Two examples of Langmuir wave collapse

L. M. Degtyarev, V. E. Zakharov, and L. I. Rudakov

Institute of Applied Mathematics, USSR Academy of Sciences
(Submitted August 12, 1974)
Zh. Eksp. Teor. Fiz. 68, 115–126 (January 1975)

We study both numerically and analytically the evolution of two kinds of distributions for the initial density in a plasma and for the Langmuir oscillation amplitude in it. In the first case we consider the motion of a one-dimensional spherical soliton towards the center, and in the second we study the nonlinear stage of an instability in the two-dimensional case of a plane soliton. We consider both small and large Langmuir wave amplitudes. We show that Langmuir-wave collapse—the concentration of plasma oscillations in regions where the plasma density is lowered—is possible in principle.

INTRODUCTION

The problem of the dissipation mechanisms for the Langmuir wave condensate produced as a result of the evolution of Langmuir turbulence is one of the central problems in the theory of plasma turbulence. One of the authors of the present paper has formulated in [1] the concept of the dissipation of the condensate due to a nonlinear dissipation mechanism—the Langmuir collapse. Langmuir collapse is the concentration of the Langmuir oscillation energy in regions where the plasma density is lowered, which appear self-consistently as the effect of the expelling force of a high-frequency field; it is the nonlinear stage of the development of the instability of the long-wavelength Langmuir turbulence.[2] The collapse is accompanied by a strong absorption of Langmuir waves owing to their Landau damping. However, the actual model of the collapse, proposed in [1]—the self-similar spherically symmetric collapse—cannot be realized in reality. In the present paper, which is a development of [3], we shall, without touching upon the problem of the role played by the collapse in the kinetics of Langmuir turbulence, study both numerically and analytically some kinds of initial distributions in the density in the plasma and in the Langmuir oscillation amplitude in it which after a finite time lead to the collapse, i.e., to a singularity in the Langmuir amplitude. In that sense the present paper is a proof that the existence of a collapse is possible in principle, both for small and for large amplitudes of the Langmuir waves.

1. BASIC EQUATIONS

We start, as in [1], from a set of equations for the complex amplitude $\psi(\mathbf{r}, t)$ of the high-frequency electrostatic potential,

$$\varphi(\mathbf{r}, t) = \mathrm{Re}\,\{\psi(\mathbf{r}, t)\exp(i\omega_p t)\}$$

and for the variation δn in the plasma density

$$\mathrm{div}\left(-2i\nabla\frac{\partial\psi}{\partial t} - \frac{3}{2}\omega_p r_D^2\nabla\nabla^2\psi + \omega_p\frac{\delta n}{n}\nabla\psi\right) = 0,$$
$$\frac{\partial^2}{\partial t^2}\delta n - c_s^2\nabla^2\delta n = \nabla^2\frac{|\nabla\psi|^2}{16\pi M}, \quad r_D^2 = \frac{T_e}{4\pi e^2 n}, \quad c_s^2 = \frac{T_e}{M}. \tag{1}$$

It is convenient to introduce as follows dimensionless variables in Eqs. (1):

$$x \to \frac{3}{2}\omega_p\frac{r_D^2}{c_s}x, \quad t \to \frac{3}{2}\omega_p\frac{r_D^2}{c_s^2}t,$$

$$\frac{\delta n}{n} \to \frac{2}{3}\frac{c_s^2}{v_{Te}^2}\delta n, \quad |\nabla\psi|^2 \to \frac{64}{27}Mc_s^2\frac{c_s^2}{v_{Te}^2}|\nabla\psi|^2.$$

They then become

$$\mathrm{div}\,(-2i\nabla\psi_t - \nabla\nabla^2\psi + \delta n\nabla\psi) = 0, \tag{2}$$

$$\frac{\partial^2}{\partial t^2}\delta n - \nabla^2\delta n = \nabla^2|\nabla\psi|^2. \tag{3}$$

Equations (2) and (3) have the integrals of motion

$$I = \int|\nabla\psi|^2 d\mathbf{r},$$
$$H = \int(|\nabla^2\psi|^2 + n|\nabla\psi|^2 + {}^1/_2 n^2 + {}^1/_2(\nabla\Phi)^2)\,d\mathbf{r}, \tag{4}$$

where

$$\nabla^2\Phi = -n_t.$$

If $w/nT \ll m/M$ (in dimensionless coordinates when $|\nabla\psi|^2 \ll 1$) we can in Eq. (3) neglect the term $\partial^2\delta n/\partial t^2$. Then

$$\delta n = -|\nabla\psi|^2, \quad \nabla\Phi_t = 0$$

and Eqs. (2) to (4) become

$$\mathrm{div}\,(2i\nabla\psi_t + \nabla\nabla^2\psi + |\nabla\psi|^2\nabla\psi) = 0, \tag{5}$$

$$H = \int(|\nabla^2\psi|^2 - {}^1/_2|\nabla\psi|^4)\,d\mathbf{r}. \tag{5a}$$

We shall call this approximation the static one.

Equations (2), (3) admit an exact solution—a plasma soliton. Let δn and ψ depend only on one coordinate x. For $E = -\psi_x$ and δn we then have

$$E = E_0\,\mathrm{ch}^{-1}\left[\frac{1}{\sqrt{2}}\frac{E_0(x - vt - x_0)}{(1-v^2)^{1/2}}\right]e^{is},$$

$$s = -\Omega t + vx + s_0 = -\left(\frac{v^2}{2} - \frac{E_0^2}{4(1-v^2)}\right)t + vx + s_0, \tag{6}$$

$$\delta n = -|E|^2/(1-v^2).$$

The soliton exists, if $v < 1$; it moves with subsonic velocity.

One can easily prove the fact that collapse exist in the static, spherically symmetric case.[1)] Introducing $E = -\partial\psi/\partial r$ we write Eq. (5) in the form

$$2i\frac{\partial E}{\partial t} + \frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial}{\partial r}r^2 E\right) + |E|^2 E = 0. \tag{7}$$

From this equation we get the relation

$$\frac{\partial^2}{\partial t^2}\int_0^\infty r^4|E|^2\,dr = 3H - \int_0^\infty r^2\left|\frac{\partial E}{\partial r}\right|^2 dr - \frac{1}{2}\int_0^\infty|E|^2\,dr. \tag{8}$$

Here H is the integral (4) of Eq. (7) which in the present case is of the form

$$H = \int_0^\infty\left(\left|\frac{\partial E}{\partial r}\right|^2 + 2\frac{|E|^2}{r^2} - \frac{1}{2}|E|^4\right)r^2\,dr. \tag{9}$$

Integrating Eq. (7) we find

$$\int_0^\infty r^4|E|^2\,dr < \frac{3}{2}Ht^2 + C_1 t + C_2. \tag{10}$$

Copyright © 1975 American Institute of Physics

If H < 0, inequality (10) can be satisfied only for not too large values of t. Hence it follows that the solutions of Eq. (7) for which H < 0 must after a finite period $t = t_0$ end up to be singular.

2. SPHERICALLY SYMMETRIC COLLAPSE

We consider a model of spherically symmetric collapse in the form of a spherical layer which converges to the center and which has the local soliton structure. Let such a layer have a radius R and a thickness δ. The characteristic value of the field in the center of the layer $E \sim 1/\delta$ and it then follows from the conservation of the integral I that

$$E^2\delta R^2 \approx I_0, \quad \delta/R \sim R/I_0. \tag{11}$$

As $R \to 0$ the condition for the applicability of the "quasi-planar" approximation thus becomes easier to satisfy.

The way the soliton behaves can in the static case be found from (8). Substituting (11) into (8) and bearing in mind that as $R \to 0$ the second term in (8) turns out to be much larger than the conserved quantity H, we get

$$\frac{d^2}{dt^2}R^2 \approx -\frac{1}{R^4}, \tag{12}$$

whence $R \sim (t_0 - t)^{1/3}$, where t_0 is the time of collapse. The soliton velocity is then $dR/dt \approx (t_0 - t)^{-2/3}$, and the soliton accelerates as $t \to t_0$. When $dR/dt \sim 1$, the static approximation breaks down so that it is necessary to use the exact Eqs. (2) to solve the problem of the spherical soliton.

We shall look for a solution of (2) in the form

$$E(r) = E_0(R)\,\mathrm{ch}^{-1}\left[\frac{1}{\sqrt{2}}\frac{(r-R)E_0(R)}{(1-v^2)^{1/2}}\right]e^{is},$$
$$s = -\left(\frac{v^2}{2} - \frac{E_0^2(R)}{4(1-v^2)}\right)t + vr + s_0, \quad v = \frac{dR}{dt}. \tag{13}$$

Substituting (13) into the conserved integrals

$$I = \int_0^\infty r^2|E|^2\,dr,$$
$$H = \int_0^\infty \left(|E_r|^2 + \delta n|E|^2 + \frac{\delta n^2}{2} + \frac{v_r^2}{2} + \frac{2}{r^2}|E|^2\right) r^2\,dr, \tag{14}$$
$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2 v_r = -\frac{\partial}{\partial t}\delta n,$$

we get after some simple transformations

$$E^2(R) = E_0^2\frac{R_0^4(1-v_0^2)}{R^4(1-v^2)}, \quad I = 2\cdot 2^{1/2}(1-v_0^2)^{1/2}E_0R_0^2,$$
$$H = I\left[v^2 - \frac{1-5v^2}{6(1-v^2)^3}\frac{E_0^2(1-v_0^2)R_0^4}{R^4}\right]. \tag{15}$$

This relation is a differential equation connecting the soliton velocity $v = dR/dt$ with its coordinate R. If $v \ll 1$ it follows from (15) that $v^2 \sim R^{-4}$, in agreement with (12). When $v \sim 1$ this approximation can no longer be applied.

If H > 0, Eq. (15) does not contain a turning point. In that case solitons can move out to infinity. One must, however, remember that for sufficiently large R the soliton, in accordance with the estimate (11), becomes "thick," and the quasi-planar approximation breaks down. When H < 0, the equation has a turning point. Taking R_0 to be that point, we get

$$H = -\frac{I}{6}E_0^2 = \frac{\sqrt{2}}{3}E_0^3R_0^2.$$

Near the turning point Eq. (15) can be simplified to read

$$v^2 = \frac{E_0^2}{6}\left(\frac{R_0^4}{R^4} - 1\right), \tag{16}$$

whence it is clear that the soliton moves from the turning point to the center. The turning point is unique, which means that a spherical soliton collapses into the center. When $v \ll 1$ the motion is performed according to Eq. (12), otherwise with a nearly constant velocity $v \to 1/\sqrt{5}$.

We give in Fig. 1 the behavior of $|E(r)|^2$ and $\delta n(r)$ illustrating the successive stages in the collapse of a spherical soliton. Initially we specify a soliton at rest—the motion started from the turning point. When the soliton accelerates its "density well" gets deeper and the plasma is expelled from it to the center. Inside the spherical soliton the plasma density increases, but this increase is small compared to the depth of the "density well" of the soliton and cannot check the soliton collapse, although it affects the way the soliton velocity approaches its limiting value $v = 1/\sqrt{5}$.[2)] In fact, of course, the soliton cannot reach the center. When its dimensionless intensity reaches a magnitude of the order of M/m ($\sim 2 \times 10^3$) strong Landau damping comes into play and the energy of the Langmuir waves is absorbed. The numerical experiment was performed up to intensities of just such an order of magnitude, which is a direct numerical proof that collapse can possibly exist as a nonlinear mechanism for the damping of Langmuir waves.

The collapse of a spherical soliton considered here is essentially a general case of a spherically symmetric collapse. At least, it refers to quasi-linear initial conditions. Using a numerical experiment we showed in[6]

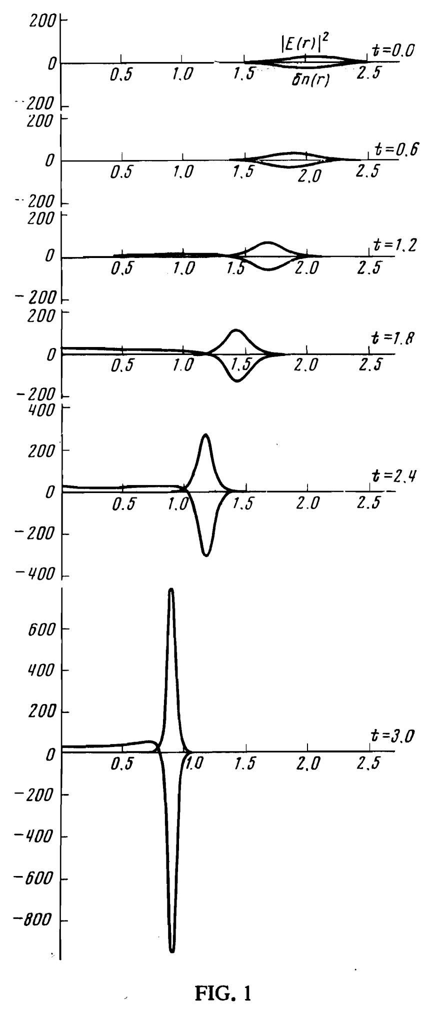

FIG. 1

that a sufficiently strong initial condition collapses in the "planar" case before the formation of a soliton. We must in the general "spherical" case expect the "splitting off" of several collapsing spherical solitons from the initial condition.

In concluding this section we discuss the problem why the self-similar supersonic collapse considered in [1] can not occur. For the case of the collapse of large amplitude waves, $w/nT \gg m/M$ we can neglect in the wave Eq. (3) the term $\nabla^2\delta n$ and use in Eq. (2) the adiabatic approximation and replace $i\psi_t$ by $-E(t)\psi$. After these simplifications Eqs. (2) and (3) permit the self-similar substitution

$$\delta n\to(t_0-t)^{-4/3}\delta n(\xi),$$
$$\nabla\psi=\frac{1}{t_0-t}\psi(\xi),$$
$$\xi=r(t_0-t)^{-2/3}.$$

After twice integrating Eq. (3) over ξ we get in the spherically symmetric case

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\xi^2\frac{\partial E(\xi)}{\partial\xi}=\frac{2E}{\xi^2}+E+\frac{9}{4}\frac{E}{\xi^{7/2}}\times\int^{\xi}\xi^{1/2}\frac{\partial}{\partial\xi}(\xi E^2)\,d\xi,$$
$$E=-\partial\psi/\partial\xi,\quad E(0)=0.$$

Considering the solution of this equation in the form of a series in odd powers of ξ:

$$E=\alpha_1\xi+\alpha_2\xi^3+\ldots,$$

we find $\alpha_2 = {}^{27}/_{14}\,\alpha_1^3$ and also $\alpha_n = C_n\alpha_1^{2n-1}$, where all $C_n > 0$. It is clear that the function $E(\xi)$, given by the series for E, cannot decrease as $\xi\to\infty$, and the self-similar spherical solution is the collapse of an exponentially growing density well and has no physical meaning. This fact is explained by a simple physical cause: due to the boundary condition $E(0) = 0$ the potential of the high-frequency force has a minimum at the origin and therefore the plasma accumulates near the center, preventing the self-similar collapse. Self-similar collapse is possible only, if $|E|^2$ has a maximum in the center of the well, which can occur only for less symmetric configurations of the oscillating field.

3. COLLAPSE OF A QUASI-PLANAR SOLITON

To realize a spherical collapse we need specially produced spherically symmetric initial conditions which normally do not occur when Langmuir waves are excited in the plasma. Moreover, it was shown in [7] that a plane plasma soliton is unstable under the development of a perturbation with a wave vector in its plane. We study in the present section in more detail this instability and show that its development is a quasi-one-dimensional Langmuir collapse which conserves the local soliton structure. To derive the equations which describe this kind of collapse we use a variational principle.

We restrict ourselves in our studies to the collapse of a "standing" soliton and we shall assume that

$$E=-\frac{\partial\psi}{\partial x}=\frac{B^2e^{i\Phi}}{\mathrm{ch}(B^2x/\sqrt{2})}.\tag{17}$$

Here B and Φ are slowly varying functions of the time and the transverse coordinates:

$$\partial B/\partial t\ll B^3,\quad|\nabla_\perp B|\ll B^3.\tag{18}$$

By virtue of inequalities (18) we may assume that variations in the plasma density are slow and we can solve the wave Eq. (3) approximately:

$$n\approx-|\nabla\psi|^2-\frac{\partial^2}{\partial t^2}\frac{1}{\Delta}|\nabla\psi|^2\approx-|\nabla\psi|^2-\frac{\partial^2}{\partial t^2}\int_0^x dx\int_0^x|E|^2\,dx.\tag{19}$$

We used here the fact that $|E|\gg\nabla_\perp\psi$. Using (19) we can rewrite Eq. (2) in the form

$$\nabla^2(2i\psi_t+\Delta\psi)+\mathrm{div}\,|\nabla\psi|^2\nabla\psi+\frac{\partial}{\partial x}\left(\frac{\partial^2}{\partial t^2}\int_0^x dx\int_0^x|\psi_x|^2dx\,\psi_x\right).\tag{20}$$

This equation can be obtained by the variation of the action

$$S=i\int(\psi^*\nabla^2\psi_t-\psi\nabla^2\psi_t^*)\,dr\,dt+\int|\nabla^2\psi|^2\,dr\,dt-\frac{1}{2}\int|\nabla\psi|^4\,dr\,dt$$
$$+\frac{1}{2}\int\left(\frac{\partial}{\partial t}|\psi_x|^2\int_0^x dx\int_0^x\frac{\partial}{\partial t}|\psi_x|^2dx\right)dr\,dt.\tag{21}$$

We choose a trial function ψ of the following form: we shall assume that

$$\nabla^2\psi=\rho=\frac{\partial E_0}{\partial x}\qquad E=\frac{B^2(r_\perp)e^{i\Phi(r_\perp)}}{\mathrm{ch}(B^2(r_\perp)x/\sqrt{2})}.\tag{22}$$

$E_0(r_\perp, x, t)$ is the longitudinal electrical field of the plane soliton which formally depends on the transverse coordinates. In first approximation in $\nabla_\perp$ the quantity E_0 satisfies the equation

$$2iE_{0t}+E_{0xx}+|E_0|^2E_0=0.\tag{23}$$

Equation (23) for ψ can be solved in the form of a series

$$\psi=\psi_0+\psi_1+\ldots,$$
$$\frac{\partial^2\psi_0}{\partial x^2}=\rho,\quad\psi_0=\sqrt{2}\,e^{i\Phi}\arcsin\mathrm{th}\frac{B^2x}{\sqrt{2}},$$
$$\partial^2\psi_1/\partial x^2+\nabla^2_\perp\psi_0=0.$$

We have for the electrical field **E**

$$E_x=E_0+\partial\psi_1/\partial x,\quad E_\perp=\nabla_\perp\psi_0.\tag{24}$$

The first term of the action (21) can be transformed to the form

$$S_1=i\int(\psi^*\nabla^2\psi_t-\psi\nabla^2\psi_t)\,dr\,dt=i\int(\psi^*\rho_t-\psi\rho_t^*)\,dr\,dt\tag{25}$$
$$\approx i\int(\psi_xE_{0t}^*-\psi_x^*E_{0t})\,dr\,dt\approx i\int(E_0E_{0t}^*-E_0^*E_{0t})\,dr\,dt+\int(\psi_{1x}E_0^*+\psi_{1x}^*E_{0t})\,dr\,dt$$

We can transform the last term in (25), using Eq. (23). Evaluating then the quantity $-{}^1\!/_2\int|\mathbf{E}|^4d\mathbf{r}$, using Eq. (24) and taking into account the first nontrivial terms, we get

$$S=i\int(E_0E_{0t}^*-E_0^*E_{0t})\,dr\,dt+\int(|E_{0x}|^2-{}^1\!/_2|E_0|^4)\,dr\,dt+\int|\nabla_\perp E_0|^2\,dr\,dt$$
$$-\int|E_0|^2|\nabla_\perp\psi_0|^2\,dr\,dt+{}^1\!/_2\int\frac{\partial}{\partial t}|E_0|^2\left(\int_0^x dx\int_0^x\frac{\partial}{\partial t}|E_0|^2dx\right)dr\,dt.\tag{26}$$

Evaluating the integrals in (26), we find finally

$$S=4\sqrt{2}\int\left(B^2\Phi_t-\frac{B^6}{12}+\frac{1}{2}\alpha_1B^2(\nabla_\perp\Phi)^2+\frac{1}{2}\alpha_2(\nabla_\perp B)^2-\frac{1}{2}\alpha_3|B_t|^2\right)dr_\perp\,dt,$$
$$\alpha_1=1-\int_{-\infty}^{+\infty}\frac{(\arcsin\mathrm{th}\,x)^2-\pi^2/4}{\mathrm{ch}^2x}dx=5.0,$$
$$\alpha_2=\int_{-\infty}^{+\infty}\frac{x^2(\mathrm{ch}^2x-3)}{\mathrm{ch}^4x}dx=2-\frac{\pi^2}{6}\approx0.32,$$
$$\alpha_3=2\int_{-\infty}^{\infty}\frac{x^2}{\mathrm{ch}^4x}dx=\frac{2}{3}\left(\frac{\pi^2}{3}-2\right)\approx0.8599.\tag{27}$$

We normalize the variables:

$$r\to r\sqrt{\alpha_2},\quad\Phi\to\Phi\sqrt{\frac{\alpha_2}{\alpha_1}},\quad t\to t\sqrt{\frac{\alpha_2}{\alpha_1}}.$$

In the new variables the equations for Φ and B have the form

$$\frac{\partial}{\partial t}B^2+\nabla_\perp B^2\nabla_\perp\Phi=0,$$

$$2B(\Phi_t+{}^1/_2(\nabla_\perp\Phi)^2)={}^1/_2B^5+\nabla_\perp^2 B-\gamma B_{tt}, \tag{28}$$
$$\gamma=\alpha_1\alpha_3/\alpha_2.$$

In the static approximation when the term B_{tt} is neglected, it is convenient to introduce the complex variable $\chi = Be^{i\Phi}$. The equation for χ has the form

$$2i\chi_t+\nabla_\perp^2\chi+|\chi|^4\chi=0. \tag{29}$$

Equations (28) allow us to solve the problem of the instability of a plane plasma soliton against long-wavelength transverse perturbations (see also[7]). Putting

$$B=B_0+\delta Be^{i\Omega t+ikx},\quad \delta B\ll B_0,$$

we find

$$\Omega^2=-\frac{k^2(2B_0^4-k^2)}{4+\gamma k^2}. \tag{30}$$

Equation (30) shows that in the whole wavenumber range $0 < k < B_0^2$ there is an instability. In the static case $B_0^2 \ll 1$ the maximum of the growth rate is reached for $k \sim B_0^2$, but the criterion for the applicability of Eq. (30) is the condition $k \ll B_0^2$, so that it only gives an estimate of the maximum growth rate. In terms of the physical variables the maximum growth rate $\gamma \sim w\omega_p/nT$ is reached for dimensions of the perturbation of the order of the width of the soliton.

In the case $B_0^2 \gg 1$ Eq. (30) is applicable only for "long-wavelength" perturbations ($k^2 \ll 1$, in terms of the physical variables $\lambda_\perp \sim r_D(M/m)^{1/2}$). When $k^2 \sim 1$ a maximum growth rate $\gamma_{max} \sim \omega_p(mw/MnT)^{1/2}$ is reached. However, as far as order of magnitude is concerned, Eq. (30) is valid up to $k \sim B_0^2$. In the range $1 < k < B_0^2$ the growth rate is practically constant: $\gamma \sim \gamma_{max}$.

We consider now the non-linear stage of the development of the soliton instability. We shall assume that $k^2 \ll 1$, $k^2 \ll B_0^4$. Introducing $n = B^2$, $v = \nabla_\perp\Phi$, we get a set of hydrodynamic equations with a negative pressure and with the adiabatic index $\gamma = 3$:

$$\frac{\partial n}{\partial t}+\mathrm{div}_\perp n\mathbf{v}=0,\quad \frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\nabla_\perp\mathbf{v}=\nabla_\perp\frac{n^2}{4}. \tag{31}$$

In the case of a single transverse coordinate, $\nabla_\perp = \partial/\partial y$, the set (31) can be transformed to the form

$$\frac{\partial z}{\partial t}+z\frac{\partial z}{\partial y}=0,\quad z=v+\frac{i}{\sqrt{2}}n. \tag{32}$$

The general analytic solution of Eq. (32) is of the form

$$z=F(y+z(t_0-t)),$$

F is an arbitrary analytical function. This solution clearly describes collapse, if we choose for F a function with a pole on the real axis. In the simplest case, choosing $F(\xi) = -C/\xi$, we find

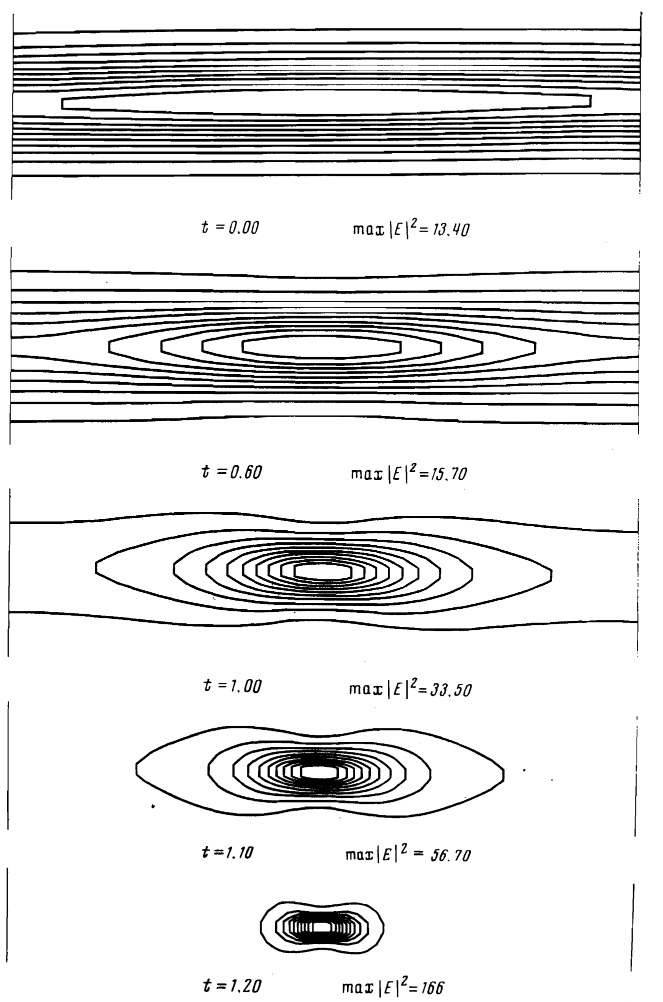

FIG. 2

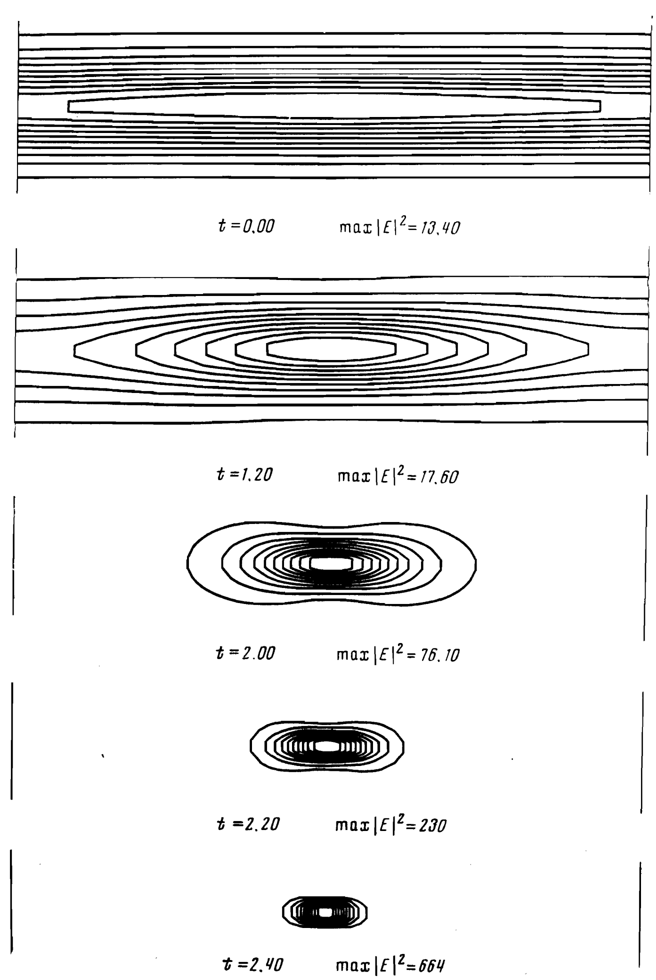

FIG. 3

$$n=\begin{cases}\frac{\sqrt{2}}{t_0-t}[4C(t_0-t)-y]^{1/2}, & y<4C(t_0-t)\\ 0, & y^2>4C(t_0-t)\end{cases} \quad (33)$$

This solution describes the collapse of a finite soliton section with half-width $y = (4Ct_0)^{1/2}$. After a time t_0 the soliton is collected together in the point $y = 0$; its amplitude in that point increases as $1/(t_0 - t)^{1/2}$.

We note that in the points $y = \pm\{4C(t_0 - t)\}^{1/2}$ the validity of Eqs. (31) breaks down, and near them it is necessary to use the more exact Eqs. (28). The role of the end points will increase as collapse is approached; this limits the time during which the solution (33) is applicable. However, increasing the initial length of the soliton we can attain collapse up to rather large amplitudes in the framework of the solution (33).

We studied the development of the instability of a plane soliton against the development of plane excitations, using an electronic computer. We show in Figs. 2 and 3 successive phases of the development of the collapse of a plane soliton. Figure 2 refers to the static subsonic case—we used Eq. (5) as starting point. Figure 3 refers to the general case and describes the collapse of a soliton in the framework of the set of Eqs. (2) and (3). In both cases we used as the initial condition a slightly perturbed soliton

$$E=\frac{E_0}{\operatorname{ch}(x/\sqrt{2})}\left(1+0.1\sin\frac{\pi x}{l}\right).$$

Both cases are close to the limit of applicability of Eqs. (31).

In the physically more interesting case of two spatial coordinates it is no longer possible to find a general solution of the set (31) in so simple a way. However, in the axially symmetric case it is possible to find a self-similar solution which is analogous to the solution (33). It is of the form

$$n=\frac{1}{(t_0-t)^{2/3}}n\left(\frac{r}{(t_0-t)^{1/3}}\right),\quad v=\frac{1}{(t_0-t)^{2/3}}v\left(\frac{r}{(t_0-t)^{1/3}}\right),$$

$$n(\xi)=\begin{cases}\left(n_0-\frac{4}{9}\xi^2\right)^{1/2}, & |\xi|<\frac{3}{2}n_0,\\ 0, & |\xi|>\frac{3}{2}n_0,\end{cases}\qquad v(\xi)=\frac{1}{3}\xi. \quad (34)$$

Solution (34) describes the collapse of a disk-shaped soliton into a point. We note also that in the process of this collapse the relation $L_\perp B^2 = \text{const}$ is satisfied so that the condition for the applicability of the quasi-planar approximation $1/L_\perp \ll B^2$ is made easier to satisfy as $B^2 \to \infty$.

As in the case of the spherical collapse, the collapse of a quasi-planar soliton can proceed only up to a level of amplitudes $w/nT \sim 1$ after which strong Landau damping leads to an absorption of the Langmuir oscillation energy. The problem of the nature of the plasma heating when collapse is taken into account remains yet to be solved.

We note also that the structures of the Langmuir collapse, studied by us, are not the only ones which are possible; for instance, in refs. [8,9] we described a two-dimensional unsymmetric dipole-type collapse with transverse and longitudinal dimensions of the same order of magnitude. We may assume that such a kind of collapse occurs in the final stage of soliton collapse.

[1)] This method is based upon a paper by Vlasov, Petrishchev, and Talanov. [4]

[2)] In this sense the results of our paper are in contrast to the conclusions given in [5].

[1] V. E. Zakharov, Zh. Eksp. Teor. Fiz. **62**, 1745 (1972) [Sov. Phys.-JETP **35**, 908 (1972)].

[2] A. A. Vedenov and L. I. Rudakov, Dokl. Akad. Nauk SSSR **159**, 767 (1964) [Sov. Phys.-Doklady **9**, 1073 (1965)].

[3] L. M. Degtyarev, V. E. Zakharov, and L. I. Rudakov, Preprint No. 35, 1974, Inst. Appl. Math. Acad. Sc. USSR, No. 1449-74.

[4] V. N. Vlasov, V. A. Petrishchev, and V. I. Talanov, Izv. vuzov, Radiofizika **14**, 1353 (1971) [Radiophys. Qu. Electron. **14**, 1062 (1974)].

[5] A. G. Litvak, G. M. Fraĭman, and A. D. Yunakovskiĭ, ZhETF Pis. Red. **19**, 23 (1974) [JETP Lett. **19**, 13 (1974)].

[6] L. M. Degtyarev, V. G. Makhan'kov, and L. I. Rudakov, Zh. Eksp. Teor. Fiz. **67**, 533 (1974) [Sov. Phys.-JETP **40**, 264 (1975)].

[7] V. E. Zakharov and A. M. Rubenchik, Zh. Eksp. Teor. Fiz. **65**, 997 (1973) [Sov. Phys.-JETP **38**, 494 (1974)].

[8] V. E. Zakharov, A. F. Mastryukov, and V. S. Synakh, ZhETF Pis. Red. **20**, 1 (1974) [JETP Lett. **20**, 1 (1974)].

[9] L. M. Degtyarev and V. E. Zakharov, ZhETF Pis. Red. **20**, 365 (1974) [JETP Lett. **20**, 164 (1974)].

Translated by D. ter Haar
17