Stimulated temperature scattering of electromagnetic waves in a plasma with collisions

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It is shown that the interaction of plane electromagnetic waves of nearly equal frequencies in a plamsa layer whose permittivity has a temperature-dependent imagniary part gives rise to effects similar to the well-known process of stimulated temperature scattering, [1] and that a periodic temperature profile whose presence has a significant effect on the character of electromagnetic-wave absorption in the plasma is established during the resonant interaction. Since to the temperature wave the medium is nonlinear, the shape of the wave is, in many cases, highly nonsinusoidal, and qualitatively alters the general pattern of spontaneous scattering of electromagnetic waves.

1. Different types of stimulated scattering (SS) of electromagnetic waves in a plasma have already been investigated quite thoroughly, but most of the results pertain to the collisionless plasma. The dominant process in this case is Raman scattering by particles or by the plasma oscillations (in a cold plasma), or scattering by longitudinal (Langmuir, ion-acoustic) waves. In the specified-phase approximation, these types of SS proceed essentially in the same way as the corresponding processes in nonlinear optics[1]. When the collisions in the plasma are sufficiently frequent, the processes of stimulated Mandel'shtam-Brillouin scattering (SMBS)—the scattering of electromagnetic waves by acoustic waves—and stimulated temperature scattering (STS) should predominate[1,2]. Which of these processes will be realized under specific conditions depends on the plasma parameters and the nature of the pumping. When the process takes place in a small volume (or when the thermal conductivity is large), we can assume that the process is isothermal and consider only SMBS. If the thermal conductivity is not high, then the dominant type of scattering will, apparently, be STS (the SMBS threshold in this case is considerably higher). In contrast to stimulated Raman scattering (SRS) and SS by plasma waves, the SMBS and STS processes in a plasma possess vital characteristics in comparison with the corresponding processes considered in nonlinear optics. This is connected, in particular, with the nonlinearity of the plasma for nonelectromagnetic waves (acoustic in the case of SMBS and thermal in the case of STS). For SMBS in a plasma with collisions, sound attenuation is not strong, and it is necessary to consider the harmonics that develop because of the hydrodynamic nonlinearity and grow in intensity in view of the absence of dispersion for the acoustic branch[2]. For the STS this nonlinearity is connected with the imaginary part of ϵ, and is due to the strong dependence of the electron collision rate on temperature. In contrast to nonresonant plasma heating, the character of electromagnetic-wave absorption qualitatively changes in the STS regime, and, furthermore, the indicated nonlinearity should lead to the establishment of a nonsinusoidal temperature profile. The development in the process of a large number of spatial harmonics affects the pattern of spontaneous electromagnetic-wave scattering; in particular, an intense scattering of the waves is possible in the frequency region where the plasma was previously transparent.

In the present paper we consider the STS of plane electromagnetic waves in a partially ionized plasma allowing a quasi-hydrodynamic description. The basic equations will be the wave equation for the field and the thermal conductivity equation for the electron component of the plasma:

$$\frac{\partial T_e}{\partial t}-\chi\frac{\partial^2 T_e}{\partial x^2}=\frac{2}{3N}\left\{\mathbf{jE}-\left(\frac{\partial\varepsilon}{\partial T}\right)_p T\mathbf{E}\frac{\partial\mathbf{E}}{\partial t}\right\}-\delta\nu(T_e-T),$$
$$\frac{\partial^2\mathbf{D}}{\partial t^2}-c^2\frac{\partial^2\mathbf{E}}{\partial x^2}=4\pi\frac{\partial\mathbf{j}}{\partial t},$$
$$\mathbf{D}=\int_{-\infty}^{0}dt'\int dx'\varepsilon(x-x',t-t')\mathbf{E}(x',t'),\tag{1}$$
$$\mathbf{j}=\int_{-\infty}^{0}dt'\int dx'\sigma(x-x',t-t')\mathbf{E}(x',t').$$

Here T_e is the electron temperature, T is the temperature of the heavy particles, χ is the thermal conductivity coefficient, and $\delta\sim m/M$.

In (1) we allow for two heating mechanisms—ohmic heating of the plasma[3] and heating connected with the electrocaloric effect[4]. The latter arises as a result of the action on the plasma of strictional forces, and is not connected with the particle collisions. The transfer of energy from one wave to the other during ohmic STS is irreversible, and the process is aperiodic in nature (similar to Raman scattering in a dissipative medium[5]). The electrocaloric STS proceeds like the other stimulated-scattering processes (e.g., SMBS) in media with damping. For low thermal conductivity, this process is typified, in particular, by a temporally periodic transfer of energy between the waves. Below we consider ohmic STS, i.e., we assume that the scattering occurs in a plasma whose nonlinearity with respect to the heating-up process is greater than the strictional nonlinearity (this is typical of, for example, the bottom layers of the ionosphere[3]). We shall henceforth assume the nonlinearity is weak and represent the field E in the form

$$E=E_p a=E_p(a_1e^{i(\omega t-kx)}+a_2e^{i(\omega t+kx)}+\text{c.c.}),\tag{2}$$

where $E_p^2=3m\delta T\omega^2/e^2$, and $a_{1,2}$ are slowly varying complex amplitudes.

2. Let us consider stationary processes in a layer of length L. Under the boundary conditions $a_1(x=0)=a_{10}$ and $a_2(x=L)=a_{2L}$, in the case when the thermal conductivity of the plasma can be neglected, an algebraic relationship between T_e and the electromagnetic-field intensity follows from the equation for T_e. Assuming that the collision rate increases with the temperature[1)] to wit, $\nu=\nu_0T_e/T$, we can find this relationship from (1)[3]:

$$|a|^2=(1+s^2b^2)(b-1);\quad s=\nu_0/\omega,\ b=T_e/T.\tag{3}$$

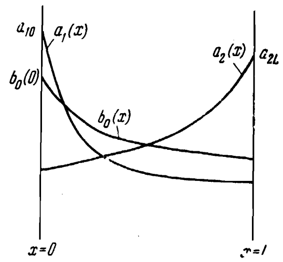

FIG. 1

For high-frequency ($\omega \gg \nu$, $sb \ll 1$) wave scattering, the equations for the intensities of the counterwaves can, when (3) is taken into account, be written in the form

$$dm_1/dx = -2\varkappa(1+m_1+2m_2)\,m_1, \qquad dm_2/dx = 2\varkappa(1+2m_1+m_2)\,m_2, \tag{4}$$

where $m_{1,2} = |a_{1,2}|^2$, $\kappa = 2\pi\sigma_0/c$, and σ_0 is the plasma conductivity for $T_e = T$. These equations have the first integral

$$m_1 m_2(1+m_1+m_2) = \text{const.} \tag{5}$$

As is well known[3,6], without allowance for STS, the scattered-field intensity $m_2 = \text{const}/m_1$. For STS in a sufficiently strong field $E \gg E_p$, it follows from (5) that electromagnetic-wave absorption increases significantly during the resonant heating of a plasma by a standing wave:

$$m_2 = \text{const}/m_1^2, \tag{6}$$

i.e., as a result of STS, the intensity of the scattered wave will be smaller by a factor of $(E/E_p)^2$.

3. Let us now consider the general case when s is arbitrary. Let us represent the dependence of the temperature profile on x in the form

$$b = b_0 + b_1 e^{-2ikx} + \text{c.c.} \tag{7}$$

where the b_i's are slowly varying complex amplitudes.

If the pump field is sufficiently strong, then allowance should be made for the fact that the mean heating b_0 of the layer significantly exceeds the resonant heating b_1, i.e., that $|b_0| \gg |b_1|$. The system of equations for the a_i and b_i can then be written in the form

$$\begin{aligned}
\dot a_1 + c a_1' &= -a_1 \frac{\omega v}{2(i+sb_0)} + a_2 b_1 \frac{\omega v s}{2(i+sb_0)^2},\\
\dot a_2 - c a_2' &= -a_2 \frac{\omega v}{2(i+sb_0)} + a_1 b_1^* \frac{\omega v s}{2(i+sb_0)^2},\\
\dot b_0 &= (|a_1|^2+|a_2|^2)\frac{\delta\omega s b_0}{1+s^2b_0^2} - \omega\delta s b_0(b_0-1),\\
\dot b_1 &= \delta\omega\left\{\frac{a_1 a_2^* s b_0}{1+s^2b_0^2} + s b_1\left[\frac{(|a_1|^2+|a_2|^2)(1-s^2b_0^2)}{(1+s^2b_0^2)^2} - 2b_0 + 1\right]\right\},
\end{aligned} \tag{8}$$

where $v = \omega_0^2/\omega^2$.

In the stationary case, assuming that $|a_1| \gg |a_2|$, we obtain from the equations for a_1 and b_0 the expressions

$$b_0' = -\frac{\omega v s}{c}\,\frac{b_0(b_0-1)}{2s^2b_0(b_0-1)+s^2b_0^2+1}, \qquad |a_1|^2 \Rightarrow (b_0-1)(1+s^2b_0^2). \tag{9}$$

Integrating (9), we find the solution in the form (the form of the solution is represented graphically in Fig. 1):

$$3b_0 - s^{-2}\ln b_0 + (1+s^{-2})\ln(b_0-1) = -x\omega v/cs + A, \tag{10}$$

where the constant A is determined from the boundary conditions.

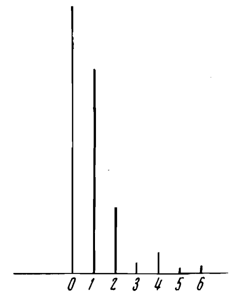

FIG. 2

From the equations for b_1 and a_2 we find

$$\frac{d}{dx}(\ln|a_2|^2) = \frac{\omega v s(2s^2b_0^3 - s^2b_0^2 + 2b_0 - 1)}{c(1+s^2b_0^2)(3s^2b_0^2 - 2s^2b_0 + 1)}, \tag{11}$$

where b_0 is a function defined by (10).

The expression (11) for any s increases along the x axis, i.e., the wave a_2 is not amplified. Notice that this result is not a priori obvious—a wave may be damped or amplified when scattered by a dissipative nonlinearity[5,7]. For low-frequency ($\omega \ll \nu$; $s \gg 1$) wave scattering, from (11) follows the nontrivial conclusion that there is for the scattered wave an additional illumination of the plasma layer by the pump field. In fact, in this case the equations for the wave intensities have, according to (1)–(3), the form

$$dm_1/dx = -\varkappa m_1^{1/3}, \quad dm_2/dx = {}^2/_3\varkappa m_1^{-1/3}\, m_2. \tag{12}$$

These equations have the solution

$$m_1 m_2^{3/2} = \text{const.} \tag{13}$$

Thus, resonant heating leads to the lowering of the absorption of the field a_2 by a factor of $m_1^{1/2}$.

4. Above we have considered the STS process in the sinusoidal temperature profile approximation. In the general case the considered scattering process and, in particular, spontaneous electromagnetic-wave scattering are influenced by the spatial harmonics of the temperature profile, which were not considered above. In the low-frequency case ($\omega \ll \nu$) we can find from (3) the spectrum of the temperature profile (Fig. 2 shows the first six harmonics):

$$b_n = \text{const}\Big/\Gamma\left(\frac{4}{3}+\frac{n}{2}\right)\Gamma\left(\frac{4}{3}-\frac{n}{2}\right), \tag{14}$$

where Γ is the gamma function and n is the number of the harmonic.

It is clear that the presence in the plasma layer of the temperature-profile harmonics leads to the spontaneous scattering of electromagnetic waves of frequencies at which the plasma was previously transparent. From (14) it follows, in particular, that the coefficient of reflection of an electromagnetic wave of frequency 2ω is only three times smaller than that of a wave of frequency ω. As can be seen from (14), the scattering of electromagnetic waves of frequencies $(2n+1)\omega$ can be neglected.

1)The approximation for $\nu(T_e)$ used here is, strictly speaking, valid for not too strong heating of the plasma. In the D-layer of the ionosphere, as a result of the Ramsauer effect, the linear approximation for $\nu(T_e)$ is quite a good approximation.

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