

Spatial anisotropy of spontaneous emission by stationary atoms in a strong field

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It is shown that the beat spectrum of a spontaneous emission by atoms in a strong field exhibits spatial anisotropy. This phenomenon is closely connected with combinational coupling, which must be taken into account in the presence of a strong field.

INTRODUCTION

The emission of radiation by an atom in the field of a strong electromagnetic wave is of particular interest. One would expect that some or all characteristics of this spontaneous emission field would exhibit axial symmetry. Rautian^[1] has investigated the spectral profile of spontaneous emission, but found no angular dependence. Henceforth we shall refer to this profile as the fundamental profile. It is determined by first-order correlations in the field (see^[2]). In general, the first-order correlator does not exhaust all the spectral properties of the field. Among the others, the most important is the second-order correlator which determines the frequency beat spectrum. This is so because modern experimental techniques enable us to record relatively easily both the fundamental spectral profiles (for example, by means of spectrographs) and the beat spectra (by means of photoelectric multipliers and spectral analyzers). The beat spectrum carries information on correlations between the Fourier components of the fundamental spectral profile. If there are no such correlations (as, for example, in the spontaneous emission by atoms in the absence of an external field), the beat spectrum has the same shape as the spectral profile. In this case, there is no point in investigating simultaneously the fundamental spectral profile and the beat spectrum. However, the emission of radiation by atoms in a strong field presents a different situation. Here the strong field leads to the appearance of combinational coupling between the weak waves and, as a result, to the appearance of correlations between the Fourier components of the spectral profile. Moreover, for extended sources, the combinational coupling is anisotropic. Consequently, the beat spectrum should also be anisotropic, and this will be demonstrated in the present paper.

We must note one further point in connection with the foregoing summary. The spontaneous emission by an atom is frequently modelled by a sinusoid with a random phase. This model provides a satisfactory description of the spontaneous emission profile, but gives a delta function for the beat spectrum. In actual fact, the beat profile has the same shape as the spectral profile, but it is centered on the zero frequency and half of all the radiated power must be assigned to it.

1. AMPLITUDE-MODULATED AND PHASE-MODULATED SPECTRAL PROFILES

In this section, we shall develop certain ideas which will help us subsequently to interpret our results. Suppose the field consists of three monochromatic waves with frequencies ω_0 , $\omega_0 - \Delta$, and $\omega_0 + \Delta$:

$$E(t) = \mathcal{E}_0 \exp(-i\omega_0 t) + \mathcal{E}_1 \exp[-i(\omega_0 - \Delta)t] + \mathcal{E}_2 \exp[-i(\omega_0 + \Delta)t] + \text{c.c.}$$

where \mathcal{E}_0 , \mathcal{E}_1 , and \mathcal{E}_2 are the complex field amplitudes.

Instead of the amplitudes of the side components \mathcal{E}_1 and \mathcal{E}_2 we introduce linear combinations of the form

$$z_a = 1/2(\mathcal{E}_1 + \mathcal{E}_2^*), \quad z_b = 1/2(\mathcal{E}_1^* - \mathcal{E}_2).$$

The field can then be written in the form

$$E(t) = [1/2\mathcal{E}_0 \exp(-i\omega_0 t) + z_a \exp(-i(\omega_0 - \Delta)t) + z_a \exp(i(\omega_0 + \Delta)t)] + [1/2\mathcal{E}_0 \exp(-i\omega_0 t) + z_b \exp(i(\omega_0 - \Delta)t) + z_b \exp(-i(\omega_0 + \Delta)t + i\pi)] + \text{c.c.}$$

It is thus clear that the triplet of waves with frequencies ω_0 , $\omega_0 - \Delta$, and $\omega_0 + \Delta$, and amplitudes \mathcal{E}_0 , \mathcal{E}_1 , and \mathcal{E}_2 can be written as the sum of two triplets of waves with the same frequencies. The amplitudes of the central components are equal in both triplets ($1/2\mathcal{E}_0$). The side components have the following properties: in one of the triplets the amplitudes of the side components are equal to z_a and the phases are the same, whilst in the other the complex amplitudes of the side components are z_b and are in antiphase. If the power carried by the side components is much less than that carried by the central component, the first triplet may be regarded as weak amplitude modulation of the central signal of frequency Δ , and the second as weak phase modulation with the same frequency. Suppose now that we are considering the spectral profile. For each pair of Fourier components placed symmetrically relative to the central frequency and the central wave, we can carry out the above resolution into the sum of two triplets of waves. The result is that the spectral profile is resolved into the sum of two spectral profiles. In one of these, each Fourier component is in phase with its symmetric partner, and in the other it is in antiphase. If we mix the original profile with a strong signal at the central frequency, one of the new profiles will determine the nonmonochromatic amplitude modulation of the strong signal (AM profile) and the other will determine the nonmonochromatic phase modulation (PM profile). Thus each of the spectral profiles is equal to the sum of the AM and PM profiles. These may have different shapes as a result of correlations in the original profile. This resolution procedure is convenient in the analysis of the beat spectrum. Beats between the side components and the central component are nonzero only for the AM profile. In the PM profile, symmetrically located beats compensate one another. The shape of the AM profile provides unambiguous information about the beat spectrum profile. This cannot in general be done on the basis of the original profile alone.

In conclusion of this section, let us apply the foregoing discussion to emission by an atom in the absence of the external field. In accordance with, for example, the results given in^[3], each pair of waves located sym-

metrically relative to the center can be described by a density matrix in the diagonal representation:

$$P_{\omega_1\omega_2}(\alpha_1\alpha_2) \sim \exp\left[-\frac{|\alpha_1|^2}{I(\omega_1)} - \frac{|\alpha_2|^2}{I(\omega_2)}\right]$$

(the matrix is defined in Section 3). The quantities α_1 and α_2 play the role of the complex field amplitudes with frequencies $\omega_1 = \omega_0 - \Delta$ and $\omega_2 = \omega_0 + \Delta$. The quantities $I(\omega_1)$ and $I(\omega_2)$ determine the strength of the components (their dependence on frequency determines the spectral profile of the spontaneous emission). For a symmetric frequency distribution we have $I(\omega_1) = I(\omega_2) = I$. In terms of the amplitudes of amplitude and phase modulation $z_a = \frac{1}{2}(\alpha_1 + \alpha_2^*)$ and $z_p = \frac{1}{2}(\alpha_1 - \alpha_2^*)$, the density matrix can be rewritten in the form [14]

$$P_{\omega_1\omega_2}(\alpha_1\alpha_2) \sim \exp\left[-\frac{2|z_a|^2}{I} - \frac{2|z_p|^2}{I}\right].$$

Therefore, while $I(\omega)$ determines the spectral profile of spontaneous emission, the AM and PM profiles are determined by $\frac{1}{2}I(\omega)$. It follows that the beat spectrum for spontaneous emission is the same as the usual spectrum, but the power carried by it is one half of the total emitted power.

2. PHYSICAL MODEL AND BASIC RESULTS

We shall suppose that the resonance medium consists of fixed and noninteracting two-level atoms. A strong field is incident on the boundary of the medium at the point $x=0$ in the positive direction of x in the form of a plane monochromatic wave with frequency equal to the atomic transition frequency ω_0 (for the sake of simplicity we are considering the one-dimensional problem). The spontaneous emission spectrum will be calculated for the same transition. We shall assume that spontaneous relaxation between working levels occurs much more slowly than relaxation to lower-lying energy levels. The formal solution is given in Sec. 3. The system is pumped only to the upper level (the steady-state population of the working levels in the absence of the strong field is $N_a = N$, $N_b = 0$).

The formal result established in Section 3 is that the density matrix in the diagonal representation for a pair of waves with frequency $\omega_1 = \omega_0 - \Delta$ and $\omega_2 = \omega_0 + \Delta$ is given by

$$P(\alpha_1\alpha_2) = \frac{1}{\pi^2 I_a I_\phi} \exp\left[-\frac{|\alpha_1 + \alpha_2|^2}{4I_a} - \frac{|\alpha_1 - \alpha_2|^2}{4I_\phi}\right]. \quad (1)$$

For thin layers of the resonance medium, when self amplification or self absorption can be ignored, the quantities I_a and I_p are given by (7)-(10). This approximation was used by Rautian, [11] and in this section we shall confine our attention to it. The matrix given by (1) describes a pair of waves travelling in the direction of the strong field. For a pair of waves travelling in the other direction we can replace both I_a and I_p by $\frac{1}{2}(I_a + I_p)$.

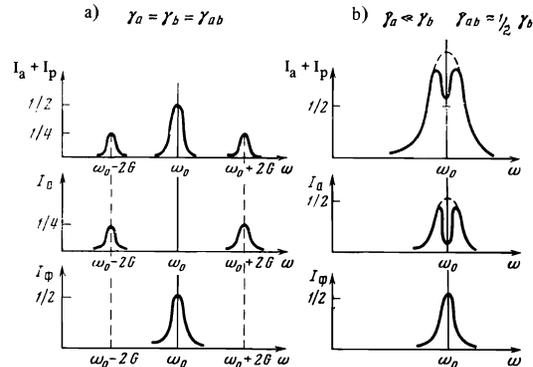
For the analysis we require the single-mode density matrix

$$P(\alpha_1) = \int \delta^2 \alpha_2 P(\alpha_1\alpha_2) = \frac{1}{\pi(I_a + I_\phi)} \exp\left[-\frac{|\alpha_1|^2}{I_a + I_\phi}\right]. \quad (2)$$

This formula is valid both for the forward and backward emission. Therefore, the fundamental spontaneous emission profile is determined by $I_a + I_p$ (which is independent of combinational coupling) and is the same as the

spectral profile calculated by Rautian [11]. In the case of equal relaxation constants ($\gamma_a = \gamma_b = \gamma_{ab} \equiv \gamma$), it consists of three profiles centered on the frequencies ω_0 and $\omega_0 \pm 2G$. In the case of very different constants ($\gamma_a \ll \gamma_b$), this is the Lorentz profile of width γ_{ab} with a narrow dip of width $\gamma_a(1 + 4G^2/\gamma_a\gamma_b)^{1/2}$ at the center. The relative width of this dip amounts to $\frac{1}{2}$ in the case of saturation (when $G^2 \gg \gamma_a\gamma_b$ and $G^2 \ll \gamma_b^2$).

To obtain the beat spectrum we must separate the AM profiles from the fundamental spectrum. In accordance with (1), we conclude that for spontaneous emission in the forward direction the AM profile is determined by the dependence of I_a on ω . For backward emission, on the other hand, the profile is replaced by $\frac{1}{2}(I_a + I_p)$. The PM profile is determined by I_p and $\frac{1}{2}(I_a + I_p)$. The figure illustrates this for two cases, namely, $\gamma_a = \gamma_b = \gamma_{ab}$ and $\gamma_a \ll \gamma_b$ ($\gamma_{ab} = \frac{1}{2}\gamma_b$). It is clear that, when the relaxation constants are equal for spontaneous emission in the forward direction, the side components of the fundamental profile (profiles centered on $\omega_0 \pm 2G$) are the AM profiles and they determine the beat spectrum. The beat spectrum is therefore the sum of two profiles of width γ and not of three, as in the case of the fundamental profile. This should, in principle, lead to an improvement of resolution, i.e., to easier detection of line splitting in a strong field. In the case of very different relaxation constants, the AM profile is similar to the fundamental profile. The difference lies only in that the depth of the dip on saturation ($G^2 \gg \gamma_a\gamma_b$) tends to unity and is not equal to $\frac{1}{2}$. For backward emission, the AM profiles (and the PM profiles) have the same shape as the fundamental profile. The beat spectrum in the backward direction, therefore, does not contain any new elements as compared with those known previously. [11]



The behavior of the PM profiles is not important from the experimental point of view, but it is very significant in principle. The role of combinational coupling is essentially to fix the initial (i.e., in the absence of the external field) shape of the PM profile. This is always the Lorentz profile centered on the frequency of the atomic transition, ω_0 , with width γ_{ab} . For forward emission this does occur (because there is combinational coupling), but for backward emission it does not. This is, in fact, the difference between the beat spectra in different directions of observation.

The foregoing may provide an explanation for the splitting of the subthreshold mode frequencies in the unidirectional traveling-wave single-mode laser described in [5]. The high-Q laser cavity forces us to consider a quasidiscrete spectral profile of spontaneous emission by the resonant medium. Just as in the case of emission into free space, this profile may be written as

the sum of discrete AM and PM profiles. Combinational coupling fixes the PM profile, and the strong field deforms the AM profile. The result is that, in the case of a weak field, instead of the single discrete line in the direction of generation there are two. This corresponds to the 'separation' of profiles in the usual spontaneous emission (for equal relaxation constants) when combinational coupling is present. Despite the qualitative agreement, there are quantitative discrepancies connected with the properties of the cavity. The high-Q cavity prevents the spectral lines from shifting by more than one resonator width, and in spontaneous emission without the cavity the splitting of the corresponding Fourier components is proportional to the intensity of the external field.

3. FORMAL SOLUTION FOR SPONTANEOUS EMISSION BY A RESONANT MEDIUM

The electromagnetic field can be approximately regarded as a set of quantum-mechanical oscillators located at points \mathbf{r} and having all the possible wave vectors \mathbf{m} [these are the (\mathbf{r}, \mathbf{m}) oscillators].^[3] In this representation we are, of course, dealing with a very rough spatial scale (smallest size l much greater than the emitted wavelength) and a very rough scale in wave-vector space (smallest size $2\pi l^{-1}$).

For each \mathbf{r}, \mathbf{m} oscillator we can introduce a density matrix $\rho_{\mathbf{m}}(\mathbf{r}, t)$ which is a solution of the equation

$$\frac{\partial \rho_{\mathbf{m}}}{\partial t} = -c \frac{\mathbf{m}}{m} \frac{\partial \rho_{\mathbf{m}}}{\partial \mathbf{r}} + F.$$

The first term on the right describes the change in the state of the field oscillator due to the transport of radiation (energy and phase) to neighboring points in space, and the second term describes the change in the state of the field due to the interaction between the (\mathbf{r}, \mathbf{m}) oscillator and the resonant medium. The quantity F is, in general, a function of the state of the resonant medium and the states of all other field oscillators at the point \mathbf{r} .

We shall only be interested in the time-independent problem ($\partial \rho / \partial t = 0$). This is sufficient for the analysis of the time-independent spectral characteristics of the field. From the formal point of view, it is more convenient to write down the equation not for the single-mode density matrix but for the two-mode matrix which describes two oscillators with frequencies $\omega_1 = \omega_0 - \Delta$ and $\omega_2 = \omega_0 + \Delta$. If the external field is assumed to be classical, the problem is completely analogous to the subthreshold mode problem considered in^[5] where the quantity F is evaluated. We shall use the equations given in that work by replacing the time derivative with the space derivative:

$$\begin{aligned} \frac{\partial \rho}{\partial x} = & -A_1 (a_1 a_1^+ \rho - a_1^+ \rho a_1 + \rho a_2 a_2^+ - a_2^+ \rho a_2) \\ & -A_2 (\rho a_1^+ a_1 - a_1 \rho a_1^+ + a_2^+ a_2 \rho - a_2 \rho a_2^+) + A_3 (a_1 a_2 \rho - a_2 \rho a_1 + \rho a_1^+ a_2^+ - a_2^+ \rho a_1^+) \\ & + A_4 (\rho a_1 a_2 - a_1 \rho a_2 + a_1^+ a_2^+ \rho - a_1^+ \rho a_2^+) + \text{h.c.} \end{aligned} \quad (3)$$

where, for the sake of simplicity, we confine our attention to the one-dimensional problem. For equal relaxation constants ($\gamma_a = \gamma_b = \gamma_{ab} \equiv \gamma$) the coefficients A_1, A_2, A_3 , and A_4 are given by^[5]

$$\begin{aligned} A_1 = & 2|g|^2 N \frac{\gamma^2 + 2G^2}{\gamma^2 + 4G^2} \frac{1}{\gamma - i\Delta} + A_4, \\ A_2 = & 2|g|^2 N \frac{2G^2}{\gamma^2 + 4G^2} \frac{1}{\gamma - i\Delta} + A_3, \end{aligned} \quad (4)$$

$$\begin{aligned} A_3 = & |g|^2 N \frac{\gamma^2}{(\gamma - i\Delta)^2 + 2\gamma G} \frac{G^2}{\gamma^2 + 4G^2} \left(\frac{1}{G} - \frac{2}{\gamma - i\Delta} \right), \\ A_4 = & -|g|^2 N \frac{\gamma^2}{(\gamma - i\Delta)^2 + 2\gamma G} \frac{G(\gamma + G)}{\gamma^2 + 4G^2} \left(\frac{1}{\gamma + G} + \frac{2}{\gamma - i\Delta} \right). \end{aligned}$$

For very different relaxation constants ($\gamma_a \ll \gamma_b$) we write down only the coefficients which we require for the analysis outlined in Section 2:

$$\begin{aligned} A_1 = & |g|^2 N \frac{\gamma_a \gamma_b}{\gamma_a \gamma_b + 4G^2} \left(\frac{1}{\gamma_{ab} - i\Delta} - \frac{4G^2}{\gamma_b^2} \frac{1}{\gamma_a' - i\Delta} \right), \\ A_4 = & -|g|^2 N \frac{4G^2}{\gamma_a \gamma_b + 4G^2} \frac{\gamma_a}{\gamma_b} \frac{1}{\gamma_a' - i\Delta} \end{aligned} \quad (5)$$

In the above equations $\gamma_a' = \gamma_a (1 + 4G^2 / \gamma_a \gamma_b)^{1/2}$ and $G^2(\mathbf{x}) = |g|^2 n(\mathbf{x})$ is the intensity of the external field at the point \mathbf{x} in frequency units [$n(\mathbf{x})$ is the mean number of photons at the point \mathbf{x}]. The quantity g determines the interaction between the atom at the point \mathbf{x} and the traveling wave:

$$g = (2m)^{-1/2} (\hat{\mathbf{d}}_{ab} \cdot \boldsymbol{\varepsilon}) \exp(i\mathbf{m} \cdot \mathbf{r}).$$

The coefficient A_1 describes the interaction between the field and the upper level of the atom, whilst A_2 describe the interaction with the lower level. The coefficients A_3 and A_4 determine the combinational interaction of weak fields in the presence of a strong field. Equation (3) describes two weak waves traveling in the direction of the strong field. To describe two sub-threshold waves traveling in the opposite direction we must set A_3 and A_4 equal to zero in (3), keep A_1 and A_2 the same, and replace x by $-x$.

We shall solve (3) in the diagonal representation of the density matrix which can be introduced by the integral^[2]

$$\rho(x) = \iint d^2 \alpha_1 d^2 \alpha_2 P(\alpha_1, \alpha_2) |\alpha_1, \alpha_2\rangle \langle \alpha_1, \alpha_2|;$$

where $d^2 \alpha_i = d\alpha_i' d\alpha_i''$, $\alpha_i' = \text{Re } \alpha_i$, $\alpha_i'' = \text{Im } \alpha_i$, $i = 1, 2$, and $|\alpha_i\rangle$ and α_i are the eigenvectors and eigenvalues of the annihilation operators a_i , i.e., $a_i |\alpha_i\rangle = \alpha_i |\alpha_i\rangle$. The vector $|\alpha_1, \alpha_2\rangle$ represents the product $|\alpha_1\rangle |\alpha_2\rangle$.

The transformation to the diagonal representation of the density matrix is dictated not only by the desire to transform from the operator equation (3) to a partial differential equation which can be solved by the usual methods of mathematical physics. There are reasons to suppose that the problem can be solved more readily in the diagonal representation than in any other way. The point is that in the semiclassical theory we have a solution for the problem of two weak waves and one strong wave.^[4] In the latter case it was found more convenient to transform from the amplitudes of the weak fields to the amplitudes of amplitude and phase modulations of the strong signal, which were found to be the normal coordinates of the problem. In quantum theory, and the usual operator approach, there is no analog of the classical modulations. The diagonal representation, on the other hand, enables us to carry out the analysis in a language very similar to the classical language. In fact, the quantities α_1 and α_2 play the role of the complex field amplitudes, and we can introduce new amplitudes

$$\begin{aligned} z_+ = & \frac{\alpha_1 + \alpha_2}{2} \exp[i(\Delta - \Omega_a)x], \\ z_- = & \frac{\alpha_1 - \alpha_2}{2} \exp[-i(\Delta - \Omega_a)x], \end{aligned}$$

which play the role of the complex amplitudes of amplitude and phase modulation. The frequencies Ω_a and

Ω_p are related to the constants of the original equation given by (3) as follows:

$$\Omega_a - \Delta = -\frac{1}{2} \operatorname{Im}(A_1 - A_2 - A_3 + A_4) = \frac{4|g|^2 N \gamma}{\gamma^2 + 4G^2} \left\{ \frac{\gamma(\Delta - 2G)}{(\Delta - 2G)^2 + \gamma^2} + \frac{\gamma(\Delta + 2G)}{(\Delta + 2G)^2 + \gamma^2} - \frac{G}{\gamma} \left[\frac{\gamma^2}{(\Delta - 2G)^2 + \gamma^2} - \frac{\gamma^2}{(\Delta + 2G)^2 + \gamma^2} \right] \right\},$$

$$\Omega_p - \Delta = -\frac{1}{2} \operatorname{Im}(A_1 - A_2 + A_3 - A_4) = \frac{4|g|^2 N \gamma}{\gamma^2 + 4G^2} \frac{\gamma \Delta}{\Delta^2 + \gamma^2}.$$

These explicit expressions are given for the case $\gamma_a = \gamma_b = \gamma_{ab} \equiv \gamma$.

As in the semiclassical theory, the new variables are the normal coordinates and, when these are used, the problem becomes separable:

$$P(\alpha_1 \alpha_2 x) = P_a(z_a x) P_p(z_p x).$$

For each of the distribution functions P_k ($k = a, p$) we have the equation

$$\frac{\partial P_k}{\partial x} = -\frac{\Gamma_k}{2} \frac{\partial}{\partial z_k} (z_k P_k) + \frac{\sigma_k}{2} \frac{\partial^2 P_k}{\partial z_k \partial z_k^*} + \text{c.c.} \quad (6)$$

For the case of equal relaxation constants ($\gamma_a = \gamma_b = \gamma_{ab} \equiv \gamma$), the two coefficients are given by [5]

$$\Gamma_a = -\frac{1}{2} \operatorname{Re}(A_1 - A_2 - A_3 + A_4) = -\frac{2|g|^2 N \gamma}{\gamma^2 + 4G^2} \left\{ \frac{\gamma^2}{(\Delta - 2G)^2 + \gamma^2} + \frac{\gamma^2}{(\Delta + 2G)^2 + \gamma^2} + \frac{2G}{\gamma} \left[\frac{\gamma(\Delta - 2G)}{(\Delta - 2G)^2 + \gamma^2} - \frac{\gamma(\Delta + 2G)}{(\Delta + 2G)^2 + \gamma^2} \right] \right\},$$

$$\Gamma_p = -\frac{1}{2} \operatorname{Re}(A_1 - A_2 + A_3 - A_4) = -\frac{2|g|^2 N \gamma}{\gamma^2 + 4G^2} \frac{2\gamma^2}{\Delta^2 + \gamma^2},$$

$$\sigma_a = \operatorname{Re}(A_1 + A_4) = \frac{|g|^2 N \gamma}{\gamma^2 + 4G^2} \left\{ \frac{\gamma^2 + 2G^2}{\gamma^2} \left[\frac{\gamma^2}{(\Delta - 2G)^2 + \gamma^2} + \frac{\gamma^2}{(\Delta + 2G)^2 + \gamma^2} \right] + \frac{G}{\gamma} \left[\frac{\gamma(\Delta - 2G)}{(\Delta - 2G)^2 + \gamma^2} - \frac{\gamma(\Delta + 2G)}{(\Delta + 2G)^2 + \gamma^2} \right] \right\}, \quad (7)$$

$$\sigma_p = \operatorname{Re}(A_1 - A_4) = \frac{|g|^2 N \gamma}{\gamma^2 + 4G^2} \frac{\gamma^2 + 2G^2}{\Delta^2 + \gamma^2}. \quad (8)$$

For the case $\gamma_a \ll \gamma_b$ we write out only the expressions for σ_a and σ_p :

$$\sigma_a = |g|^2 N \left[\frac{\gamma_a \gamma_b}{\gamma_a \gamma_b + 4G^2} \frac{\gamma_{ab}}{\Delta^2 + \gamma_{ab}^2} - \frac{8G^2}{\gamma_b^2} \frac{\gamma_a}{\gamma_a'^2 + \Delta^2} \right], \quad (9)$$

$$\sigma_p = |g|^2 N \frac{\gamma_a \gamma_b}{\gamma_a \gamma_b + 4G^2} \frac{\gamma_{ab}}{\Delta^2 + \gamma_{ab}^2}. \quad (10)$$

We shall solve (6) subject to the vacuum boundary condition

$$P(\alpha_1 \alpha_2 x = 0) = \delta^{(2)}(\alpha_1) \delta^{(2)}(\alpha_2).$$

We can readily show that the solution for the spontaneous emission in the direction of the strong field can be written in the form

$$P(\alpha_1 \alpha_2 x) = \frac{1}{\pi^2 I_a(x) I_p(x)} \exp \left[-\frac{|\alpha_1 + \alpha_2|^2}{4I_a(x)} - \frac{|\alpha_1 - \alpha_2|^2}{4I_p(x)} \right], \quad (11)$$

where

$$I_k(x) = \exp \left(\int_0^x \Gamma_k(x') dx' \right) \int_0^x \sigma_k(x') \exp \left(-\int_0^{x'} \Gamma_k(x'') dx'' \right) dx'.$$

To determine the fundamental profile, we evaluate the single-mode density matrix:

$$P(\alpha_1) = \int d^2 \alpha_2 P(\alpha_1 \alpha_2) = \frac{1}{\pi(I_a + I_p)} \exp \left[-\frac{|\alpha_1|^2}{I_a + I_p} \right].$$

It is clear that the fundamental profile is determined by the frequency dependence of the sum $I_a + I_p$ (this case is partly analyzed in [6]).

For a sufficiently thin layer of the resonant medium, when the change in the field in the resonant medium can be neglected, $I_k(x) \approx \sigma_k$ ($x = 0$, $k = a, p$) and the fundamental profile is determined by $\sigma_a + \sigma_p$ [see (7)-(10)] and is the same as that obtained by Rautian in [11].

For backward emission we must replace I_a and I_p in (11) by the quantity

$$I = \exp \left(\int_0^x \Gamma(x') dx' \right) \int_0^x \sigma(x') \exp \left(-\int_0^{x'} \Gamma(x'') dx'' \right) dx',$$

where

$$\Gamma = 1/2(\Gamma_a + \Gamma_p), \quad \sigma = 1/2(\sigma_a + \sigma_p).$$

The fundamental profile is therefore determined by the dependence of the quantity $2I$ on frequency. It is clear that when self amplification and self absorption are taken into account, the spectral profiles will be different in different directions ($I_a + I_p \neq 2I$). For thin layers, $2I \approx \sigma_a + \sigma_p$. Therefore, thin layers of the resonant medium emit the fundamental spectral profile in all directions in the same way.

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