Corrections to scaling laws in the theory of interacting pomeron

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Corrections are calculated to the scale-invariant solution [I] of the form 1/ln s of the equations of the theory of interacting pomeron. Corrections of the order 1/ξ to o^2 appear on account of the difference between the effective coupling constant from its asymptotic constant value which it attains as ξ→∞. The remaining corrections ~1/R^4, with a_1 > 0 are due to the contribution of incompletely enhanced diagrams, in particular, diagrams containing the vertices of absorption and emission of two pomeron by the particle and the vertex of direct four-pomeron interaction. The indices a_i of some of the important terms have been obtained. The article does not contain a general method for the calculation of the a_i.

In a previous paper [1, 2] we have investigated the scale-invariant solution [3] of the system of equations which describes the field of interacting pomeron. It was shown that this solution can be obtained naturally by means of the Wilson ε-expansion [4], which allows one to obtain the physical parameters which appear and to construct the unknown functions which determine the solution [2]. The theoretical model which was considered is valid only in the region of superhigh energies, for which the parameter ξ = ln(s/R^2) is very large, i.e., for ξ ≳ 1/R^2. Here r ≈ 1/12 is the three-pomeron interaction coupling constant which is now known experimentally [5]. The region of validity of this solution can be enlarged in the direction of lower energies if one finds corrections of the order 1/ξ to (1/ξ) due to terms which were not taken into account for ξ ≳ 1; here a_1 > 0 are certain powers. This problem is discussed and partially solved below.

The solution obtained in [2] takes into account only the main part of the contribution of the so-called "enhanced" reggeon diagrams (Fig. 1, a) and does not take into consideration the contribution from the incompletely enhanced ones (Fig. 1, b, c).

Therefore the corrections to this solution can be of two types:

a) Correction terms of the order ωε/2 ∼ 1/ξ ε/2, ε = 2 appear from the contribution of the enhanced diagrams. They are related to the fact that for small ωε/2 the effective value of the square of the three-pomeron vertex

\[ \lambda_{R'}(ω) = \frac{1}{R'}^{\gamma_{R'}} = \frac{1}{1 + 1/2\pi(ωε/2 - 1)/ε} \]

differs only little from its limiting value \( \lambda_{R'}(ω) \approx 2ε/3 \) which has for \( ωε/2 = 0 \).

b) Corrections of the order ωε/2 ∼ 1/ξ ε/2, where a_1 are indices, appear also from the contribution of the incompletely enhanced diagrams of Fig. 1, b, c, which contain the vertices N_m describing the simultaneous emission or absorption of n pomeron by the particle, or the vertex \( G_{nm} \) describing the direct transition of n pomeron into m. (The "enhanced" diagrams of this type \( G_{nm} \) constructed out of three-pomeron vertices, have already been taken into account in the contribution of the enhanced diagrams. For example, the vertex \( G_{13} \) is surrounded by the dashed loop on one of the diagrams in Fig. 1, a.)

1. Let us find the corrections of the first kind, which are of order ωε/2. An expansion of the function (1) into a series of powers of ωε/2 yields

\[ \lambda_{R'}(ω) = \lambda_{R'}(ωε/2) = \frac{1}{R'}^{\gamma_{R'}} = \frac{1}{1 + 1/2\pi(ωε/2 - 1)/ε} \]

where \( r_1 = 2ε/3 \) is the stable root of the Gell-Mann–Low equation, i.e.,

\[ \frac{d}{dl} F(λ_n) = 0, \quad l = \ln \frac{1}{λ_n}, \]

\[ \frac{1}{λ_n} F(λ_n) = \frac{2 + 1/2}{1} + \frac{3\pi^2}{5} \left( \frac{2 - \frac{1}{2}}{(R')^2} \right) \]

and λ = r(d) is the three-pomeron coupling constant. This constant may be considered having an arbitrary dependence on the dimension d = 4 – ε of space and it appears in \( λ_{R'}(ω) \) as ε → 0, i.e., for d = 4.

The quantity \( r(4) \) differs from the physical four-pomeron coupling constant \( r^2 = r(2) \). Denoting as in [1],

\[ r^2 = r^2(4), \quad r^2(2) = r^2 \]

and neglecting in \( λ_{R'}(ω) \) the term ~ωε/2 (compared to terms ~ωε/2/r^2(4); r(4) ≈ 1), we obtain for ε = 2

\[ \lambda_{R'}(ω) = r^2 - r'_{11}ωε/r^2. \]
For the functions
\[ \beta(t) = (\lambda/\pi)^{-\lambda}, \quad R(t) = (\lambda/\pi)^{-\lambda}, \quad \Gamma = (\lambda/\pi)^{-\lambda} \]
the taking into account of the second term in \( \lambda_2(w) \) leads to small corrections to the values (3.5) in [12]:
\[ Z_i G(w, \omega) = Z_i^{\beta}(t) = e^{\omega t^{-1} \left( 1 + \frac{2\pi \omega}{9} \right)} \]
\[ R(t) = R(w) = e^{\omega t^{-1} \left( 1 + \frac{2\pi \omega}{9} \right)} \]
\[ \Gamma(t) = Z_i^{\beta}(t) = e^{\omega t^{-1} \left( 1 + \frac{2\pi \omega}{9} \right)} \]
(3)
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Going over to the \( \xi \)-representation we obtain \( \sigma_{tot}(\xi) \):
\[ \sigma_{tot}(\xi) = \frac{8\pi^2}{Z_2 G(w, \omega)} \left( 1 + \frac{2\pi \omega}{9} \right) \]
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(4)

where the constants \( \eta \approx 1/6, \nu \approx 1/12, \gamma \approx 2/3 \) and \( Z_0, Z_2 \) have values which have been determined before (cf. [17]).

As we have seen in [1], the optimal value of the ratio \( v^2(2) \) of the constants can be obtained from a condition of smooth matching at \( \omega > r^2 \) of the power-law solutions for \( G(w, \omega) \) and \( \Gamma \) in the strong coupling region \( \omega < r^2 \) with the perturbation theory series which converge for \( \omega > r^2 \). This condition yields [1] \( v^2 \approx 0.11 \pm 0.01 \). Therefore the correction terms in (3) are small already for attainable energies, when \( \xi \leq \ln(\delta/\epsilon^2) \), and has the order of magnitude 10 and for \( 1/\epsilon^2 \xi = 15 \), if \( \epsilon^2 = 10^2 \approx 150 \).

We show how one can calculate the corrections from the incompletely enhanced diagrams and determine the main ones.

The general definition of pomeron asymptotics of an amplitude of the form (1.1), (1.2) of [12], taking all diagrams into account, is
\[ T(k, k') = \int e^{i(w, 0)T(v, \psi)\chi(0, 0)} \exp \left( i \int \mathcal{L}_i d^pd\xi \right) \]
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(5)

where \( T \) is the \( \xi \)-ordering operator, \( V \) is the operator describing the absorption of a pomeron by the particle
\[ V = V_i = \sum_\xi V_{i, \xi} \psi_\xi(\rho, \xi), \]
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(6)

and \( L_i \) is the pomeron interaction operator:
\[ L_i = \mathcal{D}_{\omega} + \sum_\xi \mathcal{D}_{\xi} \]
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(7)

Here \( \mathcal{D}_{\omega} \) denote higher order terms in the expansion of the Lagrangian in powers of the pomeron field \( \psi(\rho, \xi) \) (cf. (1.3) in [12]). It is easy to verify that an expansion of the exponential in (5) in powers of the constants \( R \) and \( \lambda^{nm} \), taking into account the terms \( \delta V_n \) in (5), (6), reproduces the contribution of all the pomeron diagrams of Fig. 1 to the scattering amplitude.

As was noted in [1], the scale invariance of the theory in the region of small \( \omega \sim k^2 \sim r^2 \) means the invariance of the \( n \)-point Green's functions with respect to the transformation
\[ \psi(\rho, \xi) \rightarrow e^{-\Lambda} \psi(\rho, \xi) \]
\[ \psi(\rho, \xi) \rightarrow e^{-\Lambda} \psi(\rho, \xi) \]
(8)

Here the field \( \psi \) has the dimension \( \Delta \) and the n-point Green's function
\[ G_n = \langle \mathcal{O}_n(\psi(\rho, \xi) \psi(\rho, \xi) \ldots \psi(\rho, \xi)) \rangle \]
has the dimension \( n \Delta \). The transition to the \( \omega, k \)-representation yields for \( G_\omega, k^2 \sim G_\omega \) and \( \Gamma \sim G_\omega \), of the constants of the type (3.8) in [1] with
\[ \eta = \frac{\delta v}{2 - 2\Delta}, \quad \gamma = \frac{\nu}{2 - 2\Delta} \]

The operators \( \delta V_n \) and \( \delta J_n \) were obtained by means of perturbation theory and the renormalization group method. They are positive and increase with \( n \) and \( m \).

We denote by \( T(n, m) / \phi_s \) the contribution from one or all the enhanced diagrams (they are all of the same order for \( \xi \rightarrow \infty \)) and let \( T(n, m) / \phi_s \) and \( T(n, m) / \phi_{nm} \) denote the contributions of the incompletely enhanced diagrams, containing respectively one vertex \( \lambda^{nm} \) or \( \lambda^{nm} \).

It follows from the definition (5) that the ratio
\[ T(n, m) / T(0, 0) \]

In the same manner one can see from (5) that the relative contribution of the diagrams containing unenhanced vertices \( \lambda^{nm} \) describing the direct transition of \( n \) pomerons into \( m \) is determined also by the quantity \( \delta J_{nn} / \phi_s \)
\[ \delta J_{nn} / \phi_s \approx \frac{\delta J_{nn} / \phi_{nm}}{\phi_{nm}} \]

As \( \xi \) increases all these ratios decrease, since in general \( \lambda^{nm} \) is always larger than \( \lambda^{nm} \) and \( \lambda^{nm} \) is larger than \( 1 + \delta \). Before calculating some of these numbers we explain how these estimates can be obtained directly in the method of reggeon diagrams. The operators \( \delta V_n \) and \( \delta J_{nm} \) in (6), (7) lead to the appearance of incompletely enhanced diagrams in Fig. 1, b, containing "dressed" vertices \( \lambda^{nm} \) describing the direct transition of \( n \) pomerons into a system particle-antiparticle and the vertices \( \delta J_{nm} \) describing a direct transition of \( n \) pomerons into \( m \). The quantities \( \lambda^{nm} \) and \( \lambda^{nm} \) are "bare" values of these vertices. In place of these vertices the completely enhanced diagrams contain the "enhanced" vertices \( \lambda^{nm} \) and \( \lambda^{nm} \) of the same type, which are the contribution of "tree-like" diagrams of Fig. 2 and also the contribution from a large number of more complicated "enhanced" diagrams. Their order of magnitude is
\[ (\lambda^{nm}) \sim (\mathcal{G} \mathcal{J})^{\lambda^{nm}} \sim \omega^{-\lambda^{nm}} \]

where \( \Delta = \eta + 1 - \gamma \) is the dimension of the operator \( \psi \)

FIG. 2

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(and $3\Delta + \gamma = 1 + d/2$). Since replacing the vertices $N_n$ and $\gamma_{nm}$ respectively by $(N(n))_{\text{enh}}$ and $\gamma_{nm}$ has the result that an incompletely enhanced diagram goes over into an enhanced diagram, the ratios of the contribution of the diagrams is directly the ratio of the vertices:

$$\frac{T(n)}{T(n,m)} = \frac{N(n)}{\Gamma_{nm}}.$$

Taking into account that $\omega \sim 1/\xi$, we obtain hence

$$N^{(0)}_{n,m} \sim \Gamma_{nm}, \quad \delta \Gamma_{nm} \sim \omega^{n+m-1}.$$

Thus, the equations (9) are equivalent to the almost obvious assertion that the dependence of the vertices $N_n$ and $\gamma_{nm}$ on $\omega \sim 1/\xi$ has a power-law character as $\omega \sim 1/\omega \rightarrow 0$.

Let us determine some of the numbers $\Delta_n,m$. In order to compute the index $\Delta_2$, we construct the vertex $N(2)$ which is analogous to the three-pomeron vertex $\Gamma_1$ in Fig. 3. Determining according to the diagrams of Fig. 3 the corrections of order $r^2$ to $N_2$ and using the renormalization group method in four-space we obtain

$$N_2 = N_1(1-1/2)^{-1/2}N_2(1+1/2)^{-1/2}.$$

Similar to analogous formulas for $\Gamma_1$ in Fig. 3. Going over to a space of dimension $d = 4 - \epsilon$ by replacing $l$ by $2(\omega \gamma/2 - 1)/\epsilon$, we obtain, for $\omega \rightarrow 0$

$$\Delta_2 = 2\Delta + \epsilon/3 + O(\epsilon^2) = \Delta_1, \epsilon.$$  

Comparing this value with (12), we obtain

$$\Delta_3 = 2\Delta + \epsilon/3 + O(\epsilon^2) \approx \Delta_1,$$

This means that the corrections (10) from the semi-enhanced diagrams (the first two on the left in Fig. 1, b) to the main contribution have the relative value

$$T^{(0)}_{3,1} \sim T^{(1)}_{3,1} \sim T^{(2)}_{3,1} \sim T^{(3)}_{3,1},$$

for $\epsilon = 2$. The correction $r^{(2)}$ from the unenhanced two-reggeon diagram of Fig. 1, c are very small indeed:

$$T^{(0)}_{3,1} \sim T^{(1)}_{3,1} \sim T^{(2)}_{3,1} \sim T^{(3)}_{3,1}.$$

It is easy to calculate the index $\Delta_1,2$ if one takes into account the fact that the corresponding corrections are due to the perturbation $\Delta^{(1)} = 1/2 \gamma_2(\gamma_2^2 + \gamma_2^2)/2$ of exactly the same form as the fundamental interaction $\gamma$ in (6). In other words, they appear from the replacement of the constant $r = r_1 + r_0$, where $r = r_1$. From $r = r_1$ all quantities have the values determined in [12], and under the substitution $r = r_1 + \lambda_2$ there appear corrections of the order $\omega/\omega^2$, obtained above in Eqs. (1) and (2). Therefore

$$\int d^2 \gamma d^d \rho - \frac{1}{\omega^4} \gamma_1^2,$$

i.e., according to (11) $\Delta_1 = 1 + \nu d/2 + \epsilon/2$.

As the numbers $n$ and $m$ increase, the quantities $\Delta_n,m$ increase rapidly, in particular $\Delta_n,m > (n + m)\Delta$ (since for $\omega \rightarrow 0$ the vertices $N^{(0)}$ and $\gamma_{nm}$ decrease). Therefore the corrections from multipomeron interactions, in particular from the four-pomeron vertices $\lambda_{13}$, $\lambda_{24}$, are small and decrease rapidly as $\xi$ increases.

Taking into account the main correction (4) from $\lambda_{12}$ and from $N^{(2)}$, we obtain

$$\sigma^{(2)}(\xi) = \frac{8\pi}{1+\eta} \xi^{1/2} \left[ 1 + \frac{C_2}{2\pi^2} \eta + \frac{C_3}{\eta^2} + \cdots \right],$$

where $C_{2,3}$ is an unknown constant, $\eta \approx 1/10$.

1Some of the results of [1-4] have also been obtained recently by Abarbanel and Bronzan [6]. The authors are grateful to M. Baker who has made them aware of these results.

2More precisely, linear combinations of operators of this type have a definite dimension. This makes the problem of determining the numbers $\Delta_{n,0}$ and $\Delta_{n,m}$ more difficult and was not taken into account in [2], where the numbers $\Delta_{n,0}$ and $\Delta_{n,m}$ are determined incorrectly. For $\xi \rightarrow \infty$ the operators $\delta V_{1}, \delta_{2}^{(2)}$ are also proportional to some powers of the quantity $\xi$.


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