with the retarded function \(G^{\text{r}}(\mathbf{p})\) and the function \(G^{<}(\mathbf{p}) = G^{(1)}(\mathbf{p}) + G^{(2)}(\mathbf{p})\) through the relations

\[
S^{>}_{\alpha\beta}(\mathbf{p}) = \text{Re} S^{>}_{\alpha\beta}(\mathbf{p}) + i/2 \text{Im} S^{>}_{\alpha\beta}(\mathbf{p}) .
\]

If we, therefore, perform a unitary transformation on

G. L. Bir et al.
where \( T \) is the hole relaxation time. We included the imaginary part of \( \Sigma^{R}(p) \), which is practically independent of \( p \), in the renormalization of the chemical potential.

Thus

\[
\Gamma^{\text{tot}}(p) = \frac{1}{\epsilon - \text{Im} \Sigma^{R}(p)}. \tag{34}
\]

The equation for \( g(p) \) is of the form

\[
\Pi^{\text{tot}}(p) = -\mathcal{N} N \sum_{q} |w(p-q')|^{2} \Gamma^{\text{tot}}(p'), \tag{35}
\]

where \( \tau \) is the hole relaxation time. We included the imaginary part of \( \Sigma^{R}(p) \), which is practically independent of \( p \), in the renormalization of the chemical potential.

The solution of (35) is

\[
\Pi^{\text{tot}}(p) = \frac{1}{\epsilon - \text{Im} \Sigma^{R}(p)}, \tag{36}
\]

\[
\Pi^{\text{tot}}(p) \equiv \Omega(p)|\Pi^{\text{tot}}(p)|^{2}, \tag{37}
\]

From (36) to (40) we find an equation for \( g(p) \):

\[
\Pi^{\text{tot}}(p) = -\mathcal{N} \sum_{q} |w(p-q')|^{2} \Gamma^{\text{tot}}(p'), \tag{40}
\]

\[
\Pi^{\text{tot}}(p) = \frac{1}{\epsilon - \text{Im} \Sigma^{R}(p)}, \tag{41}
\]

the solution of which is of the form

\[
\Pi^{\text{tot}}(p) = \frac{N}{\epsilon - \text{Im} \Sigma^{R}(p)} \sum_{q} |w(p-q')|^{2} \Gamma^{\text{tot}}(p'), \tag{42}
\]

\[
f(\epsilon) \text{ is an arbitrary function of } \epsilon. \text{ This function is determined by the inelastic collisions with phonons. When there are no external fields } f(\epsilon) \text{ is the equilibrium distribution function } f(\epsilon) = (e^{\epsilon/T} + 1)^{-1}. \text{ Using Eq. (33) we get}
\]

\[
\Pi^{\text{tot}}(p) = 2n(2\pi)^{1/2} \sum_{q} |w(p-q')|^{2} \Gamma^{\text{tot}}(p'), \tag{43}
\]

We note that, using (33), (38), and (43), we can write the Green function matrix in the form

\[
\Pi(p) = \begin{pmatrix} f(\epsilon) & f(\epsilon) \\ f(\epsilon) & f(\epsilon) \end{pmatrix} \Pi^{\text{tot}}(p) \begin{pmatrix} f(\epsilon) & f(\epsilon) \\ f(\epsilon) & f(\epsilon) \end{pmatrix} = -\mathcal{N} \Pi^{\text{tot}}(p) - f(\epsilon) \Pi^{\text{tot}}(p) f(\epsilon) \tag{44}
\]

We need this representation in what follows, \( \Pi(\epsilon) \) is the advanced Green function, \( \Pi^{\text{tot}}(p) = \Pi^{\text{tot}}(\epsilon) \).

To find the vertex part which describes the interference of the scattering of holes by impurities and by an electron we must sum the diagrams shown in Fig. 5. We can then neglect the diagrams with intersecting impurity lines as they are small in the parameter \( 1/T_{\text{eff}} \approx 1 \). The integral equation for the vertex function \( K^{\text{imm}}(p-p', \epsilon) \) is of the form

\[
K^{\text{imm}}(p-p', \epsilon) = \mathcal{N} \sum_{q} K^{\text{imm}}(p-q, \epsilon) G^{\text{imm}}(p-q) V(q) (\alpha \alpha) + N \sum_{\alpha \beta} K^{\text{imm}}(p-q, \epsilon) G^{\text{imm}}(p-q) \chi(q) (\alpha \beta), \tag{45}
\]

As the inhomogeneous term of the equation has the spin structure \( \alpha \bar{\alpha} \) the solution must be of the form

\[
K^{\text{imm}}(p-p', \epsilon) = S^{\text{mm}}(p-p', \epsilon) \tag{46}
\]

Substituting (46) into (45) we get the equation

\[
M^{\text{imm}}(p-p', \epsilon) = -\mathcal{N} S^{\text{mm}}(p-p', \epsilon) V(q) (\alpha \alpha) + N \sum_{\alpha \beta} M^{\text{imm}}(p-p', \epsilon) G^{\text{mm}}(p-q) \chi(q) (\alpha \beta), \tag{47}
\]

The matrix element \( M^{\text{imm}}(p-p', \epsilon) \) which is summed over \( p \) occurs in the kinetic equation. As the integral of two retarded functions vanishes we can look for the solution of (47) in the form

\[
M^{\text{imm}}(p-p', \epsilon) = \mathcal{N} M^{\text{imm}}(p-p', \epsilon) V(q) (\alpha \alpha) + N \sum_{\alpha \beta} M^{\text{imm}}(p-p', \epsilon) G^{\text{mm}}(p-q) \chi(q) (\alpha \beta), \tag{48}
\]

Using for \( \Pi(p) \) the representation (44), bearing in mind that \( \Delta_{\tau} \Delta = \bar{\Delta}, \bar{\Delta}_{\tau} \Delta = -\bar{\Delta}, \) while \( \Delta_{\tau} \bar{\Delta} = \bar{\Delta}_{\tau} \Delta = 0 \), and dropping terms containing products of only retarded or advanced functions, we get two equations for \( M_{1}(p+q+p) \) and \( M_{2}(p+q+p) \):

\[
M_{1}(p+q+p) = -\mathcal{N} S^{\text{mm}}(p-q) \Pi^{\text{tot}}(p) \left( 1 - N \sum_{\alpha \beta} M_{1}(p+q+p) \right) \tag{49}
\]

\[
M_{2}(p+q+p) = -\mathcal{N} S^{\text{mm}}(p-q) \Pi^{\text{tot}}(p) \left( 1 - N \sum_{\alpha \beta} M_{2}(p+q+p) \right) \tag{50}
\]

It follows from (49) and (50) that \( M_{1}(p+q+p) = M_{2}(p+q+p) \).

The matrix element has the form

\[
M^{\text{imm}}(p-q+p) \equiv \left( 1 - N \sum_{\alpha \beta} M_{1}(p+q+p) \right) \tag{51}
\]

As

\[
\phi_{\alpha}(\epsilon, \epsilon') = \sum_{\alpha \beta} \left( V(q) \right) \text{diag} \left( \sum_{k \alpha} J_{\alpha \beta}^{(1)} M^{\text{imm}}(p+q+p) G^{\text{imm}}(k-q) \right), \tag{52}
\]

we get, substituting (52) into (30) and using (30), (42), (48), and (51), summing over the hole spin states \( \alpha, \alpha' \), and integrating over \( \epsilon \), the following expression for the drift term in the collision integral:

\[
\frac{\partial f_{\alpha}}{\partial t} = -2n \sum_{\alpha \beta} \left( V(q) \right) \text{diag} \left( \sum_{k \alpha} J_{\alpha \beta}^{(1)} M^{\text{imm}}(p+q+p) G^{\text{imm}}(k-q) \right) \tag{53}
\]

where

G. L. Bir et al. 710
\[ |V(q)|^2 = |V_i(q)|^2 \sum_{\alpha \sigma} (\sigma_\alpha \sigma_i) \]

and

\[ \chi(\omega, q) = -i \frac{2 \alpha}{\pi} \sum_{p} M_i(p+q) \phi(p) \].

When obtaining (53) we used the fact that \( \Sigma_{\alpha} \text{Re} M_{i}(p + q) \) is independent of \( \epsilon \) and depends only on \( \omega, q \).

To solve Eq. (49) we shall assume that the potential for the scattering of a hole by impurities is a short-range one so that the scattering amplitude is independent of the angles. One can then easily solve Eq. (49) and after some simple transformations we get for \( \frac{1}{\chi(\omega, q)} \) the following expression:

\[ \frac{1}{\chi(\omega, q)} = \frac{3 N}{2 \epsilon_0} \eta(\omega, q) \] \hspace{1cm} (55)

where

\[ \eta(\omega, q) = \text{Re} \left[ T_\omega e^{-i\omega + \epsilon_j} \right] \]

\[ T_\omega = \frac{\gamma}{\gamma + \epsilon_j} \sum_i \frac{\omega}{\omega + \gamma_j} \eta(\omega, q) \delta(\omega - \epsilon_i + \epsilon_j). \]

As \( \omega \rightarrow \infty \) and \( \tau_{sh} \rightarrow 0 \),

\[ \eta(\omega, q) = \text{Re} T_\omega = \frac{\pi}{2} \frac{1}{\epsilon_j} \delta(\omega - q\omega). \]

Substituting (59) into (58) we get a formula which is the same as Eq. (9) introduced above (we have here in (58) and (59) dropped the term \( \frac{q^2}{2m} \) which occurs in (9) as it is small compared to \( \omega \) or \( qV_F \) in the parameter \( m_0/m \) when \( V_F \gg V_e \), or in the parameter \( k/p \) when \( V_F \ll V_e \). When \( V_F \ll V_e \), we can when \( |\omega| \ll k \), T let in (58) and (59) \( \omega \rightarrow 0 \), i.e., assume \( \eta = \delta(\omega) \). We then get, integrating over \( \omega \) and changing from a summation over \( q \) to a summation over \( k' = k - q \),

\[ \left( \frac{1}{2\pi} \right)^{-1} \frac{3 \pi N}{2 \epsilon_0} |V_i|^2 \sum_{k' \epsilon^2} \delta(\epsilon_i - \epsilon_{k'}) \] \hspace{1cm} (60)

whence (10) follows after integration.

When \( V_F \gg V_e \) in (58) and (59) \( x \approx V_e/V_F \ll 1 \) are the important values. We can thus extend the integration over \( x \) in (59) from \( \omega \rightarrow \infty \). Then \( \eta = \frac{\pi}{2qV_F} \) and

\[ \left( \frac{1}{2\pi} \right)^{-1} \frac{3 \pi N}{4 \epsilon_0 \epsilon_1} |V_i|^2 \sum_{k' \epsilon^2} \delta(\epsilon_i - \epsilon_{k'}) \left( \exp \left( \frac{\epsilon_i - \epsilon_{k'}}{T} \right) - 1 \right)^{-1} \] \hspace{1cm} (61)

from which (11) follows after simple transformations.

We now turn to the general Eqs. (58) and (57) and make more precise the regions in Fig. 1 where Eqs. (9) to (11) are valid and consider the case when these formulae are inapplicable.

We consider first the region where \( \hbar k^2 \gg 1 \), i.e., \( qV_F \gg 1/\tau \). In the case of fast holes in that region \( qV_F \gg 1/\tau \). We can thus neglect in (57) all terms except \( qV_F \). It then follows from (57) and (56) that \( \eta = \text{Re} \left[ \frac{-i\omega + Dq^2}{1 + i\omega T} \right] \)

\[ \left( \frac{1}{2\pi} \right)^{-1} \frac{3 \pi N}{2 \epsilon_0} |V_i|^2 \sum_{k' \epsilon^2} \delta(\epsilon_i - \epsilon_{k'}) \left( \exp \left( \frac{\epsilon_i - \epsilon_{k'}}{T} \right) - 1 \right)^{-1} \]

whence follows (12).

When \( m_0D \ll 1 \) the characteristic values \( \omega \approx Dk^2 \ll \epsilon_K \) and then \( \omega \rightarrow \hbar k^2 \ll 1 \). Therefore, in the whole band \( \tau/\tau_{sh} \ll \hbar k^2 \ll 1 \) we may assume, independent of the magnitude of \( \tauT \), that \( \eta \approx \delta(\omega) \) in (58) and (62) as a result of which we are again led to (60).

In the region where \( \hbar k^2 \ll \tau/\tau_{sh} \) we can, on the other hand, neglect the term \( Dq^2 \) in comparison with \( \tau_{sh} \) and, hence, in this region

\[ \eta = \text{Re} \left[ \frac{-i\omega + Dq^2}{1 + i\omega T} \right] \]

\[ \left( \frac{1}{2\pi} \right)^{-1} \frac{3 \pi N}{2 \epsilon_0} |V_i|^2 \sum_{k' \epsilon^2} \delta(\epsilon_i - \epsilon_{k'}) \left( \exp \left( \frac{\epsilon_i - \epsilon_{k'}}{T} \right) - 1 \right)^{-1} \]

whence follows (13).

When \( \tau_{sh} \gg 1/\tau \) the important values are \( \omega \approx \tau_{sh} \ll T \) and, hence, again \( \eta \approx \delta(\omega) \) as a result of which we get again (60), independent of the magnitude of \( \tauT \). Equations (60) and (10) are thus valid in the whole of the slow hole regions (a) and (b).

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1) In semiconductors with a degenerate valence band we take into account in this case only the scattering by the heavy holes, since the light-hole density is small.

2) In Sec. 5 we consider the scattering of holes by a short-range potential, equivalent to a strongly screened Coulomb potential when the drift time for relaxation is then the same as the transport time. When the scattering is by a not strongly screened Coulomb potential, we shall assume that \( \tau \) is the transport time.
119 (1973) [Sov. Phys.-Semicond. 7, 81 (1973)].

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