Effect of magnetic breakdown on the Hall effect in beryllium

N. E. Alekseevskii and V. I. Nizhankovskii

Institute of Physics Problems, USSR Academy of Sciences
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The Hall effect in beryllium is investigated under magnetic breakdown conditions. The breakdown field strength is estimated to be $H_{\text{B}} \approx 110$ kOe on the basis of the magnetic field dependence of the Hall constant. Giant oscillations of the symmetric (with respect to the field) component $\rho_{yx}$ are observed. The change in the phase shift between the oscillations of $\rho_{yx}$ and $\rho_{xx}$ which is observed on deviation of the hexagonal axis of the sample from the direction of the magnetic field, can be explained on the basis of a coherent model for the breakdown.

INTRODUCTION

Investigation of the galvanomagnetic properties in strong magnetic fields is now a very effective method of studying the behavior of electrons in metals. Such investigations, as is well known, make it possible to determine the topological features of the Fermi surface of a metal and to obtain the values of some of its parameters $^{(1,2)}$.

In a number of metals, the characteristic of the carrier motion can undergo serious changes when the magnetic field is increased. Such changes may be connected with magnetic breakdown $^{(3)}$, which has been the subject of a rather large number of theoretical and experimental studies. Modern theoretical studies of magnetic breakdown are based on two concepts, stochastic and coherent. In the stochastic approach, the transitions of an electron from one classical orbit to another are regarded as independent of one another $^{(4)}$. In the coherent approach, the phase relations are taken into account for the wave functions of the electrons that are multiply scattered by the magnetic-breakdown regions $^{(5,6)}$.

Whereas the stochastic model is a satisfactory approximation when it comes to describing the monotonic part of the resistivity tensor $^{(7)}$, some features of the behavior of the oscillating part make it necessary to resort to the recently developed coherent model $^{(8)}$. In this paper we investigate the influence of magnetic breakdown on the off-diagonal element ($\rho_{yx}$) of the resistivity tensor of beryllium; to this end, we use a method developed by us to record a signal proportional to $\rho_{yx}$ as a function of $\rho_{xx}$ (the diagonal element).

2. MEASUREMENT METHOD

All the measurements were performed on three samples of beryllium with dimensions $0.3 \times 0.8 \times 5$ mm. To ascertain the influence of the purity of the sample on the measurement results, we used single-crystal beryllium with various resistivity ratios $\rho(300K)/\rho(4.2K)$, from 150 to 400. $^{13}$ It turned out that the purity of the samples employed had practically no influence on the measurement results (only a small increase was observed in the amplitude of the oscillating components of the signals picked off samples having the larger resistivity ratio).

The mounting of the samples is shown schematically in Fig. 1. The samples were cut and mounted in such a way that the measuring current $J$ was parallel to the $[\bar{1}0\bar{1}]$ axis, and the [0001] axis was perpendicular to the plane of the sample. To obtain a uniform distribution of the current over the sample and to avoid the effects observed in $^{10}$, each current lead consisted of four wires welded with the aid of a laser. The potential terminals were welded either with a laser or with a spark; the distance between them was $\pm 1$ mm. The accuracy (equipotentiality) of the Hall terminals was monitored by the value of the voltage $U_{yx}$ in the absence of a magnetic field. To determine the contribution of the resistivity signal to $U_{yx}$ with increasing magnetic field, we used the $\rho_{xx}(H)$ plot measured with the same sample. The contribution of the resistivity signal to $U_{yx}$ in fields $\gtrsim 20$ kOe, determined by this method, did not exceed 0.5%. After welding of the current and potential terminals, the sample was mounted in a holder placed between permendur concentrators (the device used to rotate the sample in the magnetic field was described in detail earlier $^{10}$). The investigations were carried out at a temperature 4.2 K in the field of a superconducting magnet.

Figure 2 shows a typical plot of $U_{yx}(H)$ when the sample is so rotated that the angle $\alpha$ between the [0001] axis of the sample and the direction of the magnetic field is $2^\circ 30'$ (in this case, as in all other measurements, the current was perpendicular to the magnetic field). The oscillations of $U_{yx}$ begin in the same field region as the oscillations of the resistivity. When the $U_{yx}$ and $U_{xx}$ signals were recorded simultaneously, a phased shift relative to the magnetic field was observed between them.

For detailed measurements of the monotonic and oscillating parts of $U_{xx}$ and $U_{yx}$ we used the following method: we determined the angle $\sigma$ of inclination of the hexagonal axis of the sample to the direction of the magnetic field, the signals $U_{xX}$ and $U_{yx}$ were partially balanced with potentiometers, and the unbalance signals were fed through photoamplifiers to the X and Y coordinates of an automatic X–Y recorder. The magnetic field was varied in a small range ($\pm 2$–3 kOe), and an “untwisting” ellipse was obtained on the automatic recorder (the “untwisting” is due to the field dependence of the monotonic signal components). A typical plot of this type is shown in Fig. 3. Assuming that the variations of the monotonic components of the signals at such a small variation of the magnetic field are linear in the field, it was possible to eliminate the untwisting (corresponding in Fig. 3 to the dashed “non-untwisting” ellipse in Fig. 3). From the shapes and positions of the ellipse obtained by this method we determined the following quantities:

1) The values of the monotonic components of the signals $U_{yx}$ and $U_{xx}$ (from the position of the center of the ellipse).

2) The amplitudes $A_{YX}$ and $A_{XX}$ of the oscillating components of the voltages (from the lengths of the size of the circumscribed rectangle).

3) The phase difference $\theta$ between the $U_{yx}$ and $U_{xx}$ oscillations ($\sin \theta = a/b$ (Fig. 3); in addition, we determined the direction in which the ellipse is traced and the position of its major axis).

In the present study we investigated the angular de-
3. EXPERIMENTAL RESULTS AND DISCUSSION

Antisymmetrical Components of the Signal $U_{yx}$ ($E_1$)

As is well known \cite{1}, expansion of the nondiagonal element of the conductivity tensor in the magnetic field takes in general the form

$$
\sigma_{yx} = \frac{e}{H} \Delta n + \frac{a_1}{H} + \frac{a_2}{H} + \ldots,
$$

where $\Delta n = n_1 - n_2$ ($n_1$ is the number of electrons and $n_2$ is the number of holes). Since beryllium is a compensated metal ($n_1 = n_2$), the first term of the series vanishes, and the Hall constant $R_H = \sigma_{yx} D / J$ ($D$ is the sample thickness and $J$ is the measuring current) is given by

$$
R_H = \frac{E_1}{H} \approx \frac{E_2}{H} \left( \frac{a_1}{H} + \frac{a_2}{H} + \ldots \right).
$$

The dependence of the Hall constant on the magnetic field is shown in Fig. 5. In fields up to 30 kOe, $R_H$ does not depend on the field and $R_H(0) = (4.25 \pm 0.1) \times 10^{-12}$ $\Omega$-cm/Oe. In fields exceeding 30 kOe, $R_H$ decreases with increasing field. Since it is precisely in these fields that magnetic breakdown takes place and leads to the formation of open trajectories in the basal plane, it is natural to attribute this field dependence of $R_H$ to the unbalance of the electron and hole volumes of the Fermi surface, which results from the breakdown (in


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the case of breakdown $\Delta n \neq 0$ and increases with increasing breakdown probability $W$). As $W \rightarrow 1$ the unbalance is

$$\frac{\Delta n}{(\Delta n)_{W=\infty}} = \frac{d_{\text{max}}}{S_{\text{dm}}/4n^2}$$

($S$ is the cross-sectional area of the Brillouin zone, $d_{\text{max}}$ is the maximum thickness of the layer of open trajectories). For beryllium, $d_{\text{max}}$ is determined by the dimension of the neck of the hole part of the Fermi surface in the second zone; in accordance with \[11\] we have $d_{\text{max}} = 3.6 \times 10^{-6} \text{cm}^2$ and $(\Delta n)_{W=\infty} = 0.96 \times 10^{12} \text{cm}^{-3}$.

If the influence of the magnetic breakdown on $\rho_{yy}$ reduces to an unbalance, then it is easy to obtain the following dependence of $\Delta n$ on the field:

$$\frac{\Delta n}{(\Delta n)_{W=\infty}} = \frac{H^2[R_\nu(k_\nu)-R_\nu(H)]}{c\sigma_\nu(H)(\Delta n)_{W=\infty}}$$  \hspace{1cm} (3)

This dependence, obtained on the basis of the experimental data, is shown by the solid curve of Fig. 6. The dashed lines in the figure are theoretical plots for different values of the breakdown field $H_0 (W = \exp(-H_0/H))$ on the basis of calculations performed in \[21\]. It follows from Fig. 6 that the field dependence of $\rho_{yy}$ can be attributed to the unbalance due to magnetic breakdown, and that the breakdown field is $H_0 = 110 \pm 10 \text{kOe}$.

In earlier studies \[12,13\] the breakdown field $H_0$ was determined by other methods and the following values were obtained: $H_0 = 130 \text{kOe}$ from measurements of the de Haas-van Alphen effect \[12\] and $H_0 = 110 \text{kOe}$ from the temperature and field dependences of the resistivity oscillations \[13\].

The dimensions of the Fermi surface of beryllium have now been determined with sufficient accuracy \[11\]. Calculating by means of the Chambers formulas \[14\]

$$H_\nu = \frac{n e}{2\sqrt{2} \pi c}$$  \hspace{1cm} (4)

where $k_\nu$ is the minimal distance between the orbits in $k$-space and $k_\pi$ is the curvature radius of the orbits, we obtain $H_\nu^{\text{theor}} = 113 \text{kOe}$, which is in good agreement with the estimate of $H_0$ from measurements of the Hall constant \[3\].

We observed no oscillations of the antisymmetrical component of the signal $U_{yx} (E_1)$. The absence of oscillations of $E_1$ can be regarded as a consequence of the relatively weak influence of magnetic breakdown on the off-diagonal elements of the conductivity tensor \[6\].

Symmetrical Component of the Signal $U_{yx} (E_2)$

For compensated metals ($n_1 = n_2$) the signal $U_{yx}$ can contain a symmetrical component (in the field). If the current $J$ and the magnetic field $H$ are directed arbitrarily relative to the principal axes of the conductivity tensor, then $U_{yx}$ contains a term quadratic in the magnetic field; this term characterizes the anisotropy of the resistivity (the difference between the diagonal elements $\rho_{xx}$ and $\rho_{yy}$ of the resistivity tensor). Thus, for example, if the current is inclined at an angle $\alpha$ to one of the principal axes and the magnetic field is parallel to the other, then

$$E_z = \frac{J}{D} \sin \alpha \cos (\varphi_0 - \varphi_\nu) (1 + \frac{H}{H_0})^\circ,$$  \hspace{1cm} (5)

where $H_0 = m^* c/ \tau \sigma$ ($\tau$ is the relaxation time) and $\rho_{xx}(0)$ and $\rho_{yy}(0)$ are the diagonal elements of the resistivity tensor in the absence of a field \[11\].

Figure 7 shows plots of the monotonic part of $U_{yx}$ against the angle $\alpha$ between the hexagonal axis of the sample and the magnetic field, obtained after reversing the field, and Fig. 8 shows the dependence of the monotonic part $E_z$ (the symmetrical component of $U_{yx}$) on the field when the angle $\alpha = 1.5^\circ$. From Fig. 7 and 8 it follows that in our case the symmetrical component of $U_{yx}$ is described by expression (5).

The relaxation time $\tau$ which enters in the definition of $H_0$ can be determined from measurements of $R_p$ and $\rho_{xx}(H)$ in weak magnetic fields. For the investigated samples $\tau = 10^{-13} \text{sec}$ and $H_0 = 3-5 \text{kOe}$. Thus, the term ($H/H_0)^\circ$ in (5) gives an enhancement, by hundreds of times, of the relatively small $|\rho_{yx} - \rho_{yy}|/\rho_{xx}$ difference of the diagonal elements of the resistivity tensor.

It was already noted above that we observed oscillations of only the symmetrical component of $U_{yx}$ ($E_2$). Since the monotonic part $E_0$ is described by expression (5), it is natural to assume that the oscillating part of the symmetrical component $U_{yx}$ is determined by the difference of the diagonal elements of the resistivity tensor. Incidentally, this is confirmed by the experimentally observed dependence of the amplitude of the oscillations of the signals $U_{yx}$ on the angle $\alpha$ (Fig. 9).

Let

$$\rho_{yx} = \rho_{yy} - k \sin (F/H + x), \hspace{1cm} \rho_{xx} = \rho_{yy} - k \sin (F/H + y)$$

($F$ is the oscillation frequency); then elementary operations yield

$$0 = \frac{\vartheta}{\pi} \sin \theta + \frac{\vartheta}{\pi}(x - y) + N \alpha,$$  \hspace{1cm} (6)

where $\vartheta$ is the phase difference between the oscillations of $U_{yx}$ and $U_{xx}$, and $\alpha$ is the angle between the hexagonal axis of the sample and the magnetic field. Thus, when the angle $\alpha$ goes through zero, a phase jump equal to $\pi$ should be observed.

The dependence of the phase difference between the oscillations of $U_{yx}$ and $U_{xx}$ on the angle $\alpha$ is shown in Fig. 10. On going through zero, one observes a phase jump close to $\pi$ ($= 0.85 \pi$) \[9\]. With further change of the angle $\alpha$, a monotonic increase of the phase difference $\theta$ is observed. This increase cannot be understood on the basis of the stochastic model of magnetic breakdown \[14\], since variation of the angle $\alpha$ between the hexagonal axis of the sample and the magnetic field causes the cross sections of all the electron cigars in the third Brillouin zone of beryllium to vary in absolutely the same manner. It is precisely these sections that cause the giant magnetoresistance oscillations, and the Lifshitz-Onsager quantization rules produce one and the same initial phase $\pi/4$ for them ($\rho_{xx}^{\text{sc}}$ and $\rho_{yy}^{\text{sc}}$).
The measuring current was $J$ when the motion of the electrons over the cigars is synchronized with the motion in the corona in such a way that the wave function of the electron, corresponding to the open direction, is of the Bloch type, and the phase jump of the wave function on going from the corona to the cigar is determined by the breakdown probability $\gamma_1$. In this case the initial phase of the resistivity oscillations $\rho^{\text{osc}}$ remains unchanged. Thus,

$$\Delta \phi = \frac{\rho_{\text{deb}}(0^\circ)}{\cos \alpha} \frac{1}{\epsilon_0 H}$$

which is in qualitative agreement with Fig. 10, and a numerical estimate of the coefficient $1/\cos \alpha$ agrees with the experimentally observed value.

As noted above, the initial phase of the oscillations also depends on the breakdown probability. If $W \ll 1$, then

$$y_1 = \frac{\gamma}{\alpha + \gamma} = \frac{1}{\gamma} \ln \left(\frac{\gamma}{2}\right) + \arg \Gamma(\gamma/2),$$

where $\gamma = \frac{1}{\epsilon_0 c H}$. This second contribution should lead to a change in the experimentally observed oscillation frequency with changing field. If $\gamma > 1$ (for beryllium this condition is satisfied at $H < 100$ kOe), then $y_1 = 0.17/\gamma$, and an estimate for the change of the frequency in a field $\approx 30$ kOe yields $\Delta F = 10^{-6} F$.

Unfortunately, observation of such small frequency changes is complicated in the case of beryllium by low-frequency beats due to the formation of diamagnetic domains (the corresponding value is $\Delta F \approx 10^{-7} F$).

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12 N. E. Alekseevskii, V. S. Egorov, ZhETF Pis. Red. 8, 301 (1968) [Sov. Phys.-JETP Lett. 8, 185 (1968)].

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