Structure functions of deep inelastic $ep$ scattering

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Results are presented of an analysis of all existing data on deep inelastic $ep$ scattering. Various parametrizations of the functions $vW_2$ and $R = \sigma_T/\sigma_\nu$ are considered. The region of scale invariance of the function $vW_2$ is established. It is shown that the experimental data are satisfactorily described by various parametrizations of $R$.

1. INTRODUCTION

Experiments on the deep inelastic scattering of high energy electrons by protons\(^1\) have led to substantial progress in understanding of photon-hadron interactions. Scale invariance (scaling) was discovered and it was shown that in the deep inelastic region the structure functions are of the order of unity.\(^2\) The discovery of scale invariance stimulated a large number of new directions in theoretical research: the parton model,\(^3\) self-similar asymptotic behavior in local field theory,\(^4\) and so forth.

In the present article we report\(^5\) the results of analysis of all experimental data on deep inelastic $ep$ scattering.\(^1,2,6,7\) The method of analysis used was that previously used in analysis of data on elastic $ep$ scattering\(^8\) and elastic scattering of electrons by deuterons.\(^9\) To determine the structure functions by this method it is necessary to parametrize these functions from the beginning. The parameters introduced here are determined directly from the experimental data by minimization of the $\chi^2$ functional. In a combined analysis of the data of different groups, norms are introduced—variable parameters taking into account possible errors in normalization of the cross sections. This method, in contrast to that used previously,\(^2\) permits use in determination of the structure functions of the entire set of experimental data.

The analysis showed that scale invariance of $vW_2$ occurs in the region $W \geq 2.3$ BeV ($W$ is the mass of the final hadrons). Various parametrizations were considered for the ratio of the total cross sections for absorption of virtual photons with longitudinal and transverse polarizations

$$R = \sigma_T/\sigma_\nu.$$  

From the results of the analysis it follows that the existing experimental data do not permit exclusion of those expressions for $R$ which signify violation of scale invariance of the function $2MW_1$ in the region of variables where scale invariance of $vW_2$ exists. If we assume that $vW_2$ depends on the variable $\omega$ proposed by Bloom and Gilman,\(^10\) the experimental data can be described for $W \geq 1.8$ BeV.

2. CROSS SECTION OF THE PROCESS. METHOD OF DATA ANALYSIS

It is well known that the differential cross section of the process

$$e^+p \rightarrow e^+\ldots$$  

for unpolarized initial particles has the following general form in the one-photon approximation (laboratory system):

$$\frac{d\sigma}{d\Omega dE'} = \left( \frac{d\sigma}{d\Omega} \right)_{\nu} \left[ W_2 + 2W_1, \frac{q^2}{2} \right],$$  

where

$$\frac{d\sigma}{d\Omega} = \frac{a^\nu}{e^\nu} \cos^\nu(\theta/2).$$

Here $E$ and $E'$ are the initial and final electron energies, $\theta$ is the scattering angle, and $W_1$ and $W_2$ are functions of the scalars

$$\nu \equiv \frac{1}{M} p_\nu = E - E', \quad q^2,$$

where $q$ and $p$ are the 4-momenta of the photon and proton and $M$ is the proton mass. The functions $W_1$ and $W_2$ are determined by the hadron part of the matrix element. We have

$$\sum f (f'IJp\langle f|p|J\nu\rangle \delta(p' - p - q) d\omega = e^\nu M \left[ W_1(\delta_{0\nu} - \delta_{0\nu}^{+}) + \frac{4}{M} W_1 \left( p_\nu - \frac{pq}{q^2} \right) \delta_{0\nu}^{+} \right].$$

Here $J_{\mu\nu}$ is the hadron electromagnetic current operator and $p'$ is the total 4-momentum of the final hadrons.

The total cross sections $\sigma_T$ and $\sigma_\nu$ are connected with $W_1$ and $W_2$ by the relations\(^11\)

$$\sigma_T = (2\pi)^3 \alpha \frac{1}{k} W_n,$$

$$\sigma_\nu = (2\pi)^3 \alpha \frac{1}{k} \left( W_1 + \frac{q^4 + q^4}{q^2} W_2 \right).$$

Here

$$k = -q^2/2M, \quad \alpha = e^\nu/4\pi.$$

For $q^2 \rightarrow 0$ we have

$$\sigma_\nu = \sigma_n, \quad \sigma_T = 0,$$

where $\sigma_n$ is the total cross section for absorption of protons by real photons with energy $\nu$.

From Eq. (7) it follows that

$$2MW_1 = \omega \nu W_1 \frac{1 + q^4/\omega q^2}{1 + R},$$

where the variable $\omega$ is defined as follows:

$$\omega = 2MW_1 q^4.$$

Since $R \geq 0$, we find from Eq. (10)

$$\frac{1 + q^4/\omega}{uvW_1} > 2MW_1.$$

To determine $vW_2$ and $R$ (or $vW_2$ and $2MW_1$) by the method used, it is necessary to parametrize these functions. The corresponding parameters are then found from all existing data by minimization of the functional...
\[ x' = \sum_{k} \left( \frac{1}{\Delta_{ik}} \exp^{-N_k \Delta_{ik}} \right)^{\nu}. \]  

Here \[ \Delta_{ik} \] is the differential cross section for the process at the \( i \)-th point measured in the \( k \)-th experiment, \( \Delta_{ik} \) is the error in \[ \exp^{-N_k \Delta_{ik}} \] is the cross section at the \( i \)-th point calculated with Eq. (3), and \( N_k \) are normalization factors. Minimization of the \( x' \) functional is carried out by the linearization method.\(^{12}\) The method of analysis has been described in more detail in an earlier article.\(^{13}\)

Bjorken\(^{13}\) showed on the basis of current algebra that for \( q^2 \to -\infty, \nu \to -\infty \) and fixed \( \omega = 2Mv/q^2 \) the dimensionless functions \( \nu' \) and \( 2MW_1 \) depend only on the variable \( \omega \) (scale invariance). It follows from experiments on deep inelastic ep scattering that scale invariance is consistent with the data even for \( q^2 \) and \( \nu \) of the order of 1 BeV. To start with we will assume scale invariance of \( \nu' \).

In parametrization of this function we will take into account the relation obtained in refs. 10 and 15 connecting the behavior of the function \( \nu' \) for \( \omega \to 1 \) with the behavior of the proton magnetic form factor \( GM(q^2) \) for \( q^2 \to -\infty \). If we write

\[ \nu' = \left( 1 - \frac{1}{\omega} \right)^{\nu}, \quad G_M(q^2) = \frac{1}{\nu}(q^2)^{\nu}, \]  

then, as shown by Drell and Yan\(^{15}\) and Bloom and Gilman\(^{16}\),

\[ p = n + 1. \]  

From analysis of the data on elastic ep scattering it follows that \( n = 2 \). Thus, if we assume Eq. (15),

\[ p = 3. \]  

In addition, it follows from the theory of complex angular momenta\(^{16}\) that

\[ \nu' \to \text{const}, \quad 2MW_1/\omega \to \text{const}. \]  

We will consider first for the function \( \nu' \) the following expression which satisfies conditions (14), (16), and (17):

\[ \nu' = \sum_{i} a_i \left( 1 - \frac{1}{\omega} \right)^{\nu_i}. \]  

The function \( 2MW_1 \) is related to \( \nu' \) and \( R \) by Eq. (10). We will consider various expressions for the ratio \( R \).

3. RESULTS OF THE ANALYSIS

1. We will assume that

\[ R = q^2/\nu'. \]  

Then, as can been from Eq. (10), the functions \( 2MW_1 \) and \( \nu' \) are related by the equation

\[ 2MW_1 = \omega \nu'. \]  

It should be noted that this relation arises in the parton model for the case of partons with spin \( 1/2, \)\(^{17}\) It occurs if the squares of the moduli of the matrix elements for photon absorption, summed over the final states, are equal for transverse and longitudinal polarization of the photons.

The results of the analysis of the data obtained by the SLAC-MIT group\(^{1,2}\) show that a satisfactory description of the experimental data is achieved in this case if the mass of the final hadrons \( W \geq 2.5 \) BeV. Note that the variable \( W \) is related to \( q^2 \) and \( \nu \) by the equation

\[ W^2 = 2MW_1 + M^2 - q^2. \]  

For this case the following three parameters are different from zero in Eq. (18):

\[ \begin{align*} a_0 &= 1.12 \pm 0.07, \\ a_1 &= -1.00 \pm 0.20, \\ a_2 &= -1.98 \pm 0.14 \end{align*} \]  

(\( \chi^2/\nu = 162/135; \) the confidence level is 5.5%).

2. It is possible to obtain a relation between \( 2MW_1 \) and \( \nu' \) which agrees with experiment over a wider range of the variable \( W \). Let us assume that scale invariance occurs both for \( \nu' \) and for \( 2MW_1 \). This assumption means that the ratio (1 + \( R/(1 + q^2/\nu') \)) depends only on \( \omega \). We will expand this ratio in a series in \( 1/\omega \). We obtain

\[ \omega'V = 1 + \frac{d}{\omega} + \frac{d_2}{\omega^2} + \ldots \]  

Analysis of the data showed that in this expansion it is sufficient to take into account the term \( d/\omega \). We have

\[ 2MW_1 = \omega'V + d/\omega. \]  

The experimental data obtained in refs. 1 and 2 for \( W \geq 2.3 \) BeV can be described by means of Eq. (24). In the expansion (18) it is sufficient to assume that \( a_0 \) and \( a_2 \) are nonzero. We obtain the following parameter values:

\[ a_0 = 1.84 \pm 0.02, \quad a_2 = -1.50 \pm 0.02, \quad d = 0.63 \pm 0.09. \]  

The ratio \( \chi^2/\nu = 162/150; \) the confidence level is 23.9%.

3. As we have seen, the data of refs. 1 and 2 in the region \( W \geq 2.3 \) BeV are satisfactorily described by means of three parameters if we assume that scale invariance exists both of \( \nu' \) and of \( 2MW_1 \). We can convince ourselves, however, that the existing data also are satisfactorily described for those parametrizations of \( R \) which signify violation of scale invariance of the functions \( 2MW_1 \). We will consider the following expressions for \( R \):

\begin{align*} R &= c_1 q^2/M^2, \quad (26.1) \quad R = c_2 q^2/W^2, \quad (26.2) \quad R = c_3/\omega, \quad (26.3) \quad R = \text{const.} \quad (26.4) \end{align*}

The first three expressions satisfy the condition \( R = 0 \) for \( q^2 = 0 \). Note that in the Bjorken limit (\( q^2 \to -\infty, \nu \to -\infty, \omega \) fixed) \( R = c_1 q^2/M^2 \to -\infty, \quad R = c_2 q^2/W^2 \to -c_2 (\omega - 1)^2 \).

Equations (26.1) and (26.4) have been discussed previously in ref. 2.

The results of analysis of the data of Bloom et al.\(^{11}\) and Miller et al.\(^{2}\) in the region \( W \geq 2.3 \) BeV are presented in Table I. It is evident from the table that for

| TABLE I. Results of analysis of the SLAC-MIT data \(^{1,2}\) with parametrization (18) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( R = \text{const} \) | \( R = \text{const} \) | \( R = \text{const} \) | \( R = \text{const} \) |
| \( q^2/\nu \) | \( q^2/\nu \) | \( q^2/\nu \) | \( q^2/\nu \) |
| \( 2MW_1 \) | \( 2MW_1 \) | \( 2MW_1 \) | \( 2MW_1 \) |
| \( 124/150 \) | \( 145/150 \) | \( 145/150 \) | \( 145/150 \) |
| \( 162/150 \) | \( 162/150 \) | \( 162/150 \) | \( 162/150 \) |
| \( 124/150 \) | \( 162/150 \) | \( 162/150 \) | \( 162/150 \) |
| \( 145/150 \) | \( 145/150 \) | \( 145/150 \) | \( 145/150 \) |
| \( 162/150 \) | \( 162/150 \) | \( 162/150 \) | \( 162/150 \) |

satisfactory description of the experimental data it is sufficient to assume the coefficients \( a_0 \) and \( a_2 \) to be nonzero in expression (18). The values of these coefficients for various parametrizations of \( R \) agree within experimental error. Note also that the coefficients \( a_0 \) and \( a_2 \) are nearly of the same magnitude and opposite in sign. The value found by us for the coefficient \( c_1 \) agrees with the value determined in ref. 2 by another method.

For \( R = \text{const} \) the description of the experimental data is substantially improved if we assume, in addition to \( a_0 \) and \( a_2 \), that the parameter \( a_1 \) is also nonzero.

4. We also analyzed that data of refs. 1 and 2, parametrizing the functions \( v_W \) and \( 2MW_1 \). For \( v_W \) we assumed Eq. (18). For the function \( 2MW_1 \) we assumed the expression

\[
2MW_1 = a_0 \sum_i b_i (1 - w^{-1})^{r_i}.
\]  

It is evident that Eq. (27) satisfies Eq. (17). It turned out that the data can be described by means of Eqs. (18) and (27). In the expansion (18) it is sufficient in this case to retain two terms, and in the expansion (27)—three terms. As a result of analysis of the data in the region \( W \geq 2.3 \text{ BeV} \) we find

\[
a_0 = 1.68 \pm 0.02, \quad a_1 = -1.57 \pm 0.04,
\]

\[
b_0 = 0.72 \pm 0.14, \quad b_1 = 1.35 \pm 0.42, \quad b_2 = -1.84 \pm 0.31,
\]

\[
(x^2/\chi^2 = 154/148; \text{ the confidence level is } 35.6\%).
\]

The functions \( v_W \) and \( 2MW_1 \) should satisfy inequality (12), which is equivalent to the condition \( \varphi_0 \geq 0 \). We note that in the last of the cases discussed this inequality at some points is satisfied only within two standard deviations.

5. Let us turn to discussion of the results presented in Table II (\( W \geq 2.3 \text{ BeV} \)) of a combined analysis of all published data on deep inelastic ep scattering. In the last three columns we have shown the norm values obtained: \( N_1 \) is the norm for the data of the SLAC-MIT group \([1,2]\) (153 points), \( N_2 \) and \( N_3 \) are the respective norms for the DESY 1971 data \([3]\) (83 points) and the DESY 1969 data \([4]\) (31 points).

From Table II we can conclude that only introduction of norms permits description of all existing data on deep inelastic ep scattering. The norm values for different parametrizations of \( R \) agree within experimental error. A combined description of all data in this case requires a significant renormalization of the results of Mottet et al. \([5]\). Comparison of Tables I and II shows that the parameter values obtained in analysis of the data of refs. 1 and 2 and of the world set of data agree within experimental error.

6. Up to this time we have used Eq. (18) for \( v_W \). We can convince ourselves that the experimental data on deep inelastic ep scattering can be satisfactorily described also for other parametrizations of the structure function \( v_W \). Let us consider the expression

\[
v_W = \left( \frac{a_0 + a_2}{a_0} \right) (1 - r_1 + r_2 - \frac{1}{r_0}).
\]

where \( r_1, r_2 \), and \( \omega_0 \) are the parameters which can be varied. This expression was proposed by Moffat and Snell \([6]\) and is based on the theory of complex angular momenta. It is evident that Eq. (29) satisfies both the threshold relation (16) and the asymptotic condition (17). For the ratio \( R \) the same expressions were considered as before. The results of analysis of the data of refs. 1 and 2 are given in Table III (\( W \geq 2.3 \text{ BeV} \)). As can be seen, if Eq. (29) is used for \( v_W \), an acceptable description of the data is obtained for all five of the parametrizations of the ratio \( R \) considered by us.

From comparison of the results given in Tables I and III, it follows that the parameters entering into the expression for \( R \) agree within experimental error for the two parametrizations of \( v_W \). In Table IV we have given the values of the functions \( v_W \) calculated from Eqs. (18) and (29) for \( R = c q^2/M^2 \) (columns 2 and 4) and for \( R = \text{const} \) (columns 3 and 5). Figures 1 and 2 show the function \( v_W \) as a function of \( \omega \), as obtained respectively from Eqs. (18) and (29) for the various parametrizations of the ratio \( R \) in the region of variation of \( \omega \) from 1 to 100. In all cases we used the parameter values obtained as the result of analysis of the data of refs. 1 and 2.

By analyzing the SLAC results \([1,2]\) we became convinced that for \( W \geq 2.3 \text{ BeV} \) it is possible to describe the data with three parameters over the entire experimentally investigated interval of \( q^2 \). The variable \( \nu \) varies in this case over the range \( 1.87 \text{ BeV} \leq \nu \leq 16.28 \text{ BeV} \), and the variable \( \omega \)—over the range \( 1.25 \leq \omega \leq 37.8 \). Values of the variable \( \omega \) greater than 20 correspond to \( q^2 \leq 1 \text{ (BeV/c)}^2 \). The question of the behavior of \( v_W \) at large \( \omega \) presents considerable interest for the theory. We made a separate analysis of the SLAC results, selecting data with \( q^2 \geq 0.5 \text{ (BeV/c)}^2 \), \( q^2 \geq 1 \text{ (BeV/c)}^2 \), and \( q^2 \geq 1.5 \text{ (BeV/c)}^2 \). The parameter values obtained as the result of this analysis for \( W \geq 2.3 \text{ BeV} \) are given in Tables V and VI.

7. Determination of the structure function \( v_W \) permits calculation of the integrals entering into the sum rules for this function. Let us first consider the Callan-Gross sum rule \([19]\):

\[
\int \frac{v_W}{\omega} \frac{d\omega}{\omega} = \sum_i \frac{Q_i}{N} \frac{1}{N} F_N.
\]

Here \( Q_i \) is the charge of \( i \)-th parton, \( P_N \) is the probability of observing \( N \) partons in a proton. Using the values of \( a_0 \) and \( a_2 \) given in Table I for \( R = \text{const} \), we obtain

\[
\int \frac{v_W}{\omega} \frac{d\omega}{\omega} = 0.162 \pm 0.001.
\]

If we use the values of \( a_0 \) and \( a_2 \) obtained for \( R = c q^2/M^2 \),

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Ref} & a_0 & a_2 & c_1 & N_1 & N_2 & N_3 & x/\omega^2 \\
\hline
R \text{ from (26.1)} & 1.50 \pm 0.02 & -1.42 \pm 0.02 & c_1 = 0.098 \pm 0.004 & 1.04 \pm 0.005 & 0.755 \pm 0.009 & 0.907 \pm 0.003 & 249/284 \\
R \text{ from (26.2)} & 1.58 \pm 0.02 & -1.45 \pm 0.02 & c_1 = 0.47 \pm 0.01 & 1.039 \pm 0.005 & 0.783 \pm 0.009 & 1.027 \pm 0.004 & 267/284 \\
R \text{ from (26.3)} & 1.50 \pm 0.02 & -1.45 \pm 0.02 & c_1 = 0.47 \pm 0.01 & 1.040 \pm 0.005 & 0.783 \pm 0.009 & 1.047 \pm 0.002 & 286/284 \\
R = \text{const} \quad (24) & 1.54 \pm 0.02 & -1.72 \pm 0.02 & c_1 = 0.31 \pm 0.08 & 1.039 \pm 0.005 & 0.759 \pm 0.009 & 1.127 \pm 0.002 & 303/281 * \\
\hline
\end{array}
\]

*The confidence level is 3.7%.
Finally, from the results of analysis of the SLAC data for $q^2 > 1.5$ (BeV/c)$^2$ for $R = 0.26 \pm 0.04$ we obtain

$$\int v_{W_1} \frac{d \sigma}{\omega} = -1.05 \pm 0.01,$$

TABLE III. Results of analysis of the SLAC-MIT data [1,2] with parametrization (29) ($W \gg 2.3$ BeV)

<table>
<thead>
<tr>
<th>$q^2$ (BeV/c)$^2$</th>
<th>$x_{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.5$</td>
<td>$1.0$</td>
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<tr>
<td>$0.5$</td>
<td>$1.0$</td>
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</tbody>
</table>

TABLE IV. Results of analysis of the SLAC-MIT data [1,2] with parametrization (29) ($W \gg 2.3$ BeV)

<table>
<thead>
<tr>
<th>$x_{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.5$</td>
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<tr>
<td>$1.0$</td>
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<tr>
<td>$1.5$</td>
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</tbody>
</table>

TABLE V. Results of analysis of the SLAC-MIT data [1,2] with parametrization (18) ($W \gg 2.3$ BeV)

<table>
<thead>
<tr>
<th>$q^2$ (BeV/c)$^2$</th>
<th>$x_{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.5$</td>
<td>$1.0$</td>
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<tr>
<td>$0.5$</td>
<td>$1.0$</td>
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</tbody>
</table>

TABLE VI. Results of analysis of the SLAC-MIT data [1,2] with parametrization (29) ($W \gg 2.3$ BeV)

<table>
<thead>
<tr>
<th>$q^2$ (BeV/c)$^2$</th>
<th>$x_{1/2}$</th>
</tr>
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<tbody>
<tr>
<td>$0.5$</td>
<td>$1.0$</td>
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<tr>
<td>$0.5$</td>
<td>$1.0$</td>
</tr>
</tbody>
</table>

Note that in the ordinary quark model the right-hand side of Eq. (30) is equal to 1/3.

For the integral entering into the Gottfried sum rule [16] we have for $R = \text{const}$ and for all $q^2$

$$\int v_{W_2} \frac{d \sigma}{\omega} = 0.97 \pm 0.01.$$

8. Up to this time we have assumed that $v_{W_2}$ depends only on $\omega$. We convinced ourselves that for very different assumptions about $R$ in this case it is possible to describe the entire set of data on deep inelastic ep scattering. In order to estimate the magnitude of the possible violation of scale invariance, we analyzed the SLAC data with the assumption that the function $v_{W_2}$ has the form

$$v_{W_2} = \sum a_i \left(1 - \frac{1}{\omega}ight)^{n_i} + \alpha M^2 / q^2.$$  

It is evident that the second term of this expression disappears in the Bjorken limit. We will assume that the functions $2M_{W_1}$ and $v_{W_2}$ are related by Eq. (24). As the result of analysis of the data on refs. 1 and 2 we find that for $x^2/\chi^2 = 150/149$

$$a_0 = 0.05 \pm 0.02, a_1 = -1.47 \pm 0.02,$$

$$d = 0.01 \pm 0.08, \alpha = -0.02 \pm 0.01.$$

Comparing Eqs. (36) and (25), we see that inclusion in $v_{W_2}$ of a term violating scale invariance does not change the values of the parameters $a_0$, $a_1$, and $d$ within experimental error. The parameter $\alpha$ which characterizes the violation of scale invariance is equal to zero within two standard deviations.


$$\omega = (2M_{W_1}) / q^2,$$

which for $2M_{V} > M^2$ is identical with $\omega$. It was shown that $v_{W_2}$ is a function of $\omega$, over a wider range of the variable $W$. We analyzed the data of refs. 1 and 2 with parametrization of $v_{W_2}$ as follows:

$$v_{W_2} = \sum A_i \left(1 - \frac{1}{\omega}ight)^{n_i},$$

where

$$\omega = (2M_{W_1}) / q^2.$$

Here it turned out that in the region directly adjoining the resonance ($W \geq 1.8$ BeV), the existing data can be described (see Table VII). In the expansion (38) it is sufficient to take as nonzero three parameters: $A_0$, $A_1$, and $A_2$. If we assume that $R = \text{const}$, then we obtain for the parameters being varied

$$A_0 = 0.60 \pm 0.08, A_1 = 9.66 \pm 0.17, A_2 = -2.37 \pm 0.10,$$

$$x^2/\chi^2 = 169/187.$$
In this analysis we chose data with $q^2 \geq 0.5$ (BeV/c)^2. If this limitation is not imposed, then, as can be seen from Table VII, for $W \geq 1.8$ BeV Eqs. (38) and (39) can satisfactorily describe the data in the case where Eqs. (26.2) and (26.3) are used for $R$.

The values of $A_1$ and $\beta$ obtained by us are in agreement with the values found by Miller et al.$^{[2]}$

If we assume that $2MW_1$ and $\nu W_2$ are related by the equation

$$2MW_1 = \omega \nu W_2 (1 + D/\omega)^{-1}$$

and $\nu W_2$ is given by Eq. (38), then for the parameters being varied we find ($W \geq 1.8$ BeV):

$A_1 = 0.50 \pm 0.08$, $A_2 = 2.35 \pm 0.17$, $A_3 = -2.71 \pm 0.10$.

$\beta = 1.03 \pm 0.09$, $D = 0.33 \pm 0.07$

($\chi^2/\nu = 215/191$; the confidence level is 11.5%).

In conclusion we would like to mention that there are seven points of refs. 1 and 2 which give a large (greater than 9) contribution to $\chi^2$ for all of the parametrizations considered by us. If these points are not included in the analysis of the data, the quality of the description is substantially improved in all cases.

4. CONCLUSION

We have analyzed the existing data on deep inelastic ep scattering. We have separately analyzed the data of the SLAC-MIT group$^{[1,2]}$ and the world data$^{[1,2,4,7]}$. We have shown that for $W \geq 2.3$ BeV the data are satisfactorily described on the assumption that the function $\nu W_2$ depends only on the scale variable $\omega$. Various parametrizations of this function were considered. Violation of scale invariance of $\nu W_2$ was introduced. It was shown that the parameter characterizing the violation is nonzero only by two standard deviations.

Various expressions have been considered for the ratio $R$. The existing data do not permit definite conclusions to be drawn about $R$ and the scale invariance of $2MW_1$. For unique determination of the ratio $R$, data are necessary on the cross sections for deep inelastic scattering of leptons by protons over a wider range of the kinematic variables.

We have checked in detail the validity of the relation $2MW_1 = \omega \nu W_2$ which follows from the parton model in the case in which the spin of the partons is 1/2. If this relation is modified by introduction of an additional term whose contribution disappears asymptotically, then, as shown by analysis of the data, all data can be described satisfactorily. If we assume that $\nu W_2$ is a function of $\omega'$, then in this case all data for $W \geq 1.8$ BeV can be described.

The authors take pleasure in thanking A. A. Ansel'm, Z. Kunst, and Ya. A. Smorodinskii for helpful discussions of the questions considered here.

TABLE VII. Results of analysis of the SLAC-MIT data$^{[1,2]}$ with parametrization (38) and (39)

<table>
<thead>
<tr>
<th>$N$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$\beta$</th>
<th>$D$</th>
<th>$\chi^2/\nu$</th>
</tr>
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<tbody>
<tr>
<td>R from (26.1)</td>
<td>0.93 ± 0.11</td>
<td>2.06 ± 0.10</td>
<td>0.37 ± 0.04</td>
<td>0.78 ± 0.07</td>
<td>220/191</td>
<td>0.44%</td>
</tr>
<tr>
<td>R from (26.2)</td>
<td>0.70 ± 0.09</td>
<td>2.09 ± 0.11</td>
<td>0.64 ± 0.07</td>
<td>0.85 ± 0.07</td>
<td>223/191</td>
<td>0.46%</td>
</tr>
</tbody>
</table>

1) The possibility of this behavior of the structure functions was discussed many years ago by Markov.$^{[7]}$
2) Preliminary results were presented at the XVI Conference on High-Energy Physics at Chicago and Batavia in September 1972.