

Viscous resistance of vortices in type-II superconductors

I. N. Goncharov, G. L. Dorofeev, A. Nichitiu, L. V. Petrova,
D. Fricsovszky, and I. S. Khukhareva

Joint Institute for Nuclear Research

(Submitted June 23, 1974)

Zh. Eksp. Teor. Fiz. 67, 2235-2245 (December 1974)

The resistance $\rho_f(H, T)$ to viscous vortex motion is measured in recrystallized strips of the superconducting alloy Nb-80% Zr in the range from 1°K to T_c and throughout the whole magnetic-field range. The temperature dependence of the coefficient of viscosity of the moving vortices is derived from the measurement and is compared with the microscopic theory. The behavior of $\rho_f(H)$ at all magnetic field values is compared with the theory as $T \rightarrow T_c$. To explain the dependence of the depth and position of the minimum on the $\rho_f(T)$ curve on the magnetic field strength, the thermal viscosity coefficient is calculated for a system of independent vortices and compared with the experimental results.

INTRODUCTION

From the point of view of the dependence of the differential resistivity (DR) on the external magnetic field at constant temperature, the entire interval from 0 to $H_{c2}(t)$ (where $t = T/T_c$) can be arbitrarily divided into three parts. In weak fields, a linear relation is observed, adequately described by a theory that considers the motion, under the influence of the transport current, of vortices that do not interact with one another. For this region of fields an expression for the effective conductivity $\sigma_{\text{eff}} \equiv \rho_f^{-1} = (dI/dj)^{-1}$ in terms of the temperature and the average magnetic field in the sample was obtained in theoretical papers^[1-5] under the simplifying assumption that there are no pinning forces.

In the other limiting case $H \rightarrow H_{c2}(t)$, there is also a linear dependence of the DR on H . The microscopic theory that describes the behavior of the effective conductivity in this region has been constructed under the assumption of gapless superconductivity^[6] and is therefore applicable, strictly speaking, only in a very narrow vicinity of $H_{c2}(t)$. In experiment, the linear section occupies a rather noticeable field interval, but its slope varies with temperature, in good agreement with the theoretically predicted dependence^[7-11].

The least investigated, both experimentally and theoretically, is the region of medium fields. In a number of studies^[8-11], the measurements of ρ_f/ρ_n were carried out in the entire range of magnetic fields, and in this case a detailed study was made of the laws governing the behavior only in one of the limiting cases, either $H \rightarrow 0$ or $H \rightarrow H_{c2}(t)$. In the immediate vicinity of T_c , for fields satisfying the condition $1 \gg 1 - H/H_{c2}(t) \gg 1 - t$, the following relation was obtained^[4,12]:

$$\sigma_{\text{eff}} \propto [1 - H/H_{c2}(t)]^n.$$

However, the condition for the applicability of this formula, even at $t \approx 0.995$, does not make it possible to cover the entire interval of average fields, since the limitation $1 - H/H_{c2}(t) \ll 1$ corresponds at best to $H/H_{c2}(t) \approx 0.7$. Danilov, Kupriyanov, and Likharev^[13] have calculated, for temperatures on the order of T_c , the values of $\rho_f(h)$ (for all values of H from $H_{c1}(t)$ to $H_{c2}(t)$) from the non-stationary Ginzburg-Landau equations, neglecting the so-called anomalous terms. In the case of dirty superconductors ($l \ll \xi$) without paramagnetic impurities, the values of ρ_f for weak fields coincide with those calculated in^[2], while at $H \approx H_{c2}(t)$, the result of Maki is obtained^[6]. Allowance for anomalous terms, in the opinion of the authors of^[13], should lead to a decrease of ρ_f at a given field.

So far we have dealt only with the electromagnetic mechanism of the dissipation due to the motion of the fluxoids under the influence of the transport current. Clem^[14] has proposed an additional thermal dissipation mechanism, which explains the appearance of a minimum on the $\rho_f(T)$ curve in the case of superconducting alloys with small mean free paths^[15], but considered only a single isolated vortex. The model was subsequently improved by taking into account the interaction of the vortices via the electric fields that are connected with their motion^[16], thus providing a qualitative explanation of the experimentally obtained dependence of the temperature of the $\rho_f(T)$ minimum on H ^[17,18].

We have measured the DR of an Nb alloy with a large Zr content at temperatures from 1.0°K to T_c in the entire interval of the magnetic fields from 0 to $H_{c2}(t)$. At fixed values of the temperature, we plotted ρ_f/ρ_n against the field, or else the temperature was scanned with the magnetic field fixed. The results were used to determine the temperature dependence of the viscosity coefficient of the moving vortices, and to compare it with the theory. The dependence of ρ_f/ρ_n on H was also compared with the theory at various temperatures. Extending the idea of the thermal dissipation mechanism to the aggregate of moving vortices, and neglecting the electromagnetic interaction of the vortices, we have calculated the dependence of the depth of the minimum on the $\rho_f(t)$ curve on the magnetic field, and compared the results with the experimental data.

EXPERIMENTAL PROCEDURE

The samples were foil strips ($0.01 \times 0.16 \times 5$ cm) of an alloy of Nb with 80% Zr, subjected after mechanical working to a recrystallization annealing at 1000°C, followed by rapid quenching^[19]. To measure the current-voltage characteristics, the sample was secured in a holder with clamped potential contacts; the holder was placed in the magnetic field perpendicular to the plane of the sample and to the transport current. The current-voltage characteristics were plotted with an x-y recorder using a photoelectronic amplifier of the F-118 type. The DR was determined graphically from the slope of the linear section of the current-voltage characteristic.

In some cases, however, the presence of a linear section does not exclude integral heating of the sample as a result of the power released in the sample, nor on the influence of this heating on the value of the DR^[20]. The corresponding values of the DR were therefore subjected to corrections that depended principally on I_c , $\partial I_c/\partial T$,

and the heat-conduction and heat-transfer coefficients. When calculating the corrections for HeII, we used the thermal resistance value $R_K^I = 25T^{-3}[\text{K}\cdot\text{cm}^2/\text{W}]$, which is close to the maximum possible for Nb-80% Zr [21]. For He I we used a thermal-resistance coefficient that is linear in the temperature and takes on the values $R_K^I = 5^\circ\text{K}\cdot\text{cm}^2/\text{W}$ at $T = 4.2^\circ\text{K}$ and $R_K^I = 20^\circ\text{K}\cdot\text{cm}^2/\text{W}$ at $T = 2.2^\circ\text{K}$ [22]. In the region $T > 4.2^\circ\text{K}$, where the measurements were performed in vapor, we used the value $R_K^{\text{vap}} = 220^\circ\text{K}\cdot\text{cm}^2/\text{W}$, calculated from a comparison of the current-voltage characteristics of the sample for $T = 4.2^\circ\text{K}$, plotted in liquid and in vapor. Table I lists the relative values of the thermal corrections for three temperatures in different magnetic fields.

RESULTS AND DISCUSSION

1. Dependence of the Differential Resistance in the Field

Figure 1 shows the results of the measurements of ρ_f/ρ_n as a function of H at different fixed values of the temperature. A number of characteristic singularities can be observed in these measurements. First, in weak fields (Fig. 1a), $\rho_f(H)$ is quite linear at all temperatures. Second, in the field interval from 0 to 60 kOe, a characteristic "hump" is observed at low temperatures (curves 6 and 7 in Fig. 1b); this hump was noted earlier by Kim [8] in alloys having high values of the parameter κ . Third, near $H_{C2}(t)$ there is observed a small plateau on the dependence of ρ_f/ρ_n on H (curves 3, 4, and 6 in Fig. 1b), which corresponds to the peak effect on the $j_C(H)$ curves. The gaps on the curves correspond to places where the determination of ρ_f is extremely difficult in the case of our samples.

The presence of a linear dependence of ρ_f/ρ_n on H in weak fields makes it possible to determine, on the basis of these measurements, the viscosity coefficient η of the moving vortices and its temperature dependence. Indeed, η and the DR are connected by the simple relation [8] $\eta(t) = \Phi_0 H / c^2 \rho_f(t)$. If $\rho_f(t)/\rho_n = b_0(t)H / H_{C2}(0)$, then

$$\eta(t) = \frac{1}{b_0(t)} \frac{\Phi_0 H_{C2}(0)}{c^2 \rho_n}$$

The value of b_0 at fixed temperature is determined

T, K	Correction %			
	H=24kOe	18 kOe	14 kOe	8.3kOe
1	3.5	5.5	7.0	9.5
2.2	3.0	4.0	4.5	6.0
4.3	5.0	—	20.0	35.0

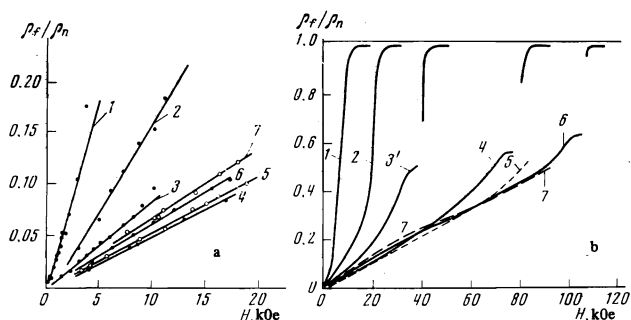


FIG. 1. Experimental plot of ρ_f/ρ_n against H for fixed values of the temperature (in $^\circ\text{K}$): 1— $T = 7.6$, 2— $T = 7.1$, 3— $T = 6.5$, 3'— $T = 6.0$, 4— $T = 4.2$, 5— $T = 3.4$, 6— $T = 2.0$, 7— $T = 1.37$.

graphically from the slope of the initial linear section in the experimental plot of ρ_f/ρ_n against H ; $b_0(t)$ describes completely the temperature variation of η .

A different form is frequently used in the literature, namely $\rho_f(t)/\rho_n = b(t)H/H_{C2}(t)$. Then

$$\eta(t) \sim H_{C2}(t)/b(t).$$

In the theoretical papers [1-5] that consider the motion of single vortices under the influence of the transport current, the $\sigma_{\text{eff}}(H, T)$ dependence is represented in the general form

$$\sigma_{\text{eff}}/\sigma_n = \beta(t)H_{C2}(t)/H.$$

The coefficient $\beta(t)$ was determined in two limiting cases: $T \rightarrow T_c$ and $T = 0$. It is easily seen that

$$\beta(t) = \frac{1}{b(t)} = \frac{1}{b_0(t)f(t)} = \frac{\eta(t)}{f(t)}, \quad f(t) = \frac{H_{C2}(t)}{H_{C2}(0)}.$$

Curve 1 in Fig. 2a was calculated from the formula [3] $\beta(t) = 1, 1(1-t)^{-1/2}$, curve 2 corresponds to $\beta(t) = 1.47$ [2]; the point on the $t = 0$ axis is $\beta(0) = 0.9$ [5].

Figure 2a shows also the experimental values of $1/b(t)$ obtained by different workers. We note that some of the points were obtained by us from published $\rho_f(H)/\rho_n$

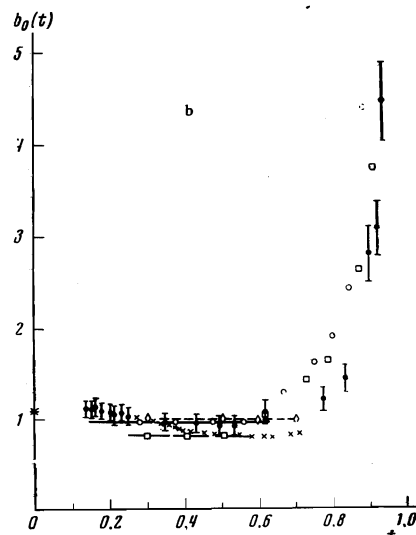
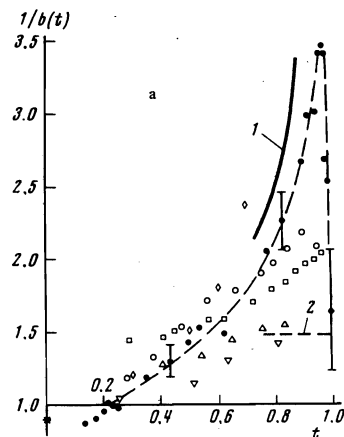


FIG. 2. Dependence of the viscous-friction coefficient of moving vortices on the temperature: a) $1/b(t) \propto \eta(t)/H_{C2}(t)$, b) $b_0(t) \propto \eta(t)$. Points \bullet —our experimental data for $\text{Nb}_{20}\text{Zr}_{80}$, \diamond — $\text{Nb}_{50}\text{Ta}_{50}$ [8], \square — $\text{Pb}_{76}\text{In}_{24}$ [9], \triangle —V-B [10], $-V-A$ [10], \circ — $\text{Nb}_{80}\text{Mo}_{20}$ [11], \times — $\text{Pb}_{60}\text{Tl}_{40}$ [18]. Curve 1—theoretical from [3], curve 2— from [2]; asterisk—theoretical estimate [5] at $t = 0$.

curves, and this explains their scatter to some degree. However, the difference between the temperature dependences of $1/b(t)$ of different alloys exceeds this scatter and may be due to the difference in the temperature dependence of $H_{c2}(t)$, which enters in the definition of $b(t)$ together with $\eta(t)$. In the region $t > 0.6$ we see a certain correlation between the position of the $1/b(t)$ curve and the value of the parameter $\kappa_1(1)$, the values of which for the given samples are presented below:

Sample:	Nb ₂₀ Zr ₈₀	Pb ₇₅ In ₂₅	Nb ₈₀ Mo ₂₀	V-B	V-A
$\kappa_1(1)$:	64	4.8	4.1	2.3	1.9
Source:	present work [9]	[9]	[11]	[10]	[10]

As seen from Fig. 2a, the experimental plot of $1/b(t)$ for Nb-80% Zr agrees well with the theory [3,5] at $t \rightarrow 0$ and $t \lesssim 0.95$. In the immediate vicinity of T_c , an abrupt decrease is observed on the $1/b(t)$ curve [23], so that as $t \rightarrow 1$ the experimental points approach the limiting value $1/b = 1.47$. Recognizing that the formula that determines the curve 1 was obtained with only the anomalous terms in the expression for the conductivity taken into account, and that the limiting estimate was made by discarding these terms, it must be assumed that when T_c is approached there comes into play a mechanism that suppresses strongly (at least in the case of our samples) the contribution of the anomalous terms.

Figure 2b shows the plot of $b_0(t) \propto 1/\eta(t)$ for various samples. As mentioned above, for a fixed temperature b_0 is determined experimentally from the slope of the initial linear section of the $\rho_f(H)$ curve. A characteristic feature of the behavior of $b_0(t)$, first noted by Kim et al. [8], is the tendency to assume a constant value at low temperatures. For Nb₅₀Ta₅₀, for example, the $b_0(t)$ remains constant all the way to $t \approx 0.7$. For all the samples, the constant value of b_0 is close to unity. A theoretical estimate yielded $b_0(0) = 1.1$. [5] Gilchrist and Monceau [17] obtained values $b_0 = 0.82 \pm 3\%$ and $b_0 = 0.89 \pm 4\%$ from high-frequency measurements of the DR of samples of Pb₅₀In₅₀ and Pb₉₀In₁₀.

Figure 3 shows plots of ρ_f/ρ_n against the relative field, for different temperatures, in the entire interval from 0 to 1. Curve 1 duplicates the results of the calculation of ρ_f from the nonstationary Ginzburg-Landau equations, by neglecting the anomalous terms for temperatures on the order of T_c [13]. This curve has asymptotes with slopes 0.68 ($b = 1.47$) in weak fields [2] and 2.5 in strong fields [6]. Curves 2a, 2b, and 2c were calculated respectively for the temperatures $t = 0.924$, 0.977, and 0.989 in accordance with the formula

$$\frac{\sigma_{\text{eff}}}{\sigma_n} - 1 = 0.18x^{1/2}(1-t)^{-1/2} + 1.25x + 0.27[x(1-t)]^{1/2} \ln \frac{5x}{1-t} + \alpha_1 x \left(\frac{x}{1-t}\right)^{1/2} + 0.1\alpha_2 x \left(\frac{x}{1-t}\right)^{1/4},$$

which is valid for $1-t \ll x \ll 1$, where $x = (1 - H/H_{c2})/(1 - 1/2\kappa^2)$, and α_1 and α_2 is of the order of unity.

In the region of weak fields, for the same temperatures, the figure shows the asymptotic forms (2a', 2b', and 2c') calculated from the formula

$$\frac{\langle H \rangle}{H_{c2}} \frac{\sigma_{\text{eff}}}{\sigma_n} = 1.1(1-t)^{-1/2} + 0.81[1 + \alpha_3(1-t)^{-1/2} + \alpha_4(1-t)^{-1/4}],$$

where α_3 and α_4 are of the order of unity [4]. The corresponding values of b lie above curve 1 of Fig. 2a (see [23]). Unfortunately, the large scatter of the experimental points in fields close to $H_{c2}(t)$ (this is partially due to the manifestation of the peak effect) does not per-

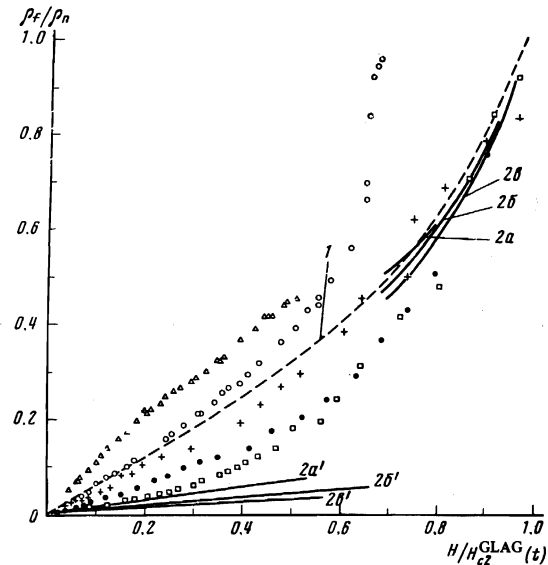


FIG. 3. Plot of ρ_f/ρ_n against $H/H_{c2}^{\text{GLAG}}(t)$: 1—theoretical curve [13], 2—theoretical curves for t equal respectively to 0.924 (a) 0.977 (b), and 0.989 (c) [4]. The experimental points for Nb-80%Zr were obtained at the following temperatures: +— $t = 0.989$, □— $t = 0.977$, ●— $t = 0.924$, ○— $t = 0.535$, △— $t = 0.175$.

mit a comparison with the theory in this region. It should be noted, however, that to reconcile the calculated curves 2 with the corresponding asymptotic forms in weaker fields, it is obviously necessary to use as lower bounds the still higher values $H/H_{c2}^{\text{GLAG}}(t)$ than in the figure, where it is assumed that

$$1 \gg (1 - H/H_{c2}^{\text{GLAG}}(t))_{\text{max}} = 0.3.$$

The systematic deviation towards higher values of ρ_f/ρ_n in the case $t = 0.535$ and $t = 0.175$ in fields $H/H_{c2}^{\text{GLAG}}(t) < 0.8$ is due to the paramagnetic pair-breaking effect [19,24]. The deviation of the experimental points for $t = 0.977$ and $t = 0.924$ from curve 1 towards lower values of ρ_f/ρ_n is in agreement with the predictions made in [13]. Indeed, as seen from Fig. 2a, the contribution of the anomalous terms increases with increasing temperature (up to the maximum), and accordingly, at a given field, the points for $t = 0.977$ deviate from curve 1 more strongly than for $t = 0.924$.

2. Temperature Dependence of the Differential Resistivity

Since the entropies of the superconducting and the normal phases are not equal, the transition from one phase to the other under the influence of the magnetic field leads to a change in the temperature. Therefore the motion of the normal regions in a superconductor (for example, of a magnetic spot with dimensions much larger than the coherence length) is accompanied by the appearance of temperature gradients with irreversible losses [25]. It can be assumed that in the case of dirty type-II superconductors these gradients are produced on moving vortices, since the geometrical uncertainty of the heat sources is equal to the mean free path of the normal electrons, and in this case the condition $l \ll \xi$ is satisfied. We should then have a thermal dissipation mechanism similar to that occurring when large regions of the normal phase move in type-I superconductors. To estimate the contribution of the thermal dissipation, Clem has considered a model in which a single moving vortex is represented in the form of a cylinder [14]. The equation of the thermal conductivity for one cylinder, ex-

pressed accurate to terms linear in the velocity, has the solution

$$T(r) = T_0 - \frac{T_0 a (S_n - S_s)}{K_n + K_s} \begin{cases} rv/a, & |r| \leq a \\ (rv)/a/|r|^2, & |r| > a \end{cases} \quad (1)$$

where \mathbf{r} is the radius vector from the center of the cylinder, a is the radius of the cylinder, \mathbf{v} is its velocity, S_n and S_s are the entropy per unit volume inside and outside the cylinder, T_0 is the average temperature of the sample, and K_n and K_s are the thermal conductivities inside and outside the cylinder.

The irreversible thermal losses per unit cylinder length and per unit time are equal to

$$W_q = 2 \int_{-\pi/2}^{+\pi/2} \frac{T_0 a^2 (S_n - S_s)^2 v^2 \cos^2 \varphi d\varphi}{K_n + K_s} = \frac{\pi a^2 T_0 (S_n - S_s)^2 v^2}{K_n + K_s} = \eta_q v^2, \quad (2)$$

where η_q is the coefficient of thermal viscosity per unit cylinder length, and φ is the angle between the radius vector from the center of the cylinder and the velocity in a plane perpendicular to the cylinder. An increase of the magnetic field leads to an increase of the density of the vortices, and consequently to a decrease in the temperature gradients on the vortex due to the decrease of the distances between the sources and the absorbers of the heat, namely the surfaces of the cylinders. Consequently, formula (2) determines the upper bound of the thermal dissipation in the limit as $H \rightarrow 0$.

Let us examine the dependence of the coefficient of thermal viscosity on the magnetic field. We put first, for simplicity, $K_s = K_f$. Then, in view of the linearity of the equation of the thermal conductivity over the sources, the temperature distribution for the lattice of normal cylinders will be the sum of the distributions for the individual cylinders. If the condition $d_\varphi^2/4a^2 \gg 1$ is satisfied (where d_φ is the distance between the centers of the cylinders), we can restrict the sum to the closest cylinders. We then obtain for the temperature on the cylinder surface the expression

$$T(\varphi) = T_0 - \frac{T_0 a v (S_n - S_s)}{2K_n} \left\{ 1 - 4 \frac{a^2}{d_\varphi^2} \right\} \cos \varphi, \quad (3)$$

from which it follows that

$$\eta_q = \frac{\pi a^2 T (S_n - S_s)^2}{2K_n} \left\{ 1 - \frac{\sqrt{3}}{\pi} \frac{H}{H_{c2}(t)} \right\}. \quad (4)$$

We put $K_s = 0$, then the temperature distribution on a given cylinder will not depend on the neighboring cylinders, and the thermal coefficient of viscosity should accordingly be independent of the magnetic fields, just as in the case of a single vortex and arbitrary values of K_n and K_s . Consequently, when the thermal conductivity K_s varies from zero to K_n , the coefficient of thermal viscosity η_q will change in the range

$$\frac{\pi a^2 T (S_n - S_s)^2}{K_n} \text{ to } \frac{\pi a^2 T (S_n - S_s)^2}{2K_n} \left(1 - \frac{\sqrt{3}}{\pi} \frac{H}{H_{c2}(t)} \right).$$

To estimate the relative contribution of the thermal mechanism, let us consider the quantity $\eta_q/\eta(0)$, where $\eta(0) = \Phi_0 H/c^2 \rho_f = \Phi_0 H_{c2}(0)/c^2 \rho_n$ is the electromagnetic viscosity coefficient determined from the empirical relation $\rho_f \approx \rho_n H/H_{c2}(0)$, which is valid in weak fields at low temperatures. Assuming $a = \xi$ and recognizing that

$$H_c(t) = H_c(0) (1-t)^2, \\ \xi^2 = \frac{\Phi_0}{2\pi H_{c2}(t)}, \quad H_{c2}(t) = H_{c2}(0) (1-t)^2, \\ S_n - S_s = -\frac{H_c(t)}{4\pi} \frac{dH_c(t)}{dt} \frac{1}{T_c}$$

and that for dirty type-II superconductors we have

$$H_c(0) = 2.42 \gamma^2 T_c, \quad H_{c2}(0) = 3.06 \cdot 10^4 \rho_n \gamma T_c, \quad \kappa = 7.5 \cdot 10^3 \rho_n \gamma^2,$$

and furthermore expressing ρ_n in terms of the thermal conductivity of the normal electrons with the aid of the Wiedemann-Franz law, we obtain

$$\frac{\eta_q(t)}{\eta(0)} = 2.115 \frac{(1-t)^2 K_{en}}{K_n + K_s} \quad \text{as } K_s \rightarrow 0, \quad (5) \\ \frac{\eta_q(t)}{\eta(0)} = 2.115 \frac{(1-t)^2 K_{en}}{K_n + K_s} \left[1 - 0.55 \frac{H}{H_{c2}(t)} \right] \quad \text{as } K_s \rightarrow K_n.$$

In the case of our alloy, the total thermal conductivity¹¹, measured at $T = T_c$, practically coincided with K_{en} calculated in accordance with the Wiedemann-Franz law. On this basis, we have put $K_n \approx K_{en}$ also for lower temperatures. In addition, for estimates of the minimum value of $\eta_q(t)$ we assumed $K_s = K_n \approx K_{en}$.

It is seen from (5) that the relative contribution of the thermal dissipation mechanism, which is equal to zero at $t = 0$ and $t = 1$, can reach appreciable values (50%) at medium temperatures.

Figure 4 shows a plot of ρ_f/ρ_n against T for different values of the magnetic field. The dashed curves were drawn through the experimental points after introducing the appropriate corrections referred to above. The solid lines are the results of calculation by means of the formula

$$\frac{\rho_f}{\rho_n} = \frac{H}{H_{c2}(0)} \frac{1}{1 + \eta_q(t)/\eta(0)},$$

where $\eta_q(t)/\eta(0)$ is calculated from formula (5) for $K_s = K_n$. As seen from the figure, the experimental minima of ρ_f are located at lower temperatures than the calculated ones. This discrepancy, obviously, is due to the fact that in the calculation no account was taken of the temperature dependence of that part of ρ_f which is due to the electromagnetic dissipation.

Figure 5 shows a comparison of the relative experimentally-obtained depth of the minimum^[9, 15, 17, 18] with the calculated value. The abscissas represent the relative field and the ordinates the quantity $\Delta\eta/\eta(0, 1)$, which was determined from the experimental dependence of

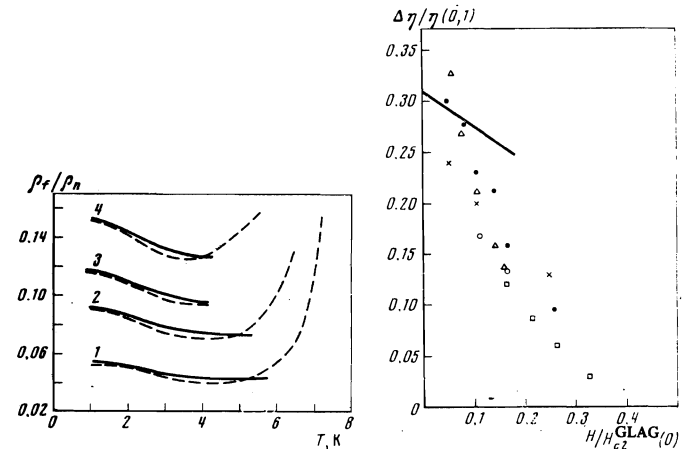


FIG. 4.

FIG. 5.

FIG. 4. Temperature dependence of ρ_f/ρ_n : 1) $H = 8.34$ kOe, 2) $H = 13.9$ kOe, 3) $H = 18$ kOe, 4) $H = 24$ kOe. Dashed lines—experiment, solid—calculated from formula (5) as $K_s = K_n$.

FIG. 5. Dependence of the depth of the minimum on the magnetic field: \bullet — $\text{Nb}_{20}\text{Zr}_{80}$ —present paper, \square — $\text{Pb}_{76}\text{In}_{24}$ from [9], \times — $\text{Pb}_{60}\text{Tl}_{40}$ from [15, 18], \triangle — $\text{Pb}_{50}\text{In}_{50}$ from [17], \circ — $\text{Pb}_{90}\text{In}_{10}$ from [17].

$\rho_f(t)$ by the formula $\Delta\eta/\eta(0, 1) = \rho_f(0, 1)/\rho_{f \min} - 1$. The calculated value is

$$\frac{\Delta\eta}{\eta(0, 1)} = \frac{\eta_{g \max} - \eta_g(0, 1)}{\eta_g(0, 1) + \eta(0)},$$

where $\eta_q(t)$ was obtained from (5). We see that the thermal-mechanism model accounts well for the absolute value of the depth of the minimum $\rho_f(t)$ in the region of weak fields, and deviates strongly from the experimental $\Delta\eta/\eta(0, 1)$ dependence with increasing field. This is due principally to the fact that (as indicated above), in stronger H that part of ρ_f which is due to the electromagnetic dissipation mechanism begins to increase with increasing temperature, by the same token offsetting in part the decrease of the total DR as the result of the thermal mechanism (starting with a certain field, no minimum is observed at all on the $\rho_f^{\text{exp}}(t)$ curve at $H = \text{const}$. In our calculation, however, this increase was not taken into account, so that its results should be compared only in the case of relatively small $H/H_{c2}^{\text{GLAG}}(0)$.

CONCLUSION

Our detailed investigation of $\rho_f(H, T)$ in samples with large κ allows us to draw the following conclusions:

First, we have shown that in the region of relatively weak $H/H_{c2}^{\text{GLAG}}(t)$, where the interaction between the vortices is weak and it is possible to determine the viscosity coefficient of the moving vortices, all the way to temperatures that differ from T_c by several per cent, the modern microscopic theory^[3,4] yields the correct $\eta(t)$ dependence, but the numerical coefficients of the theory call for refinement. At still higher temperatures, an appreciable discrepancy is observed between experiment and theory in the temperature dependence of η . The latter circumstance is possibly due to the presence in our samples^[19] of ω -phase particles that differ in composition from the matrix. For $T \rightarrow 0$, we have observed good quantitative agreement between the theoretical and experimental values of η .

Second, it was reliably established that in the region of the lowest temperatures, in fields $H/H_{c2}(0) > 0.2-0.3$, the $\rho_f(H)$ curve dips downward from the initial straight line with a slope $b_0 = (\rho_f/\rho_n)/[H/H_{c2}(0)] \approx 1.1$, and the tangent to the $\rho_f(H/H_{c2}(0))$ curve, passing through the origin, has a slope ~ 0.9 . This behavior has found no theoretical explanation as yet.

Third, it was observed that near $H_{c2}^{\text{exp}}(t)$, where there is a peak on the $j_c(H)$, the growth of the resistance $\rho_f(H)$ slows down appreciably. This behavior of ρ_f reveals that important feature of the dynamic connection between the moving vortices and the pinning centers of the vortices.

Fourth, to describe the observed experimental minimum on the $\rho_f(T)$ curve at $H = \text{const}$, within the framework of the phenomenological model (14) that describes the additional dissipation mechanism due to the appearance of a local temperature gradient, the calculation was performed for a single moving vortex with account taken of the influence of the neighboring vortices. Assuming that the thermal conductivity is the same outside and inside the vortex, explicit expressions were obtained for $\rho_f(T)$, including the dependence on the magnetic field. A comparison with experiment has shown fair agreement in relatively weak fields, $H/H_{c2}^{\text{GLAG}}(0)$, and at stronger fields the calculated depth of the minimum turns out to be larger than the observed one. This indicates the need for taking into account the temperature dependence of

also that part of ρ_f which is due to the electromagnetic dissipation mechanism.

In conclusion, the authors are sincerely grateful to B. T. Geilikman, V. V. Danilov, N. P. Kopnin, V. Z. Kresin, M. Yu. Kupriyanov, K. K. Likharev, and Yu. N. Ovchinnikov for fruitful discussions.

¹Measurements of the thermal conductivity were performed by L. P. Mezhev-Deglin and co-workers at the Solid-State Physics Institute of the USSR Academy of Sciences, for which the authors are deeply grateful.

- ¹L. P. Gor'kov and N. P. Kopnin, Zh. Eksp. Teor. Fiz. **60**, 2331 (1971) [Sov. Phys.-JETP **33**, 1251 (1971)].
- ²M. Yu. Kupriyanov and K. K. Likharev, ZhETF Pis. Red. **15**, 349 (1972) [JETP Lett. **15**, 247 (1972)].
- ³L. P. Gor'kov and N. P. Kopnin, Zh. Eksp. Teor. Fiz. **64**, 356 (1973) [Sov. Phys.-JETP **37**, 183 (1973)].
- ⁴Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **65**, 290 (1973) [Sov. Phys.-JETP **38**, 143 (1974)].
- ⁵L. P. Gor'kov and N. P. Kopnin, Zh. Eksp. Teor. Fiz. **65**, 396 (1973) [Sov. Phys.-JETP **38**, 195 (1974)].
- ⁶C. Caroli and K. Maki, Phys. Rev. **159**, 306, 1967.
- ⁷E. N. Goncharov and I. S. Khukhareva, ZhETF Pis. Red. **17**, 85 (1973) [JETP Lett. **17**, 58 (1973)].
- ⁸Y. B. Kim, C. F. Heampstead, and A. R. Strnad, Phys. Rev. **139**, A1163, 1965; Phys. Rev. Lett. **13**, 794, 1964.
- ⁹N. Ya. Forel', Zh. Eksp. Teor. Fiz. **63**, 1371 (1972) [Sov. Phys.-JETP **36**, 725 (1973)].
- ¹⁰N. Usui, T. Ogasawara, K. Yasukochi, and S. Tomada, J. Phys. Soc. Japan **27**, 574, 1969; Phys. Lett. **27A**, 529, 1968.
- ¹¹K. Noto, K. Mori, and Y. Muto, Paper at Soviet-Japanese Conf. on Low Temperature Physics, Novosibirsk (1969); Physica **55**, 362 (1971).
- ¹²A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **64**, 1096 (1973) [Sov. Phys.-JETP **37**, 557 (1973)].
- ¹³V. V. Danilov, M. Yu. Kupriyanov, and K. K. Likharev, Fiz. Tverd. Tela **16**, 935 (1974) [Sov. Phys.-Solid State **16**, 602 (1974)].
- ¹⁴John R. Clem, Phys. Rev. Lett. **20**, 735, 1968.
- ¹⁵Carl Y. Axt and W. C. H. Joiner, Phys. Rev. Lett. **21**, 1168, 1968.
- ¹⁶W. S. Chow, Phys. Rev. **188**, 783, 1969; Phys. Rev. **B1**, 2130, 1970.
- ¹⁷J. Gilchrist and R. Monceau, J. Phys. Chem. Solids, **32**, 2101, 1971.
- ¹⁸W. C. H. Joiner and J. Thompson, Solid St. Comm. **11**, 1393, 1972.
- ¹⁹I. N. Goncharov and I. S. Khukhareva, Zh. Eksp. Teor. Fiz. **62**, 627 (1972) [Sov. Phys.-JETP **35**, 33 (1972)].
- ²⁰I. N. Goncharov, G. L. Dorofeev, L. V. Petrova, and I. S. Khukhareva, JINR Preprint R8-6260 (1972).
- ²¹N. S. Snyder, Cryogenics **10**, 89 (1970); K. N. Zinov'eva, Zh. Eksp. Teor. Fiz. **60**, 2243 (1971) [Sov. Phys.-JETP **33**, 1205 (1971)].
- ²²N. D. Reeber, J. Appl. Phys. **34**, 481 (1963); A. P. Dorey, Cryogenics **5**, 146 (1965); I. N. Goncharov, Khvan Zon Su, and F. Khovanets, JINR Preprint R8-4558, Dubna (1969); Cryogenics **10**, 316 (1970).
- ²³I. N. Goncharov and I. S. Khukhareva, JINR Preprint 8-7718 (1974); Phys. Stat. Solidi (b) **65**, K67 (1974).
- ²⁴E. Helfand and N. R. Werthamer, Phys. Rev. **147**, 288, 1966.
- ²⁵A. F. Andreev and Yu. K. Dzhikaev, Zh. Eksp. Teor. Fiz. **60**, 298 (1971) [Sov. Phys.-JETP **33**, 163 (1971)].

Translated by J. G. Adashko
238