Suppression of inelastic channels under hyperfine splitting conditions. Interference phenomena

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The specifics of diffraction of resonant γ quanta by perfect crystals in the presence of hyperfine splitting of nuclear levels are considered for γ-quantum energies at which the suppression effect is realized simultaneously for two polarizations. It is shown that in this case some peculiar interference phenomena arise and are manifest in the energy dependence of the integral characteristics. Various possibilities of experimental observation of the effects are considered.

INTRODUCTION

In a preceding paper [1] we considered resonant interaction of γ quanta with Mossbauer-type nuclei in regular crystals, when the nuclei are subject to the action of internal magnetic and electric fields. As shown in [2] (see also the references in [1]), the regularity in the arrangement of the nuclei in the crystal lattice can radically change the character of the resonant nuclear interaction. Under certain conditions, the γ quanta will propagate through the crystal without being absorbed, although under ordinary conditions (polycrystalline samples) the same quanta are absorbed in the strongest manner by the nuclei, and the absorbed energy is consumed principally by the inelastic internal-conversion process. This phenomenon has been called the effect of suppression of the inelastic channels of the nuclear reaction (SE) and has now been observed experimentally (see the references in [1]). In [1] they were investigating the conditions for the realization of the complete SE in the case of hyperfine splitting of the nuclear levels. It turned out that, unlike the case of an unsplit line, when the SE takes place only for γ quanta that are polarized in a definite manner [2], in the presence of splitting there are possible situations when the SE takes place simultaneously for both polarizations. This means that two linearly independent superpositions of electromagnetic waves can propagate simultaneously through the crystal without absorption (or with a very small absorption coefficient). These superpositions can have different phase velocities, and this leads to rather unique interference phenomena. The present paper is devoted to this question. We shall make extensive use of the results of [1], which will be referred to as I.

2. GENERAL FORMULATION OF PROBLEM

Assume that resonant γ quanta are incident on a crystal at an angle close to the Bragg angle. We assume that the Bragg condition is satisfied only for one of the reciprocal-lattice vectors. The motion of the γ quanta in the crystal is specified by a system of dynamic equations (2), I, with coefficients $g_{hh'}^{ss'}$ defined by formulas (3), I and (15), I–(18), I, while the wave field inside the crystal is determined by formula (9), I.

As shown in I, if the energy of the γ quanta falls in a spectral region where there are two adjacent lines (or one doubly degenerate line) of the hyperfine structure; then if the Bragg condition is exactly satisfied,

$$\alpha = \mathrm{K}(\mathrm{K}+2\varkappa)/\varkappa^2 = 0,$$

two wave superpositions can propagate simultaneously through the crystal without absorption. On the other hand, the two remaining superpositions of the wave would experience strong absorption with a coefficient approximately double the usual coefficient of nuclear absorption μ_{nuc}. Therefore even at a relatively small depth of the crystal t, at $\mu_{\text{nuc}}t/\gamma_0 \gtrsim 1$ and at $|\alpha| \ll |g_h^{ss}|$, the wave field will be described by an expression of the type

$$\mathbf{E}(\mathbf{r},\alpha) = \exp(i\varkappa\mathbf{r})\sum_{m=1}^{2}\exp\left(i\frac{\varkappa t}{\gamma_0}\varepsilon_m(\alpha)\right)[\mathbf{E}_{0m}^{(s)}(\alpha)+\mathbf{E}_{1m}^{(s)}(\alpha)\exp(i\mathbf{K}\mathbf{r})]. \quad (1)$$

Here $\epsilon_m(\alpha)$ are the roots of the dispersion equation (8), I, which in the considered approximation of two lines vanish rigorously at $\alpha = 0$; $\mathbf{E}_{hm}^{(s)}(\alpha)$ are the solutions of the system of equations (2), I corresponding to these roots, with allowance for the boundary conditions (10), I under the assumption that radiation with a polarization vector $\mathbf{e}_0^{(s)}$ is incident on the crystal.

The intensities of the γ quanta with polarization $\mathbf{e}_{0,1}^{(f)}$ passing through the crystal (J_0) and diffracted in the crystal (J_1) are given by the following expression ($h = 0, 1$):

$$J_h^{(f,s)}(\alpha) = \sum_{m=1}^{2}\exp\left(-\frac{2\varkappa t}{\gamma_0}\operatorname{Im}\varepsilon_m(\alpha)\right)|\mathbf{e}_h^{(f)*}\mathbf{E}_{hm}^{(s)}(\alpha)|^2$$
$$+2\operatorname{Re}\left\{\exp\left[-\frac{\varkappa t}{\gamma_0}\operatorname{Im}(\varepsilon_1(\alpha)+\varepsilon_2(\alpha))+i\frac{\varkappa t}{\gamma_0}\operatorname{Re}(\varepsilon_1(\alpha)-\varepsilon_2(\alpha))\right]\right.$$
$$\left.\times(\mathbf{e}_h^{(f)}\mathbf{E}_{h2}^{(s)*}(\alpha))(\mathbf{e}_h^{(f)*}\mathbf{E}_{h1}^{(s)}(\alpha))\right\}. \quad (2)$$

The second term in (2) leads to an oscillatory dependence of the intensities $J_h^{(f,s)}(\alpha)$ on the crystal thickness t at a fixed value of α (or on α at a fixed crystal thickness), i.e., in this case we have a typical manifestation of interference between two anomalously transmitted waves. (It should be noted here that the type of interference phenomena considered by us differs significantly from the pendellosung effect known in the diffraction of x-rays. Indeed, the pendellosung effect takes place only in weakly absorbing crystals, whereas in our case we are dealing with the appearance of interference under strong-absorption conditions.) It is easily seen that interference can also appear in the dependence of $J_h^{(f,s)}(\alpha)$ on the energy of the incident γ quanta. Indeed, the change in energy will be accompanied by a change in the magnitude and the character of the interaction of the γ quanta with the nuclei, and consequently also in $\operatorname{Re}(\epsilon_1(\alpha) - \epsilon_2(\alpha))$. A change in the γ-quantum energy is equivalent in a certain sense to a change in the crystal thickness. The latter circumstance, naturally, greatly facilitates experimental observation of the interference effects. Moreover, in many cases interference phenomena in the form of unique energy dependences can also become manifest in integral characteristics (with respect to the angles). Such cases are the simplest from the experimental point of view, and their detection will be the principal task of the present paper (see Sec. 3).

It is clear from general physical considerations that to realize interference phenomena it is necessary that definite phase relations exist between the amplitudes of the waves $\mathbf{E}_{h1}^{(s)}(\alpha)$ and $\mathbf{E}_{h2}^{(s)}(\alpha)$ (see formula (2)). This phase connection can be established experimentally by exposing the crystal to radiation that is polarized in a definite manner. This class of cases will be considered in Sec. 4. In addition, it turns out that in many cases such phase relations can "develop" over a sufficient thickness in the crystal even when unpolarized radiation is incident on the crystal. This effect, which will become manifest in a nontrivial energy dependence of the integral intensities, seems to us to be the most interesting from the physical point of view, since it will be realized in simplest fashion experimentally. It will be considered in Sec. 5.

3. INTERFERENCE EFFECTS IN INTEGRAL CHARACTERISTICS

We turn again to expression (2). At small α ($|\alpha| \ll |g_{hh'}^{ss'}|$), the roots of $\epsilon_m(\alpha)$ are expanded in a series

$$\varepsilon_m(\alpha) \approx d_m\alpha + c_m\alpha^2 \quad (m = 1, 2). \tag{3}$$

As follows directly from the solution of the dispersion equation, the coefficients d_m are real and ~1, while the coefficients c_m are complex and $\sim 1/g_0$. In the general case the difference $d_1 - d_2 \sim 1$, and therefore the interference term in (2) oscillates rapidly and, upon integration with respect to the angle α, makes practically no contribution to the integral characteristics. It is therefore clear that the interference term can become manifest in the integral characteristics in those cases when $|d_1 - d_2| \lesssim (\mu_{nuct}/\gamma_0)^{-1/2}$. We shall reveal first in turn all the possible situations when the equality $d_1 = d_2$ is rigorously satisfied. We shall call them cases of second-order degeneracy.

Let the energy of the γ quanta fall in a spectral region where there are only two nondegenerate lines. The coefficients of the system of dynamic equations are then given by

$$g_{hh'}^{ij} = -g_0 \sum_{p=1}^{2} A_{hp}^{i} R_p A_{h'p}^{j*}, \tag{4}$$

where $R_p = [2(E - E_p)/\Gamma + i]^{-1}$, and the quantities g_0 and A_{hp}^{i} are defined in I.

A direct analysis shows that equality of the coefficients d_m is equivalent to satisfaction of the condition

$$(\Delta_{01}^{\pi\pi} + \Delta_{01}^{\pi\sigma} + \Delta_{01}^{\sigma\pi} + \Delta_{01}^{\sigma\sigma})^2 = 4\Delta_{00}^{\pi\sigma}\Delta_{11}^{\sigma\pi}, \tag{5}$$

where

$$\Delta_{hh'}^{ss'} = |A_{h1}^{(s)}A_{h'2}^{(s')} - A_{h2}^{(s)}A_{h'1}^{(s')}|^2, \tag{6}$$

and $A_{hp}^{(s)} = \mathbf{e}_h^{(s)} \cdot \mathbf{A}_{hp}$. In accordance with (6), the condition (5) does not depend on the energy of the γ quanta in the considered region of the spectrum, and consequently its satisfaction is determined only by the geometry of the experiment. Moreover, the condition

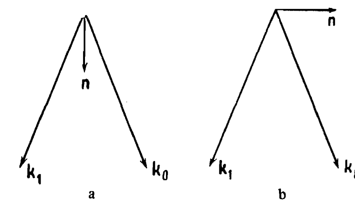

FIG. 1

(5) does not contain the parameter β, which determines the asymmetry of the reflection.

Let us consider by way of example the case when there is an axially symmetrical electric field gradient (EFG) at the nucleus, and the nuclear transition is of type E1 (or M1). In this case all the lines will be doubly degenerate (with the exception of the case when $|M| = |M_0| = 1/2$), and for E1 transitions we have

$$\mathbf{A}_{h1} = \mathbf{A}_{h2}^{*} = \mathbf{n}_x + i\mathbf{n}_y, \tag{7}$$

and for M1 transitions*

$$\mathbf{A}_{h1} = \mathbf{A}_{h2}^{*} = \varkappa^{-1}[\mathbf{k}_h, \mathbf{n}_x + i\mathbf{n}_y], \tag{8}$$

where $\mathbf{n}_x$ and $\mathbf{n}_y$ are two arbitrarily chosen mutually perpendicular unit vectors, which in turn are perpendicular to the unit vector $\mathbf{n}$ in the direction of the EFG axis. Taking (7) and (8) into account, we easily find that both in the case of E1 transitions and in the case of M1 transitions the condition (5) is satisfied exactly when $\mathbf{n}$ lies in the scattering plane $(\mathbf{k}_0, \mathbf{k}_1)$ symmetrically with respect to the vectors $\mathbf{k}_0$ and $\mathbf{k}_1$ (see Fig. 1). A similar situation also holds in the case of a collinear antiferromagnet if the magnetic field lies in the $(\mathbf{k}_0, \mathbf{k}_1)$ plane symmetrically with respect to the vectors $\mathbf{k}_0$ and $\mathbf{k}_1$.

4. INTERFERENCE IN EXPERIMENT WITH POLARIZED γ QUANTA

Assume that γ quanta of definite polarization $\mathbf{e}_0^{(s)}$ are incident on the crystal, and assume that after the radiation passes through the crystal one measures the intensity of γ quanta having only a definite polarization ($\mathbf{e}_0^{(f)}$ in a transmitted beam or $\mathbf{e}_1^{(f)}$ in a diffracted beam). We also assume that the condition (5) of second-order degeneracy is rigorously satisfied, i.e., at small α the roots $\epsilon_{1,2}$ corresponding to the anomalously transmitted waves are described by formula (3) with $d_1 = d_2$.

In addition, we put for simplicity $R_1 = R_2$. The coefficients c_m are then given by

$$c_m = \frac{1}{2g_0} p_m (x + i), \tag{9}$$

where $x = 2(E - E_1)/\Gamma$ and p_m are certain, generally speaking unequal, real constants.

Inasmuch as under the assumptions made the vectors $\mathbf{E}_m^{h(s)}(\alpha)$ which enter in formula (2) do not depend on the parameter x, and their dependence on α at $|\alpha| \ll g_0$ can be neglected, we obtain the following expression for the intensities integrated over the angles

$$J_h(x) = \int_{-\infty}^{\infty} d\alpha\, J_h^{(f,s)}(\alpha) = g_0 \left(\frac{\pi\gamma_0}{\varkappa g_0 t}\right)^{1/2} \left\{ I_h^{(1,1)} \frac{1}{\sqrt{p_1}} + I_h^{(2,2)} \frac{1}{\sqrt{p_2}} + \frac{2}{\sqrt{p_1 + p_2}} [F_1(y)\,\mathrm{Re}\, I_h^{(1,2)} + F_2(y)\,\mathrm{Im}\, I_h^{(1,2)}] \right\}, \tag{10}$$

where

$$y=\frac{p_2-p_1}{p_2+p_1},\quad F_1(y)=\left(\frac{\sqrt{1+y^2}+1}{1+y^2}\right)^{1/2},$$

$$F_2(y)=\frac{y}{[(1+y^2)(\sqrt{1+y^2}+1)]^{1/2}},\tag{11}$$

$$I_h^{(m,n)}=(\mathbf{e}_h^{(f)*}\mathbf{E}_{hm}^{(s)})(\mathbf{e}_h^{(f)}\mathbf{E}_{hn}^{(s)*}).\tag{12}$$

As seen from the obtained formulas, the character of the energy dependence of the integral intensities $J_h(x)$ is determined to a significant degree by the polarization of the incident radiation and by the detected polarization. Figure 2 shows plots of the integral intensities $J_0(x)=J_1(x)$ against the parameter y for the case of symmetrical Laue diffraction in quadrupole splitting of nuclear levels, when the orientation of the EFG axis corresponds to Fig. 1a, and $\cos\theta=0.75$. Curve 1 corresponds to the case when the polarizations of the incident and detected radiations are equal, and curve 2 corresponds to the case of orthogonal polarizations. Finally, curves 3 and 4 are given for cases when one of the polarizations, $\mathbf{e}_0^{(s)}$ or $\mathbf{e}_h^{(f)}$ is linear, and the other is circular. We note that the interference effect is maximal at the "largest" difference between the polarizations of the incident and detected radiations and the intrinsic polarizations (i.e., for cases when each of the polarizations $\mathbf{e}_0^{(s)}$ and $\mathbf{e}_h^{(f)}$ is either circular or linear, making a 45° angle with the scattering plane), and that the curves in Fig. 2 are given precisely for these cases. The interference term vanishes completely if at least one of the polarizations, $\mathbf{e}_0^{(s)}$ or $\mathbf{e}_h^{(f)}$, is the intrinsic polarization, when unpolarized radiation is incident on the crystal, and also if one measures the intensity summed over the final polarizations.

We note that the need for working with polarized γ quanta partly pays for itself in that it is possible to work with a noncollinear beam (the integral intensity is measured).

5. CASE OF UNPOLARIZED BEAM

In the preceding section it was noted that when averaging over the initial polarizations, or summing over the final polarizations, the dependence of the integral intensities on the γ-quantum energy disappears completely. This result was essentially connected with the special choice of the parameters of the problem ($d_1=d_2$ and $R_1=R_2$), wherein the polarizations of the natural waves in the crystal do not depend on x and are orthogonal to one another. If the condition $d_1=d_2$ is disturbed somewhat, leaving the difference $d_1-d_2 \sim(\mu_{\text{nuct}}/\gamma_0)^{-1/2}$, then the roots ϵ_m and the polarization eigenvectors $\mathbf{e}_{hm}$ exhibit unique dependences on x, and this leads in the final analysis to a dependence

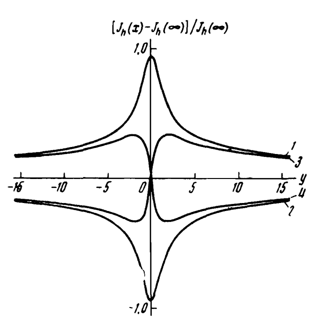

FIG. 2

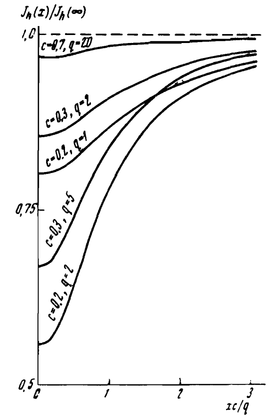

FIG. 3

of the intensities $J_h(x)$ on the energy of the γ quanta even in the case of unpolarized beams.

In the analysis that follows we confine ourselves for simplicity to the case of quadrupole splitting, which is different from that shown in Fig. 1 in that the EFG axis deviates slightly from the scattering plane, so that the component $n_\perp$ of the unit vector n in the direction of the EFG that is perpendicular to the scattering plane is small ($n_\perp \ll 1$). To determine the roots ϵ_m and the vectors $\mathbf{e}_{hm}$ it is necessary to solve the secular equation accurate to terms of second order in α and $n_\perp$. Simple calculations lead to the following expressions:

$$\varepsilon_{1,2}=-\frac{\alpha}{4}+\frac{\alpha^2}{16g_0}(x+i)\left[1+\frac{1}{2}\operatorname{tg}^2(\varphi+\theta)\right] \mp\frac{\alpha\sin(\varphi+\theta)}{4}\left\{n_\perp^2+\left[\frac{\alpha\sin(\varphi+\theta)}{8g_0\cos^2(\varphi+\theta)}(x+i)\right]^2\right\}^{1/2};\tag{13}$$

$$\mathbf{e}_{01}=\frac{n_\perp\mathbf{e}_\sigma+\xi\mathbf{e}_{0\pi}}{(n_\perp^2+\xi^2)^{1/2}},\quad \mathbf{e}_{11}=\frac{-n_\perp\mathbf{e}_\sigma-\xi\mathbf{e}_{1\pi}}{(n_\perp^2+\xi^2)^{1/2}},$$

$$\mathbf{e}_{02}=\frac{n_\perp\mathbf{e}_{0\pi}-\xi\mathbf{e}_\sigma}{(n_\perp^2+\xi^2)^{1/2}},\quad \mathbf{e}_{12}=\frac{-n_\perp\mathbf{e}_{1\pi}+\xi\mathbf{e}_\sigma}{(n_\perp^2+\xi^2)^{1/2}},\tag{14}$$

where

$$\xi=(z^2+n_\perp^2)^{1/2}-z,\quad z=\frac{\alpha\sin(\varphi+\theta)}{8g_0\cos^2(\varphi+\theta)}(x+i),\tag{15}$$

and $\varphi=0$ and $\varphi=\pi/2$ for the cases given respectively in Figs. 1a and 1b. Inasmuch as at $\alpha\neq 0$ the parameter ξ is complex, the Hermitian product of the polarization vectors

$$\mathbf{e}_{h2}^*\mathbf{e}_{h1}=n_\perp(\xi-\xi^*)\,/\,|n_\perp^2+\xi^2|\tag{16}$$

does not vanish, generally speaking, and the interference term does not vanish after averaging over the polarizations. The expression for the intensities, averaged over the initial polarizations and summed over the final polarizations, then takes the form

$$J_h(\alpha)=\frac{1}{8}(\mathbf{e}_{01}^*\mathbf{e}_{01})^2\left[\exp\left(-\frac{2\varkappa t}{\gamma_0}\operatorname{Im}\varepsilon_1\right)+\exp\left(-\frac{2\varkappa t}{\gamma_0}\operatorname{Im}\varepsilon_2\right)\right] +\frac{1}{4}|\mathbf{e}_{01}^*\mathbf{e}_{02}|^2\exp\left(-\frac{\varkappa t}{\gamma_0}\operatorname{Im}(\varepsilon_1+\varepsilon_2)\right)\cos\left(\frac{\varkappa t}{\gamma_0}\operatorname{Re}(\varepsilon_1-\varepsilon_2)\right).\tag{17}$$

It should be noted that now, in contrast to the case considered in the preceding section, the eigenvectors of the polarization and the imaginary parts of the roots $\epsilon_{1,2}$ depend essentially on the energy of the incident γ quanta. It is then impossible to obtain an analytic expression for the intensities integrated over the angles in the general case. Figure 3 shows plots of the integral intensity against the energies of the incident γ quanta for several sets of the parameters $c=\cos(\theta-\varphi)$ and $q=8n_\perp c^2[\kappa g_0 t/\gamma_0(1-c^2)]^{1/2}$. It is seen from the results that the relative magnitude of the effect increases with increasing q and with decreasing c. At $c \ll 1$, the condition $q \gg 1$ imposes essential lower bounds on the crystal thickness, and this leads to a noticeable decrease in the absolute value of the effect. We note that the resultant $J_h(x)$ dependence has the form of a dip near $x = 0$. A similar dip can be connected with the presence of defects in a crystal; at first glance, therefore, it can be assumed that the proposed effect will be masked by a dip due to the nonideal character of the crystal lattice. It is easily seen however, that the additional dip, which is connected with the interference phenomena, depends essentially on the quantity $n_\perp$, whereas the dip due to the defects is practically independent of $n_\perp$ in the case of a small deviation of the EFG axis from the scattering plane. Therefore the interference effect considered by us can readily be observed in experiment.

*$[k_h, n \times + in_y] \equiv k_h \times (n_x + in_y)$.

[1]A. M. Afanas'ev and Yu. Kagan, Zh. Eksp. Teor. Fiz. **64**, 1958 (1973) [Sov. Phys.-JETP **37**, 987 (1973)].
[2]A. M. Afanas'ev and Yu. Kagan, Zh. Eksp. Teor. Fiz. **48**, 327 (1965) [Sov. Phys.-JETP **22**, 215 (1965)].

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