Effect of spatial dispersion of carrier electric conductivity on the light- (radio-) electric effect in a strong magnetic field

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The light- (radio-) electric effect is studied in a strong electric field and at oscillation frequencies exceeding the collision frequency. Without restricting the analysis to the hydrodynamic approximation, allowance is also made for another effect of the wave vector of the propagating wave, viz., spatial dispersion of the carrier complex dielectric constant. Helicoidal and Alfven waves and also magnetoplasma waves with electric fields parallel and perpendicular to the external magnetic field are considered. It is assumed that the carrier spectrum and their scattering are isotropic. It is found that the light-electric field component that depends on spatial dispersion of the dielectric constant exceeds the component that does not depend on it for both the ordinary and extraordinary waves.

1. An electromagnetic wave propagating in a conducting crystal produces in this crystal, in the approximation quadratic in the wave field $\mathbf{E}_1$, a constant current $\mathbf{j}_2$ if the circuit is closed, or a constant electric field $\mathbf{E}_2$ if the circuit is open. This effect was pointed out by Barlow[1] and subsequently called the light-electric[2] or radioelectric effect[3]. It was interpreted as a high-frequency Hall effect[1]. It turned out subsequently that this interpretation is valid only in the hydrodynamic approximation; beyond the limits of applicability of this approximation, there is superimposed on the Hall effect another mechanism, interpreted in[4] as "dragging" of the electrons by the photons. In terms of classical physics, this denotes the influence of the "momentum" of the propagating waves, which is proportional to the square of the field and to the wave vector) on the carriers. The dependence on the wave vector becomes manifest here in two circumstances: first, the magnetic field of the wave (6) is dependent on the wave vector (and the concept of the high-frequency Hall effect is connected with this circumstance), and second, in the presence of spatial dispersion, the complex dielectric constant of the plasma can depend appreciably on the wave vector. For a weakly-damped wave, when the imaginary part of the wave vector $\mathbf{k}''$ is much smaller than the real part $\mathbf{k}'$, this has been considered in[5], and in the case $\mathbf{k}'' \gtrsim \mathbf{k}'$ it was considered in[6].

The present paper is devoted to a study of the light-electric effect in the presence of an external constant magnetic field and at different ratios of the wave frequency ω to the cyclotron frequency Ω.

We consider the different types of electromagnetic waves that propagate in this case, namely, helicons and Alfven waves if $\mathbf{k} \parallel \mathbf{H}$, and magnetoplasma waves if $\mathbf{k} \perp \mathbf{H}$ and the electric field of the wave is either parallel (ordinary wave) or perpendicular (extraordinary wave) to the external magnetic field $\mathbf{H}$.

We disregard in the present article the anisotropy of the carrier spectrum and of the carrier scattering, and in this case we have (for transverse waves)

$$\mathbf{j}_2 = \hat{\chi}\mathbf{I} + \hat{\sigma}\mathbf{E}_2,$$

where I is the Poynting vector; if the circuit is open, then $\mathbf{j}_2 = 0$, whence

$$\mathbf{E}_2 = -(\hat{\sigma}^{-1})\hat{\chi}\mathbf{I} = \hat{\gamma}\mathbf{I}$$

or in expanded form

$$\mathbf{E}_2 = \gamma\mathbf{I} + \gamma_1[\mathbf{IH}] + \gamma_2\mathbf{H}(\mathbf{IH}). \quad (1)^*$$

Here $\hat{\sigma}$ and $\hat{\chi}$ are tensors having the same form as the tensor $\hat{\gamma}$ in the preceding formula. In the presence of carriers of both signs we have $\hat{\sigma} = \hat{\sigma}_+ + \hat{\sigma}_-$ and $\hat{\chi} = \hat{\chi}_+ + \hat{\chi}_-$, and different combinations of quantities with plus and minus signs play an important role in the tensor $\hat{\gamma}$ for the various waves.

Our work differs from that of Kaganov and Peshkov[7] in two respects. First, they consider only the waves which we call extraordinary, whereas in the present article we investigate also a few other waves, as mentioned above. Second, we introduce the spatial dispersion of the complex dielectric constant, which was not taken into account in[7]. We have confined ourselves in the present article to the case of weak spatial dispersion, when $\mathbf{k}\cdot\mathbf{v} \ll \nu, \omega, \Omega$, where $\mathbf{v}$ is the average carrier velocity and ν is their collision frequency. This does not mean at all, however, that the influence of spatial dispersion constitutes only a small correction to Barlow's effect. To the contrary, we show that both terms in the expression for the light-electric field contains one and the same small parameter $\mathbf{k}\cdot\mathbf{v}/\omega$, and are of the same order of magnitude with respect to this parameter, but on the other hand they can differ significantly with respect to the parameter $\Omega^2\tau^2$. In the case of the ordinary wave, when $\omega = \Omega$, the light-electric field term that depends on the spatial dispersion is larger than the Barlow term by a factor $\Omega^2\tau^2$, and is furthermore of opposite sign. For the extraordinary wave, both terms in all the components are comparable in magnitude and differ only in numerical factors that are determined by the carrier scattering mechanism; spatial dispersion predominates in the light-electric field component parallel to the Poynting vector, and the Barlow effect predominates in the case of scattering by phonons. Finally, for helicoidal waves and Alfven waves, the first term is smaller than the second by a factor $\Omega^2\tau^2$.

2. In the kinetic equation for the carrier distribution function

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \frac{e}{m}\mathbf{E}_1\frac{\partial f}{\partial \mathbf{v}} + \frac{e}{mc}[\mathbf{vH}]\frac{\partial f}{\partial \mathbf{v}} = -\left[\frac{\partial f}{\partial t}\right]_{st},$$

we assume that the collisions are elastic and put $f = f_0 + f_1 + f_2$, where f_0 is the equilibrium distribution function, and f_1 and f_2 are respectively the waves linear and quadratic in the field.

The function f_1 can be expanded in powers of $\mathbf{k}\cdot\mathbf{v}$, i.e., we can put $f_1 = f_{10} + f_{11} + \ldots$; we confine ourselves to these terms only. Putting $f_1 \sim E_1 \sim \exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$, we obtain for them the kinetic equations

$$i\omega f_{10} + e\mathbf{E}_1\mathbf{v}\frac{\partial f_0}{\partial \varepsilon} + [\mathbf{v}\boldsymbol{\Omega}]\frac{\partial f_{10}}{\partial \mathbf{v}} - \left[\frac{\partial f_{10}}{\partial t}\right]_{st} = 0, \quad (2)$$

$$i\omega f_{11} + [\mathbf{v}\boldsymbol{\Omega}]\frac{\partial f_{11}}{\partial \mathbf{v}} - \left[\frac{\partial f_{11}}{\partial t}\right]_{st} = i\mathbf{k}\mathbf{v}f_{10}. \quad (3)$$

It is convenient to put

$$f_{10}(\mathbf{v}) = \{\varphi_1(\varepsilon)\mathbf{E}_1 + \varphi_2(\varepsilon)[\mathbf{E}_1\mathbf{H}]\}\mathbf{v};$$

Substituting this expression in the collision integral, we obtain after a simple calculation the well-known result:

$$f_{10}(v) = \frac{\partial f_0}{\partial \varepsilon}\frac{e\tau_{10}}{(1 - ix_{10})^2 + y_{10}^2}\{(1 - ix_{10})\mathbf{E}_1 - [\mathbf{E}_1\mathbf{y}_{10}]\}\mathbf{v},$$

$$\tau_{10}^{-1} = \int W(\mathbf{v}', \mathbf{v})(1 - \cos\theta)\,do' \quad (4)$$

where $W(\mathbf{v}', \mathbf{v})$ is the scattering probability, θ is the scattering angle, and we have introduced the notation $\omega\tau_{10} = x_{10}$ and $\Omega\tau_{10} = y_{10}$. As to the function f_{11}, its form differs for the waves of the different types.

In the function f_2, we consider only the part independent of the time; the kinetic equation for this function then takes the form

$$-[\mathbf{v}\boldsymbol{\Omega}]\frac{\partial f_2}{\partial \mathbf{v}} + \left[\frac{\partial f_2}{\partial t}\right]_{st} = \frac{e}{2m}\mathrm{Re}\left(\left\{\mathbf{E}_1^* + \frac{1}{c}[\mathbf{v}\mathbf{H}_1^*]\right\}\frac{\partial f_1}{\partial \mathbf{v}}\right), \quad (5)$$

where

$$\mathbf{H}_1 = \frac{c}{\omega}[\mathbf{k}\mathbf{E}_1] \quad (6)$$

is the magnetic field of the wave. The form of the function f_2 is also different for waves of different types.

3. We start with a helicoidal wave, for which we assume $\mathbf{k} \parallel \mathbf{H} = (0, 0, H)$ and $\mathbf{E}_1 = (E_1, \pm iE_1, 0)$. In this case we can put

$$f_{11}(v) = (\mathbf{k}\mathbf{v})\{\psi_1(\varepsilon)\mathbf{E}_1 + \psi_2(\varepsilon)[\mathbf{E}_1\boldsymbol{\Omega}]\}\mathbf{v}, \quad (7)$$

where $\psi_i(\epsilon)$ are functions of the energy and are to be determined. Substituting (7) in the collision energy of (3), substituting (4) in the right-hand side, and then comparing coefficients, we obtain

$$f_{11}(v) = i\frac{\partial f_0}{\partial \varepsilon}\frac{e\tau_{10}\tau_{11}}{[(1 - ix_{10})^2 + y_{10}^2][(1 - ix_{11})^2 + y_{11}^2]}$$
$$\times(\mathbf{k}\mathbf{v})(\{[y_{10}y_{11} - (1 - ix_{10})(1 - ix_{11})]\mathbf{E}_1 + [(1 - ix_{10})\tau_{11} + (1 - ix_{11})\tau_{10}][\mathbf{E}_1\boldsymbol{\Omega}]\}\mathbf{v}), \quad (8)$$
$$\tau_{11}^{-1} = {}^3/_2\int W(\mathbf{v}', \mathbf{v})\sin^2\theta\,do',$$

where we have introduced $\omega\tau_{11} = x_{11}$ and $\Omega\tau_{11} = y_{11}$[8]. The relaxation times τ_{10} and τ_{11} are functions of the particle energy ϵ. We substitute (8) and (4) in the right-hand side of (5) and recognize that it is necessary to put $f_1 = f_{11}$ in the first term on the right, and $f_1 = f_{10}$ in the second term. Putting

$$f_2 = (\mathbf{k}_1\mathbf{v})[\zeta_1(\varepsilon)E_1^2 + \zeta_2(\varepsilon)(\mathbf{E}_1\mathbf{v})^2 + \zeta_3(\varepsilon)([\mathbf{E}_1\boldsymbol{\Omega}]\mathbf{v})^2 + \zeta_4(\varepsilon)(\mathbf{E}_1\mathbf{v})([\mathbf{E}_1\boldsymbol{\Omega}]\mathbf{v})], \quad (9)$$

where $\mathbf{k}_1$ is a unit vector in the $\mathbf{k}$ direction and where we take into account the fact that $\mathbf{k}'' \parallel \mathbf{k}'$ in the considered cases. Substituting this formula in (5) and calculating the collision integral, we determine the coefficients ζ_i and obtain an expression for f_2. It turns out to be proportional to $\exp(-2k''z)$. Then the light-electric field is determined from the condition that the current density vanish:

$$\mathbf{j}_2 = \int \mathbf{v}f_2\,d^3v + \sigma\mathbf{E}_2 = 0.$$

Here $\sigma = ne^2\langle\tau_{10}\rangle/m$ is the static electric conductivity at $\mathbf{E}_2 \parallel \mathbf{H}$. From this we find, assuming $\Omega \gg \omega \gg \langle\tau_1^{-1}\rangle$ and taking into account the reflection of the wave:

$$\frac{E_2(z)}{I} = \gamma^{(0)}\frac{m}{eH^2\langle\tau_{10}\rangle}\exp(-2k''z),$$

$$\langle\tau_{10}\rangle = \int_0^\infty d\varepsilon\varepsilon^{3/2}\frac{\partial f_0}{\partial \varepsilon}\tau_{10} \Big/ \int_0^\infty d\varepsilon\varepsilon^{3/2}\frac{\partial f_0}{\partial \varepsilon},$$

where $\gamma^{(0)}$ is the numerical coefficient that depends on the carrier distribution function. For a Maxwellian function we have $\gamma^{(0)} = 4\pi$; in the case of degenerate electrons, $\gamma^{(0)} = 32\sqrt{2}\pi/3$. We have used the following expressions for the helicoidal wave:

$$k' = \frac{\omega_0}{c}\left(\frac{\omega}{\Omega}\right)^{1/2}, \quad k'' = \frac{\omega}{\Omega c\langle\tau_{10}\rangle}\left(\frac{\omega}{\Omega}\right)^{1/2},$$

I is the Poynting vector of a wave incident from the outside[1)]. In the case of two types of carriers, the light-electric coefficient is

$$\gamma = \gamma^{(0)}\frac{1}{H^2}\left(\frac{1}{\mu_+} - \frac{1}{\mu_-}\right).$$

If the dimensions of the crystal in the wave-propagation direction are much larger than the reciprocal absorption length k″, then the total light-electric potential difference is

$$U_2 = E_2(0)/2k''. \quad (10)$$

4. We consider now an Alfven wave ($\mathbf{k} \parallel \mathbf{H}$, $\mathbf{E}_1 = (E_1, 0, 0)$) in crystals with equal electron and hole densities. Proceeding in analogy with the foregoing, substituting (6) and the expressions for the real and imaginary parts of the wave vector[10]

$$k' = \omega/v_a, \quad k'' = 1/2v_a\langle\tau_{10}\rangle,$$
$$v_a = H[4\pi n(m_+ + m_-)]^{-1/2},$$

in (5), we obtain for the light-electric field

$$\frac{E_2(z)}{I} = -\frac{2^{11/2}\pi^2}{3}\frac{e}{H^2(\sigma_+ + \sigma_-)}\exp(-2k''z)(A_+ - A_-),$$

where

$$A_\pm = \int_0^\infty d\varepsilon\varepsilon^{3/2}\left(-\frac{\partial f_0}{\partial \varepsilon}\right)\frac{1}{m_\pm^{3/2}}\left(1 - \frac{\tau_{10}}{2}\frac{m_+\langle\tau_{10+}^{-1}\rangle + m_-\langle\tau_{10-}^{-1}\rangle}{m_+ + m_-}\right),$$

and the quantities with the subscripts minus and plus pertain respectively to electrons and holes. Under the previous conditions, the potential difference is obtained from (10).

5. We proceed now to magnetoplasma waves. The functions f_1 and f_2 for the ordinary wave $\mathbf{H} = (0, 0, H)$, $\mathbf{E}_1 = (0, 0, E_1)$, $\mathbf{k} = (0, k, 0)$ take the form

$$f_{11} = (\mathbf{E}_1\mathbf{v})(\{\psi_1(\varepsilon)\mathbf{k} + \psi_2(\varepsilon)[\mathbf{k}\boldsymbol{\Omega}]\}\mathbf{v}),$$
$$f_2 = E_1^2\{\zeta_1(\varepsilon)\mathbf{k}_1 + \zeta_2(\varepsilon)[\mathbf{k}_1\boldsymbol{\Omega}]\}\mathbf{v} + (\mathbf{E}_1\mathbf{v})^2(\{\zeta_3(\varepsilon)\mathbf{k}_1 + \zeta_4(\varepsilon)[\mathbf{k}_1\boldsymbol{\Omega}]\}\mathbf{v}).$$

Calculating the collision integral with this function f_{11} and comparing coefficients on the right and left sides of (3), we obtain

$$f_{11} = -i\frac{\partial f_0}{\partial \varepsilon}\frac{e\tau_{10}\tau_{11}}{(1 - ix_{10})[(1 - ix_{11})^2 + y_{11}^2]}(\mathbf{E}_1\mathbf{v})(\{(1 - ix_{11})\mathbf{k} - [\mathbf{k}\mathbf{y}_{11}]\}\mathbf{v}).$$

The light-electric field is determined from the condition for the vanishing of the expression

$$\mathbf{j}_2 = \sigma\mathbf{E}_2 + \sigma_1[\mathbf{E}_2\mathbf{H}] + \chi\mathbf{I} + \chi_1[\mathbf{I}\mathbf{H}] = 0,$$

whence

$$\mathbf{E}_2 = -\frac{\chi\sigma + \chi_1\sigma_1H^2}{\sigma^2 + \sigma_1^2H^2}\mathbf{I} + \frac{\chi\sigma_1 - \chi_1\sigma}{\sigma^2 + \sigma_1^2H^2}[\mathbf{I}\mathbf{H}]. \quad (11)$$

The expressions for χ_1 and χ_2 are

$$\chi=-\frac{2^{3/2}\pi^2}{3}\frac{e^3}{m^{1/2}c\omega}\int_0^\infty d\varepsilon\,\varepsilon^{3/2}\left(-\frac{\partial f_0}{\partial\varepsilon}\right)\frac{\tau_{10}^2}{(1+y_{10}^2)(1+x_{10}^2)}$$

$$\times\{k'[1+(1-y_{10}^2)A-2y_{10}y_{11}B]+k''[x_{10}-(1-y_{10}^2)C-2y_{10}y_{11}D]\},$$

$$\chi_1=\frac{2^{3/2}\pi^2}{3}\frac{e^4}{m^{3/2}c^2\omega}\int_0^\infty d\varepsilon\,\varepsilon^{3/2}\left(-\frac{\partial f_0}{\partial\varepsilon}\right)\frac{\tau_{10}^2}{(1+y_{10}^2)(1+x_{10}^2)}$$

$$\times\{k'[1+(1-y_{10}^2)A-2y_{10}y_{11}B]+k''[x_{10}+(1-y_{10}^2)C-2y_{10}y_{11}D]\},$$

$$A=\alpha x_{11}\frac{x_{11}(1+x_{11}^2-y_{11}^2+x_{10}x_{11})+x_{10}(1+y_{11}^2)}{(1+y_{10}^2)[(1-x_{11}^2+y_{11}^2)^2+4x_{11}^2]},$$

$$B=\alpha x_{11}\frac{x_{10}(1-x_{11}^2+y_{11}^2)+2x_{11}}{(1+y_{10}^2)[(1-x_{11}^2+y_{11}^2)^2+4x_{11}^2]},$$

$$C=\alpha x_{11}\frac{1+x_{11}^2+y_{11}^2-x_{10}x_{11}(1+x_{11}^2-y_{11}^2)}{(1+y_{10}^2)[(1-x_{11}^2+y_{11}^2)^2+4x_{11}^2]},$$

$$D=\alpha x_{11}\frac{1-x_{11}^2+y_{11}^2-2x_{10}x_{11}}{(1+y_{10}^2)[(1-x_{11}^2+y_{11}^2)^2+4x_{11}^2]}\qquad \alpha=\frac{2}{5}\frac{\partial\ln\tau_{10}}{\partial\ln\varepsilon}$$

When these expressions are substituted in (11), it is seen that if τ_1 does not depend on the energy then the field $\mathbf{E}_2$ is parallel to $\mathbf{I}$ at all values of the magnetic field $\mathbf{H}$.

We shall apply these expressions to the high-frequency case $\omega \gg \langle\tau_1^{-1}\rangle$, but at different values of Ω.

A. $\omega \gg \langle\tau_1^{-1}\rangle \gg \Omega$. In this case the light-electric field is

$$\mathbf{E}_2(y)=\frac{e}{mc^2\omega^2}\left(\frac{\gamma^{(0)}}{\langle\tau_{10}\rangle}\mathbf{I}+\gamma_1^{(0)}[\mathbf{I}\boldsymbol{\Omega}]\right)\exp(-2k''y),$$

where $\gamma^{(0)}$ and $\gamma_1^{(0)}$ are numerical coefficients that are different for weakly- and strongly-damped waves, for different scattering mechanisms, and for different distribution functions. For a weakly-damped wave ($\epsilon_0 > \omega_0^2/\omega^2$, where $\omega_0^2 = 4\pi ne^2/m$ is the square of the plasma frequency and ϵ_0 is the static dielectric constant of the lattice) we have in the case of scattering by phonons ($\tau_{11} = \tau_{10}$[6]):

a) for a nondegenerate electron gas

$$\gamma^{(0)}=18\pi^{3/2}/5,\quad \gamma_1^{(0)}=11\pi/5;$$

b) for a degenerate gas

$$\gamma^{(0)}=\gamma_1^{(0)}=8\pi/5.$$

In the case of scattering by impurity ions ($\tau_{11} = \tau_{10}/3$[6]) we have

a) $\gamma^{(0)}=17\pi/5,\quad \gamma_1^{(0)}=\pi;$ б) $\gamma^{(0)}=136\pi/5,\quad \gamma_1^{(0)}=-72\pi/5;$

The sign of the second coefficient has become negative, since in this case, unlike the preceding case, $\chi_1\sigma > \chi\sigma_1$. For a weakly damped wave we have

$$k''=\omega_0^2/2\varepsilon_0^{1/2}\omega^2c\langle\tau_{10}\rangle.$$

In the case of a strongly damped wave

$$\varepsilon_0<\omega_0^2/\omega^2,\quad k''=\omega_0/c,$$

in the case of scattering by phonons

a) $\gamma^{(0)}=24\pi/5,\quad \gamma_1^{(0)}=-2\pi/5;$ б) $\gamma^{(0)}=24\pi/5,\quad \gamma_1^{(0)}=-4\pi/5;$

in scattering by impurity ions

a) $\gamma^{(0)}=8\pi/5,\quad \gamma_1^{(0)}=189\pi^{3/2}/16;$ б) $\gamma^{(0)}=8\pi/5,\quad \gamma_1^{(0)}=12\pi/5.$

The main contribution to the reversal of the sign of $\gamma_1^{(0)}$ is made here by the terms that appear when spatial dispersion is taken into account. In the presence of two types of carriers of equal density, the main contribution is made by the electrons.

B. $\omega \gg \Omega \gg \langle\tau_1^{-1}\rangle$. Now

$$\mathbf{E}_2(y)=\frac{e}{mc^2\omega^2\langle\tau_{10}\rangle}\left(\gamma^{(0)}\mathbf{I}+\frac{\gamma_1^{(0)}}{\Omega^2\langle\tau_{10}\rangle}[\mathbf{I}\boldsymbol{\Omega}]\right)\exp(-2k''y).\qquad(12)$$

In the case of scattering by phonons and $\epsilon_0 < \omega_0^2/\omega^2$ we have

a) $\gamma^{(0)}=32\pi/5,\quad \gamma_1^{(0)}=-11;$ b) $\gamma^{(0)}=32\pi/5,\quad \gamma_1^{(0)}=8\pi/5.$

For a strongly damped wave

a) $\gamma^{(0)}=32\pi/3,\quad \gamma_1^{(0)}=-256/9;$ b) $\gamma^{(0)}=4\pi,\quad \gamma_1^{(0)}=-4\pi/5.$

In the case of two types of carriers that are of equal density

$$\gamma=-\chi_-/\sigma_+,\quad \gamma_1=-\chi_{1-}/\sigma_+.$$

C. $\omega = \Omega \gg \langle\tau_1^{-1}\rangle$. In this case $\gamma_1 = \chi/\sigma_1 H^2$, where $\chi \sim \partial\ln\tau_{10}/\partial\ln\epsilon$, i.e., the main contribution to the transverse electric field is made by terms that are due to allowance for spatial dispersion of the dielectric constant. In addition, the sign of the field depends on the carrier-scattering mechanism

$$\mathbf{E}_2(y)=\frac{e}{mc^2\Omega^2}\left(\frac{\gamma^{(0)}}{\langle\tau_{10}\rangle}\mathbf{I}+\gamma_1^{(0)}[\mathbf{I}\boldsymbol{\Omega}]\right)\exp(-2k''y),$$

and the coefficient γ_1 increases in comparison with the preceding case by a factor $\Omega^2\tau_1^2$. In scattering by phonons we have for a weakly-damped wave

a) $\gamma^{(0)}=256\sqrt{\pi}/15,\quad \gamma_1^{(0)}=8\pi/5;$ б) $\gamma^{(0)}=48\pi/5,\quad \gamma_1^{(0)}=8\pi/5.$

For a strongly-damped wave,

$$\mathbf{E}_2(y)=\frac{e}{mc^2\Omega^2\langle\tau_{10}\rangle}\left(\gamma^{(0)}\mathbf{I}+\frac{\gamma_1^{(0)}}{\Omega^2\langle\tau_{10}\rangle}[\mathbf{I}\boldsymbol{\Omega}]\right)\exp(-2k''y),$$

a) $\gamma^{(0)}=3\pi,\quad \gamma_1^{(0)}=-6\sqrt{\pi};$ б) $\gamma^{(0)}=16\pi/5,\quad \gamma_1^{(0)}=48\pi/5.$

In the presence of electrons and holes of equal density, in the case of a weakly-damped wave we have

$$\omega=\Omega_-:\quad \gamma=-\chi_-/\sigma_+,\quad \gamma_1=-\chi_{1-}/\sigma_+;$$
$$\omega=\Omega_+:\quad \gamma=-\chi_+/\sigma_+,\quad \gamma_1=-\chi_{1-}/\sigma_+;$$

and for a strongly damped wave

$$\omega=\Omega_-:\quad \gamma=-(\chi_++\chi_-)/\sigma_+,\quad \gamma_1=-\chi_{1-}/\sigma_+;$$
$$\omega=\Omega_+:\quad \gamma=-\chi_-/\sigma_+,\quad \gamma_1=-\chi_{1-}/\sigma_+.$$

D. $\Omega \gg \omega \gg \langle\tau_1^{-1}\rangle$. In this case the spatial dispersion of the dielectric constant can be neglected, and E_2 is described by the same formula (12); in the case of scattering by phonons and a weakly-damped wave

a) $\gamma^{(0)}=64\sqrt{\pi}/3,\quad \gamma_1^{(0)}=-64/3;$ b) $\gamma^{(0)}=8\pi;$

For a strongly-damped wave

a) $\gamma^{(0)}=32\pi/3,$ б) $\gamma^{(0)}=4\pi;$

For the last three cases, $\gamma_1 \sim e/mc^2\omega^2\Omega^4\langle\tau_{10}\rangle^4$.

In the case of two types of carriers of equal density

$$\gamma=-\chi_-/\sigma_+,\quad \gamma_1=-\chi_{1-}/\sigma_+.$$

6. In the case of the extraordinary wave $\mathbf{H} = (0, 0, H)$, $\mathbf{E}_1 = (E_1, 0, 0)$, and $\mathbf{k} = (0, k, 0)$, we consider the case of two types of carriers of equal density, but the electric field of the extraordinary wave is now transverse. It is convenient to express the functions f_{11} and f_2 in the form

$$f_{11}(v)=\psi_1(\varepsilon)(\mathbf{E}_1\mathbf{v})(\mathbf{k}\mathbf{v})+\psi_2(\varepsilon)[(\mathbf{E}_1\mathbf{v})([\mathbf{k}\boldsymbol{\Omega}]\mathbf{v})-{}^1/_3v^2(\mathbf{E}_1[\mathbf{k}\boldsymbol{\Omega}])]$$
$$+\psi_3(\varepsilon)[([\mathbf{E}_1\boldsymbol{\Omega}]\mathbf{v})(\mathbf{k}\mathbf{v})-{}^1/_3v^2([\mathbf{E}_1\boldsymbol{\Omega}]\mathbf{k})]$$
$$+\psi_4(\varepsilon)([\mathbf{E}_1\boldsymbol{\Omega}]\mathbf{v})([\mathbf{k}\boldsymbol{\Omega}]\mathbf{v}+\psi_5(\varepsilon){}^1/_3v^2(\mathbf{k}[\mathbf{E}_1\boldsymbol{\Omega}]);$$
$$f_2=E_1^2[\zeta_1(\varepsilon)(\mathbf{k}_1\mathbf{v})+\zeta_2(\varepsilon)([\mathbf{k}_1\boldsymbol{\Omega}]\mathbf{v})]+(\mathbf{E}_1\mathbf{v})^2[\zeta_3(\varepsilon)(\mathbf{k}_1\mathbf{v})$$
$$+\zeta_4(\varepsilon)([\mathbf{k}_1\boldsymbol{\Omega}]\mathbf{v})]+([\mathbf{E}_1\boldsymbol{\Omega}]\mathbf{v})^2[\zeta_5(\varepsilon)(\mathbf{k}_1\mathbf{v})+\zeta_6(\varepsilon)([\mathbf{k}_1\boldsymbol{\Omega}]\mathbf{v})]$$
$$+{}^1/_3v^2(\mathbf{E}_1[\mathbf{k}_1\boldsymbol{\Omega}])[\zeta_7(\varepsilon)(\mathbf{E}_1\mathbf{v})+\zeta_8(\varepsilon)([\mathbf{E}_1\boldsymbol{\Omega}]\mathbf{v})].$$

Proceeding as before and assuming $\Omega \gg \omega \gg \langle\tau_1^{-1}\rangle$ for the extraordinary wave [10], we obtain a light-electric field to which the main contribution is made by the holes:

$$\mathbf{E}_2 = \frac{2^{11/2}\pi^2}{3}\frac{e^3}{m_+^{1/2}c^2\sigma_+}\int_0^\infty d\varepsilon\,\varepsilon^{3/2}\left(-\frac{\partial f_0}{\partial\varepsilon}\right)\frac{1}{\Omega_+^4\tau_{10+}^2}\cdot$$
$$\times\left\{\left[\frac{5}{2}\frac{\tau_{10+}}{\langle\tau_{10+}\rangle}-1-\frac{\partial\ln\tau_{10+}}{\partial\ln\varepsilon}\frac{4}{5}\left(2+\frac{\tau_{10+}}{8\langle\tau_{10+}\rangle}\right)\right]\mathbf{I}\right.$$
$$\left.-\tau_{10+}\left[1-\frac{\tau_{10+}}{2\langle\tau_{10+}\rangle}+\frac{4}{5}\frac{\partial\ln\tau_{10+}}{\partial\ln\varepsilon}\right][\mathbf{I}\boldsymbol{\Omega}]\right\}\exp(-2\mathbf{k}''y).$$

Attention should be called to the fact that the sign of the coefficient γ is different here from the sign in the preceding cases; this is due to the appearance of a term that takes into account the dispersion of the complex dielectric constant.

The theory developed by us can be applied to InSb and qualitatively also to Bi. For quantitative calculations of these phenomena in the second case, when an important role is assumed by the spatial dispersion of the electric conductivity, which is not accounted for in [7], it is necessary to develop a theory in which the crystal anisotropy is taken into account. We are not convinced, however, that allowance for the anisotropy of the electron spectrum is sufficient in this case and that the scattering anisotropy can be neglected.

*[IH] ≡ I × H.

[1)]The Poynting vector of the medium differs from the Poynting vector I of the wave incident from the outside by a factor $4n'/(n'+1)^2+n''^2$, where $n'+in''=c\omega^{-1}(k'+ik'')$ [9].

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