Nonequilibrium beta processes and the role of excited states of nuclei

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The process of electron capture by heavy nuclei is calculated on the basis of the gas model of the nucleus, with account taken of excited states of the final nucleus. Results of experiments on muon capture by nuclei in mesic atoms are used to determine the matrix element. The cross section for the capture of neutrinos by heavy nuclei is calculated on the basis of the same model and it is shown that at energies $E_\nu \gtrsim 30$ MeV, capture of a neutrino with appreciable probability is accompanied by emission of a neutron from the excited nucleus. This may turn out to be essential for recording neutrinos.

1. INTRODUCTION

Processes in which neutrinos take part are very important for the consideration of astrophysical problems[1]. The high penetrability of neutrinos often makes them the principal factor determining energy losses[2]. Neutrino processes occurring against the background of a state stationary in the hydrodynamical sense lead to a loss of energy and to a change in the chemical state of the substance. In a stationary state such processes, as a rule, are significant only at high temperatures ($T > 3\times 10^8$ K) and are accompanied by cooling.

If the substance is cold, so that its temperature is much lower than the temperature at which degeneracy occurs, then the only important process in which neutrinos take part is the capture of electrons by nuclei under the condition that the Fermi energy of the electrons exceeds the threshold:

$$e^- + (A, Z) \to (A, Z-1) + \nu, \tag{1}$$

$$E_e > \Delta = E(A, Z-1) - E(A, Z). \tag{2}$$

This reaction of neutronization of matter is important under astrophysical conditions was first calculated by Frank-Kamenetskiĭ[3]; the emission of neutrinos in the reaction (1) under the condition of collapse has been investigated by Zel'dovich and Guseĭnov[4,5].

If the threshold Δ is much lower than the energy for degeneracy of ultrarelativistic electrons ($\epsilon_{fe} = (\rho/10^6\mu_e)^{1/3} m_e c^2$, ρ is the density, μ_e is the number of nucleons per electron), then the process (1) occurs under nonequilibrium conditions and can be accompanied by an increase in the entropy and the temperature of the substance. In[6] it was shown that the heating of matter associated with nonequilibrium β capture is important both at the stage of gravitational collapse, and also during the stationary stage of evolution for white dwarfs of mass close to the Chandrasekhar limit. The heating determined by nonequilibrium β processes has a characteristic nature: it is always accompanied by emission of energy in the form of neutrinos, but the store of ordered energy[1)] turns out to be so large that part of it turns into heat, i.e., into energy of random motion. In[7] the thermal effect of nonequilibrium β processes is investigated and, in particular, the maximum temperature is found up to which a substance that was originally cold can be heated. In[8] and[9] calculations have been carried out of the heating of the material of a star during collapse as a result of nonequilibrium β capture.

In the process of collapse the Fermi energy of electrons increases so much that as a result of electron capture excited nuclei are formed. The total probability of capture in this case increases significantly, while the fraction of the energy lost into neutrinos is diminished. In the present paper we have calculated the probability of β capture taking into account the excitation of the final nucleus within the framework of the model of the nucleus as a mixture of Fermi gases of protons and neutrons. It is shown that the total capture probability W_e increases significantly when excited states are taken into account and for average values of Fermi energy of electrons (but with $\epsilon_{fe} \gg \Delta$) is proportional to ϵ_{fe}^6 instead of ϵ_{fe}^5 in the presence of only a single level. We have also obtained the average values of the energy of the emitted neutrino and of the energy of excitation of the product nucleus.

Using the same approximate model of the nucleus a calculation is made of the capture of energetic neutrinos by nuclei. It is shown that if the neutrino energy exceeds ~30 MeV, then a highly excited nucleus losing a neutron is produced with a significant probability.

The model considered here is applicable only in the case when the excited levels of the final neucleus are not pure quantum states but are completely mixed, and the density of the number of levels is approximately the same as in the Fermi gas model. This circumstance depends strongly on the specific nuclear type. For certain nuclei a situation is possible when the first level, transition to which is superallowed, lies sufficiently low; then right up to high energies ~60 MeV the capture of e^- or ν occurs primarily into this state and neutrons will not be emitted[2)]. But if the first superallowed transition corresponds to an energy $\gtrsim 10$ MeV, then β capture of e^- or ν in approximately half the cases can be accompanied by the emission of a neutron. A more exact taking into account of the role played by excited states of nuclei in β processes requires, of course, a separate investigation for each particular nucleus.

2. ELECTRON CAPTURE BY NUCLEI IN THE GAS MODEL OF THE NUCLEUS

If the electron energy is not sufficient to excite the nucleus, then the probability of capture of such an electron is determined by its capture into the ground state and is equal to[3]:

$$W = \frac{\ln 2}{ft_{1/2}}\left[F(\varepsilon_{fe}) - F(\Delta)\right], \tag{3}$$

where F(x) is the Fermi function

$$F(x) = \int_1^x (\varepsilon-\Delta)^2 \varepsilon (\varepsilon^2-1)^{1/2}\, d\varepsilon \approx \frac{1}{5}x^5 - \frac{1}{2}x^4\Delta + \frac{1}{3}x^3\Delta^2 \tag{4}$$

with $x, \Delta \gg 1$.

The magnitudes of ϵ and Δ are here expressed in units of $m_e c^2$.

As the electron energy increases, its capture into increasingly higher excited levels of the final nucleus becomes possible. The number of levels of medium and heavy nuclei (for example, Mn^{56}) increases very rapidly. Even at $\epsilon_{fe} - \Delta >$ MeV the number of allowed levels is very great, and their characteristics (angular momentum and parity) are so poorly known[10] that the evaluation of the total probability for the capture of a high energy electron by a heavy nucleus by a simple summation of formulas of the type of (3), (4) over all levels does not appear to be possible. We note that the choice of the Fe^{56} nucleus in the case of collapse is not accidental, since it has the greatest binding energy and is formed at the end of the thermonuclear evolution of a star. For a calculation of the probability of capture of electrons of energy $\gtrsim 10$ MeV arising in the course of collapse we therefore use the gas model of the nucleus in which it is assumed to consist of free neutrons and protons obeying Fermi statistics at zero temperature. Such an investigation is analogous to calculations of muon capture[11,12]. The nuclear radius R as a function of the atomic weight A is approximately equal to[12]

$$R \approx \frac{e^2}{2m_e c^2} A^{1/3} = 1.4 \cdot A^{1/3} 10^{-13} \text{ cm}. \tag{5}$$

The limiting values of the momenta of the protons p_0 and of the neutrons q_0 in the nucleus as a function of the charge Z and of the number of neutrons N are equal to

$$p_0 = 3\left(\frac{\pi}{3}\right)^{1/3} \frac{\hbar c}{e^2} \left(\frac{2Z}{A}\right)^{1/3} m_e c \approx 417 \left(\frac{2Z}{A}\right)^{1/3} m_e c, \tag{6}$$
$$q_0 \approx 417 \left(\frac{2N}{A}\right)^{1/3} m_e c.$$

The total probability W_e for the capture of an electron by one nucleus as a function of the phase volumes of the electrons dN_e, of the protons dN_p, of the neutrinos dN_ν and of the neutrons dN_n is equal to

$$W_e = \frac{2\pi}{\hbar} |H|^2 \int dN_\nu \, dN_p \, dN_e \, dN_n \delta\left(\sum_i p_i\right) \delta\left(\sum_i E_i\right). \tag{7}$$

In this model the laws of conservation of energy and momentum are satisfied in the interaction with a single proton, while the matrix element $|H|^2$ associated with the spin and isospin parts of the nucleon wave functions is constant. Integrating over the neutron momenta we obtain

$$W_e = \frac{2\pi}{\hbar} |H|^2 V_n \int dN_\nu \, dN_p \, dN_e \delta(E_e + E_p - E_\nu - E_n), \tag{8}$$
$$E_n = E_p + \frac{p_{e\nu}^2}{2m} + \frac{p_p p_{e\nu} z_p}{m}, \quad p_{e\nu} = |\mathbf{p}_e - \mathbf{p}_\nu|,$$
$$z_p = \cos(\widehat{\mathbf{p}_{e\nu}, \mathbf{p}_p}), \quad V_n = \frac{3Z}{8\pi} \left(\frac{2\pi\hbar}{p_0}\right)^3,$$

m is the effective mass of a nucleon in the nucleus[12], $m = m_p/2$, V_n is the nuclear volume. In (8) one can carry out the integration over the azimuthal angles in all the distributions, and then:

$$W_e = \frac{2\pi}{\hbar} |H|^2 \frac{2^2 (2\pi)^3}{(2\pi\hbar)^9} V_n \int p_e^2 \, dp_e \, dz_e p_\nu^2 \, dp_\nu \, dz_\nu \cdot$$
$$\cdot p_p^2 \, dp_p \, dz_p \delta\left(E_e - E_\nu - \frac{p_{e\nu}^2}{2m} - \frac{p_p p_{e\nu} z_p}{m}\right); \tag{9}$$

Here we have taken into account the fact that the statistical weights of the electrons and the protons are equal to 2, and of the neutrinos to 1.

The problem now consists of determining the limits of integration in formula (9). We can at once indicate the following limits[3]:

$$p_e < p_{fe}, \quad p_p \leqslant p_0, \quad p_n \geqslant q_0. \tag{10}$$

In future we shall consider only ultrarelativistic electrons, $E_e = cp_c$, while the nucleons in the nucleus have already been assumed to be nonrelativistic. In determining the limits of integration one should take into account the fact that in the model being used both the laws of conservation of energy, and of momentum for the system of four fermions must be satisfied. The energy threshold for electron capture is equal to $\epsilon_0 = \Delta = E(A, Z-1) - E(A, Z)$. From the conservation of momentum it follows that the momentum of the threshold electron must satisfy the condition $p_{e0} + p_{\nu 0} = q_0 - p_0$, where $p_{\nu 0} = p_{e0} - \Delta/c$. Then the threshold value

$$p_{e0} = \frac{q_0 - p_0}{2} + \frac{\Delta}{2c},$$

where $\Delta = (q_0^2 - p_0^2)/2m$ in the free particle model without taking the Coulomb field into account. In subsequent discussion the value of p_{e0}, and also the other limits of integration will be obtained in a more formal manner.

In order to obtain the lower limit of integration over p_p we note that from the law of conservation of energy it follows that

$$p_p^2 = p_n^2 - 2mc(p_e - p_\nu), \tag{11}$$

and since $p_n > q_0$, then

$$p_p^2 > q_0^2 - 2mc(p_e - p_\nu) = p_1^2. \tag{12}$$

On the other hand, from the delta-function in (9) we have

$$p_p = \frac{mc(p_e - p_\nu)}{p_{e\nu} z_p} - \frac{1}{2} \frac{p_{e\nu}}{z_p}. \tag{13}$$

From this it can be seen that the minimum value of p_p is reached at $z_p = 1$, and therefore we obtain

$$p_p > mc(p_e - p_\nu)/p_{e\nu} - p_{e\nu}/2 = p_2. \tag{14}$$

In integrating over dp_p one should choose the largest of the limits in (12), (14). After integration over dz_p taking the delta-function into account and over dp_p taking the limits (10), (12), (14) into account we obtain

$$W_e = \frac{2\pi}{\hbar} |H|^2 \frac{2^2 (2\pi)^3}{(2\pi\hbar)^9} V_n \int p_e^2 \, dp_e \, dz_e p_\nu^2 \, dp_\nu \, dz_\nu \cdot$$
$$\cdot \frac{m}{2p_{e\nu}} \begin{cases} p_0^2 - p_1^2 & \text{for } p_1 > p_2 \\ p_0^2 - p_2^2 & \text{for } p_1 < p_2 \end{cases}. \tag{15}$$

We now obtain the limits of integration over z_ν, p_ν, p_e. The upper integrand in (15) is realized under the condition $p_1 > p_2$. This imposes a limitation on the value of

$$z_\nu \equiv \cos(\widehat{\mathbf{p}_e, \mathbf{p}_\nu}) < z_{\nu 1} = (p_e^2 + p_\nu^2 - x_1^2)/2p_e p_\nu,$$
$$x_1 = q_0 - [q_0^2 - 2mc(p_e - p_\nu)]^{1/2}. \tag{16}$$

From $p_2 < p_0$ we obtain the condition

$$z_\nu < z_{\nu 0} = (p_e^2 + p_\nu^2 - x_0^2)/2p_e p_\nu,$$
$$x_0 = -p_0 + [p_0^2 + 2mc(p_e - p_\nu)]^{1/2}, \tag{17}$$

and from $p_1 < p_0$ the restriction follows

$$p_\nu < p_{\nu 2} = p_e - (q_0^2 - p_0^2)/2mc. \tag{18}$$

Restrictions on p_ν from below are obtained if we note that in (17) it is necessary to assume $z_{\nu 0} > -1$, and this yields

$$p_\nu > p_{\nu 0} = [(mc + p_0)^2 + 4mcp_e]^{1/2} - p_e - mc - p_0, \tag{19}$$

while in (16) it is also necessary to take $z_{\nu 1} > -1$, and this yields

$$p_\nu > p_{\nu 1} = -[(mc+q_0)^2 - 4mcp_e]^{1/2} - p_e + mc + q_0. \qquad (20)$$

The lower limit of integration over p_e is determined from the condition (12) where we must set $p_p = p_0$, $p_\nu = p_{\nu 0}$; we then obtain

$$p_e > p_{e0} = (q_0 - p_0)/2 + (q_0^2 - p_0^2)/4mc. \qquad (21)$$

It can be easily verified that $p_{\nu 1} > p_{\nu 0}$, with the equality occurring for $p_e = p_{e0}$. Thus, the upper expression in (15) is realized for

$$-1 < z_\nu < z_{\nu 1}, \quad p_{\nu 1} < p_\nu < p_{\nu 2}, \qquad (22)$$

while the lower one is realized under the condition

$$-1 < z_\nu < z_{\nu 0}, \quad p_{\nu 0} < p_\nu < p_{\nu 1}, \qquad (23)$$

and also for

$$z_{\nu 1} < z_\nu < z_{\nu 0}, \quad p_{\nu 1} < p_\nu < p_{\nu 2}.$$

In the range of values $p_{\nu 0} < p_\nu < p_{\nu 1}$ the inequality $z_{\nu 1} < -1$ holds, and therefore in this range of values of p_ν the upper integral in (15) is absent, while for the lower integral z_ν varies in the range $-1 < z_\nu < z_{\nu 0}$.

The limits (22) and (23) are realized only in the case when $p_{\nu 2} > p_{\nu 1}$ and $p_{\nu 2} > p_{\nu 0}$. Utilizing (18)—(20) it can be easily shown that

$$p_{\nu 2} > p_{\nu 1} \text{ for } p_{e0} < p_e < \frac{q_0 + p_0}{2} + \frac{q_0^2 - p_0^2}{4mc} = p_{e1},$$
$$p_{\nu 2} > p_{\nu 0} \text{ for } p_e > p_{e0}. \qquad (24)$$

As a result of (24) in integrating over dp_e in the range $p_{e0} < p_e < p_{e1}$ all the limits (22) and (23) are realized, while for $p_e > p_{e1}$ only one type of limits remains:

$$-1 < z_\nu < z_{\nu 0}, \quad p_{\nu 0} < p_\nu < p_{\nu 2}. \qquad (25)$$

Utilizing the limits (16)—(25) and integrating in (15) over dz_ν and dz_e, we obtain the following expression

$$W_e = \frac{2\pi}{\hbar} |H|^2 \frac{2^2 (2\pi)^3}{(2\pi\hbar)^9} V_n m \int_{p_{e0}}^{p_{fe}} p_e \, dp_e \left[\int_{p_{\nu 0}}^{p_{\nu 1}} p_\nu \, dp_\nu F_1 + \int_{p_{\nu 1}}^{p_{\nu 2}} p_\nu \, dp_\nu (F_2 + F_3) \right],$$
$$p_{fe} < p_{e1},$$

$$W_e = \frac{2\pi}{\hbar} |H|^2 \frac{2^2 (2\pi)^3}{(2\pi\hbar)^9} V_n m \left\{ \int_{p_{e0}}^{p_{e1}} p_e \, dp_e \left[\int_{p_{\nu 0}}^{p_{\nu 1}} p_\nu \, dp_\nu F_1 \right.\right.$$
$$\left.\left. + \int_{p_{\nu 1}}^{p_{\nu 2}} p_\nu \, dp_\nu (F_2 + F_3) \right] + \int_{p_{e1}}^{p_{fe}} p_e \, dp_e \int_{p_{\nu 0}}^{p_{\nu 2}} p_\nu \, dp_\nu F_1 \right\}, \; p_{fe} > p_{e1}$$

$$F_1 = [p_0^2 + mc(p_e - p_\nu)](p_e + p_\nu - x_2) + m^2c^2(p_e - p_\nu)^2 \cdot \qquad (26)$$
$$\times \left(\frac{1}{p_e + p_\nu} - \frac{1}{x_0} \right) - \frac{1}{12} [(p_e + p_\nu)^3 - x_0^3],$$

$$F_2 = [p_0^2 + mc(p_e - p_\nu)](x_1 - x_0) + m^2c^2(p_e - p_\nu)^2 \left(\frac{1}{x_1} - \frac{1}{x_0} \right) - \frac{1}{12}(x_1^3 - x_0^3),$$
$$F_3 = 2mc(p_{\nu 2} - p_\nu)(p_e + p_\nu - x_1).$$

In order to obtain the value of the square of the matrix element of the interaction $|H|^2$ for different nuclei we utilize the results of experiments on the capture of mu mesons by nuclei[13].

Calculating the probability of capture of a mu meson by a nucleus utilizing the same gas model of the nucleus we obtain (cf.,[11,12]):

$$W_\mu = \frac{2\pi}{\hbar} \frac{2(2\pi)^2}{(2\pi\hbar)^6} V_{n\mu} |H|^2 \frac{1}{\pi} \left(\frac{Z_e m_\mu e^2}{\hbar^2} \right)^3 m^2 c p_{k\mu} \varphi_\mu,$$

$$\varphi_\mu = \frac{1}{3} p_{k\mu}^2 + p_{\nu 1\mu} p_{k\mu} + \frac{1}{2}(p_0^2 - m^2c^2 + mm_\mu c^2) \frac{p_{\nu 1\mu}^2 - p_{\nu 0\mu}^2}{mcp_{k\mu}}$$
$$+ 2mm_\mu c^2 \frac{p_{\nu 1\mu} - p_{\nu 0\mu}}{p_{k\mu}} - \frac{p_{\nu 1\mu}^3 - p_{\nu 0\mu}^3}{3p_{k\mu}} - \frac{p_{\nu 1\mu}^4 - p_{\nu 0\mu}^4}{16mcp_{k\mu}} - \frac{mm_\mu^2 c^3}{p_{k\mu}} \ln \frac{p_{\nu 1\mu}}{p_{\nu 0\mu}},$$
$$V_{n\mu} = \frac{3Z_e}{8\pi} \left(\frac{2\pi\hbar}{p_0} \right)^3, \qquad (27)$$
$$p_{k\mu} = m_\mu c - (q_0^2 - p_0^2)/2mc - p_{\nu 1\mu},$$
$$p_{\nu 1\mu} = mc + q_0 - [(mc + q_0)^2 - 2mm_\mu c^2]^{1/2},$$
$$p_{\nu 0\mu} = -mc - p_0 + [(mc + p_0)^2 + 2mm_\mu c^2]^{1/2}.$$

Here $m = 206.76m_e$ is the rest mass of a muon, Z_e is the effective charge of the nucleus determined by the formula[13]:

$$Z_e = Z[1 + (Z/42)^{1.47}]^{-1/1.47}. \qquad (28)$$

Integration in (26) can be carried out to the end, but as a result one obtains expressions which are very awkward and difficult to examine. In the practically interesting case of medium electron energies when the inequalities

$$p_{e\nu}^2 \ll mc(p_e - p_\nu) \ll p_0^2, q_0^2, \qquad (29)$$

are satisfied expression (26) is considerably simplified. In this case in (26) one can restrict oneself only to the contribution of F_3, and this after integration over dp_ν and dp_e yields

$$W_e = \frac{2\pi}{\hbar} \frac{2^2 (2\pi)^3}{(2\pi\hbar)^9} |H|^2 V_n m^2 c \frac{8mcq_0^2}{3(mc + q_0)^3} p_{ke}^4 \varphi_e, \quad p_{ke} = p_{fe} - p_{e0}, \qquad (30)$$
$$\varphi_e = \frac{1}{6} p_{ke}^2 + \frac{1}{5} p_{e0} p_{ke} \left(2 - \frac{q_0}{mc} \right) + \frac{1}{4} p_{e0}^2 \left(1 - \frac{q_0}{mc} \right),$$
$$p_{e0} = \frac{q_0^2 - p_0^2}{4} \left(\frac{1}{mc} + \frac{1}{q_0} \right).$$

From (27) and (30) we obtain the final expression for the probability of capture of an electron by a nucleus from a Fermi distribution with the limiting energy ϵ_{fe}:

$$W_e = W_\mu \frac{1}{2\pi} \left(\frac{137}{Z_e m_\mu c} \right)^3 \frac{Z}{Z_e} \frac{8mcq_0^2}{3(mc + q_0)^3} \frac{p_{ke}^4}{p_{k\mu}} \frac{\varphi_e}{\varphi_\mu}. \qquad (31)$$

Here Z_e is defined in (28); p_{ke}, p_{e0}, φ_e are defined in (30); $p_{k\mu}$, φ_μ are defined in (27); q_0, p_0 are defined in (6).

Expression (29) holds for not too high electron energy when the inequalities (27) are satisfied and for not too low electron energy when the inequality $\epsilon_{fe} \gg \Delta$ is satisfied. The threshold Δ for the capture of an electron by a nucleus contained in formulas (28) and (29) is obtained using a model in which the laws of conservation of momentum and of energy of the four fermions are satisfied in the process of β-capture. Such an assumption is, apparently, satisfactory, if the electron energy considerably exceeds the proton binding energy. In the case of $\epsilon_{fe} < p_{e0}c$ the assumption of conservation of momentum during the interaction is not satisfied, but β-capture occurs as a result of the fact that the nucleus in reality is not a mixture of Fermi gases. For those nuclei for which the Fermi gas model is satisfactory the dependence of the rate of the process on the electron energy must increase sharply in going over from $\epsilon_{fe} < p_{e0}c$ to $\epsilon_{fe} > p_{e0}c$. The actual threshold for the Fe^{56} nucleus, for example, will be lower by a factor of 1.6 than in (30). In calculations of heating the capture of electrons into the first several levels with $\Delta_i < p_{e0}c$ must be taken into account separately using formulas of the type (3), (4). Since the Mn^{56} nucleus has no levels with $\Delta < 2.5$ MeV into which the transitions would be superallowed[10], then at $p_e > p_{e0}$ the probability W_e from (31) will quite rapidly exceed the probability of capture into the lower levels.

3. AVERAGE ENERGIES OF EMITTED NEUTRINOS AND OF NUCLEAR EXCITATION

The calculation of the intensity of neutrino emission L_ν taking into account the excited states of the nucleus is carried out using the same nuclear model in analogy with the calculation of W_e. In order to obtain the result one must multiply the integrand in (9) by cp_ν. As a result of this for intermediate electron energies (29) we obtain

$$L_\nu=\frac{1}{2\pi}\left(\frac{137}{Z_e mc}\right)^3\frac{Z}{Z_e}\frac{8m^2c^3q_0^2}{3(mc+q_0)^4}\frac{p_{ke}^4}{p_{k\mu}}\frac{\varphi_L}{\varphi_\mu}W_\mu\left(\frac{\text{эрг}}{\text{сек}}\right),\tag{32}$$

$$\varphi_L=\left(\frac{1}{7}+\frac{x^2}{35}\right)p_{ke}^3+\left(\frac{1}{2}-\frac{x}{3}+\frac{x^2}{30}\right)p_{e0}p_{ke}^2$$

$$+\left(\frac{3}{5}-\frac{4}{5}x+\frac{x^2}{5}\right)p_{e0}^2p_{ke}+\frac{1}{4}(1-x)^2p_{e0}^3,\quad x=\frac{q_0}{mc},$$

where φ_μ is defined in (31).

Comparing (32) and (31) we obtain that for $p_{ke} \gg p_{e0}$ the average value of the emitted neutrino is equal to

$$\varepsilon_\nu=\frac{6m^2c^2+0{,}2q_0^2}{7mc(mc+q_0)}\varepsilon_{fe}\approx0{,}6\varepsilon_{fe}\tag{33}$$

instead of $\bar{\epsilon}_\nu = 5\epsilon_{fe}/6$ in the case of a single level. In this case heating occurs for two reasons. The entropy increases, first of all, as a result of the nonequilibrium distortion of the electron spectrum, and secondly, due to the fact that the excited nuclei go over into the ground state liberating the excitation energy. In such processes both a direct transition of the nucleus into the ground state with emission of gamma quanta and emission of a neutron which is captured by another nucleus with emission of the binding energy in the form of gamma quanta are possible. When the excitation energy for normal nuclei in the valley of stability exceeds 10 MeV the second process predominates[14], while for $Q < 10$ MeV the first process predominates. The excitation energy in the case of capture of an electron of energy E_e and emission of a neutrino of energy E_ν is equal to

$$Q=E_e-E_\nu-(q_0^2-p_0^2)/2m.\tag{34}$$

In order to obtain Q_{max} one must in (34) utilize the maximum electron energy ϵ_{fe} and the minimum neutrino energy $\epsilon_{\nu0} = cp_{\nu0}$ from (21). As a result taking the inequalities (29) into account we obtain

$$Q_{max}=2cp_{fe}\frac{p_0}{mc+p_0}-\frac{q_0^2-p_0^2}{2m}=\frac{cp_{fe}}{1{,}6}-\frac{q_0^2-p_0^2}{2m}.\tag{35}$$

In the case $\epsilon_{fe} \gg \Delta$, utilizing (33), we obtain for the average excitation energy

$$\bar{Q}\approx\frac{mc+7q_0}{14p_0}Q_{max}\approx0.66Q_{max}.\tag{36}$$

It is of interest to compare the value of Q_{max} with the maximum excitation energy of the nucleus in this model in the case of muon capture. Utilizing (27), we obtain

$$Q_{\mu\,max}=\Delta_\mu-cp_{\nu0\mu}-\frac{q_0^2-p_0^2}{2m}=\Delta_\mu\frac{p_0}{mc+p_0}-\frac{q_0^2-p_0^2}{2m}.\tag{37}$$

As can be seen from a comparison of (36) and (37) the fraction of energy going into excitation of the nucleus in the case of an ultrarelativistic electron is approximately twice as great as in the case of capture of a muon at rest. This is associated with the fact that the electron possesses a momentum and with the assumption that the law of conservation of momentum is satisfied in each act of interaction with a single nucleon. Without taking the excited levels into account the calculation of the heating of matter due to the nonequilibrium e^- capture in the case of collapse has been carried out in[8,9], with the role played by nuclear excitation to produce heating being noted in[9], and also in[6]. The increase of entropy per unit mass in the case of nonequilibrium β capture for degenerate electrons is determined by the expression[6]

$$TdS/dt=[(\varepsilon_{fe}-\Delta)W_e-L_\nu]/Am_p.\tag{38}$$

Here Δ is determined by the difference of the energies of the ground states of consecutive nuclei, since in the case of capture into an excited state the difference $\Delta_i-\Delta$ will in any case be liberated in the form of heat, while L_ν is determined by formula (32). One can neglect the change in the internal energy of the nuclei as a result of a possible emission of neutrons from excited nuclei and their subsequent capture by other nuclei. In the process of collapse consecutive capture of electrons with the formation of increasingly more neutron-rich nuclei occurs and therefore in the calculation in the right hand side of (38) one should take into account the sum over all the β captures taking place. One must add to (38) the equations for the change in the number of nuclei of different kinds and to take into account the law for the variation of density.

For $T > m_ec^2/k$, and even somewhat earlier, statistical equilibrium is established between nuclei and the substance can then be regarded as hot. The difference in chemical composition between the hot and the cold substance being compressed is quite significant[15,16]. The results of[8,9] show that always the hot variant of compression is realized.

The temperature $T = m_ec^2/k$ is attained for $\log\rho = 10.5$–11.5 ($\epsilon_{fe} = 15$–30 MeV) depending on the rate of collapse. The smaller is the rate of compression, the greater is the increase in temperature depending on density. For $\epsilon_{fe} = 30$ MeV the rate of β captures via excited states of nuclei is greater than the rate of β captures into the lowest level. In this process for each capture reaction into excited levels the average amount of energy liberated as heat is equal to $\bar{Q} = \epsilon_{fe} - \bar{\epsilon}_\nu - \Delta \approx 0.4\epsilon_{fe} - \Delta$, and this for $\epsilon_{fe} \gg \Delta$ is greater by a factor of $0.4 : 1/6 \approx 2.4$ than the heating in the case of captures only into the single lowest level. Therefore the substance of a collapsing star attains the hot regime even faster.

4. INTERACTION OF HEAVY NUCLEI WITH ENERGETIC NEUTRINOS

The process of interaction of an energetic neutrino with a nucleus is completely analogous to the capture by a nucleus of an ultrarelativistic electron; the following reaction takes place

$$(A,\,Z)+\nu\to(A,\,Z+1)+e^-.\tag{39}$$

In medium and heavy stable nuclei the number of neutrons exceeds the number of protons, and therefore in the consideration of the interaction of a neutrino with a nucleus using the gas model the threshold is formally absent, and is even negative. In actual fact due to the contribution of Coulomb energy the binding energy is lower in the final nucleus, and the threshold exists anyway, for example, $E(Co^{56}) - E(Fe^{56}) \approx 5.1$ MeV. As a result of this the calculation of the reaction (39) utilizing the gas model of the nucleus is meaningful only for $E_\nu \gg \Delta$. In analogy with (35), the maximum excitation energy of the final nucleus will be equal to

$$Q_{max}=2cp_\nu p_0/(mc+p_0)-\Delta, \tag{40}$$

while the cross section for the interaction of a neutrino with a nucleus for $E_\nu \gg \Delta$ obtained by the method described in Sec. 2 is equal to

$$\sigma_\nu=\frac{2\pi^2}{c}\left(\frac{\hbar^2}{e^2Z_e m_\mu}\right)^3\frac{N}{Z_e}\frac{W_\mu}{p_\nu}\frac{8mcp_0^2}{3(mc+p_0)^3}$$
$$\times\frac{(p_\nu-p_{\nu0})^4-(1-p_0/mc)p_{\nu0}(p_\nu-p_{\nu0})^3}{p_{k\mu}\varphi_\mu}. \tag{41}$$

Here the threshold $p_{\nu 0} = \Delta/c$ instead of the formally negative one has been replaced by a positive one which in fact occurs. Since according to (6) and equation $m = m_p/2$ the value $mc/p_0 = 2.2$, for a neutrino of energy $E_\nu \geq 30$ MeV and for the Fe^{56} nucleus the excitation energy Q can from (40) attain 30/1.6—5.1 ≈ 14 MeV, this leads to the emission of a neutron. Thus, the recording of neutrinos of energies of several tens of MeV which must be produced in a relativistic collapse can be carried out by means of detecting both the emitted electrons and neutrons[17]. In accordance with formulas (35) and (36) the average excitation energy is close to the maximum possible one; therefore in approximately half of the interaction events the excitation energy will be sufficient for the emission of a neutron.

The model adopted here is quite crude, but it gives satisfactory results for the description of the absorption of muons by heavy nuclei[12]. Calculations by another method carried out in[18] for the interaction of a C^{12} nucleus with a neutrino have shown that excitation energy sufficient for the emission of a neutron is acquired in half of the events of the absorption of a neutrino at $E_\nu = 60$ MeV. For heavy nuclei the mixing of levels becomes more significant and the statistical approach is better applicable. The results obtained here show that already at $E_\nu = 30$ MeV the excitation energy sufficient for the emission of a neutron can be attained in approximately half of the events of interaction. For more confident conclusions it is necessary to investigate in detail the structure of highly excited levels in nuclei and to calculate the interaction with neutrinos by utilizing more accurate nuclear models.

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1)The Fermi energy can be regarded as ordered, since it corresponds to zero entropy.

2)The value of $ft_{1/2}$ for such a level can exceed the corresponding value for a free nucleon.

3)More accurately one should take $p_n > q_0(A, Z-1)$, but in future we shall neglect the difference between the Fermi momenta of neutrons in nuclei (A, Z) and (A, Z-1).

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