Mode locking in gas lasers

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Locking of three parallel-polarized modes in a He-Ne laser is investigated experimentally and theoretically at a wavelength of 0.63 μ. It is shown that for symmetric arrangement of the modes relative to the line center and at certain pressures mode locking may be absent; for a rigorously symmetric position of the modes the relative phase angle in the locking region is either zero or π. The presence of a combination tone is the cause of mode intensity oscillations outside the locking region and of variation of its curvature as a function of the detuning in the locking region.

INTRODUCTION

The properties of laser radiation in the case of the generation of three or more longitudinal modes differ fundamentally from the properties of single- or two-mode lasers by the phenomenon of locking or self-sunchronization of modes due to the presence of combination tones.

When the laser generates more than two modes, some of the combination tones lie near the fundamental modes. When the mode frequency is altered, so that the modes are located symmetrically relative to the line center, the separation between the tones and the corresponding modes is reduced and may at a certain stage suddenly become zero, i.e., locking sets in. In the locked state the phases of the generated modes are rigidly coupled, the mode separation is constant, and the resultant radiation power as a function of time takes the form of a sequence of pulses whose shape depends on the number of locked modes and the relative phase angle.

There is a large number of papers in the literature on the time dependence of laser parameters under the locked conditions (see the review in[1]). However, most of the work on the spectral properties has been theoretical[2-8] and, frequently, the results have been contradictory. For example, there is some controversy about the character of locking in the region of symmetric mode tuning relative to the line center. The effect of the laser parameters on mode locking has practically been ignored in these papers.

At the same time, our preliminary studies[9] have shown that the pressure of the working mixture in the laser is of paramount importance for the character of the locking process. This shows that atomic collisions have an important influence on the emission properties of the multimode laser. This has not been allowed for in theoretical analyses which we used in previous work to study the locking process in the gas laser, and may have led to incorrect conclusions.

In this paper we report the results of experimental and theoretical studies of the spectral properties of the radiation emitted by a gas laser when three longitudinal modes are generated. The theoretical analysis is based on[10] and takes into account level degeneracy, elastic collisions between the atoms, and trapping of resonance radiation.

EXPERIMENTAL RESULTS

In the experimental part of our investigation we used the $He-Ne^{20}$ laser at a wavelength of 0.63 μ, which ensured the generation of three longitudinal modes with a separation of 380 MHz. The optical length of the laser cavity was varied by scanning one of the mirrors with the aid of a piezoceramic.

The laser beam was intercepted by a germanium photodiode which isolated the intermode beats. These were then amplified and fed into a radiofrequency mixer. The resulting signal was in the form of small beats (at the difference frequency between the mode beats) and was amplified and then displayed directly on an oscilloscope screen. The radiation was also intercepted by a scanning Fabry-Perot interferometer which was used to monitor the laser spectrum. Moreover, the use of the interferometer as an optical filter[11] enabled us to measure the intensity of many of the modes within the frequency interval under consideration.

It was found that mode locking in the laser usually occurs in the region of symmetric tuning $x = \nu_0 - \nu_2 = 0$ (ν_0 and ν_2 are the central frequency of the atomic transition and the generation frequency of the middle mode) in a range of about 40 MHz. At the same time, the small beat signal in the locking region has a complicated form with a characteristic narrow resonance at x = 0 which is directed upward at high pressures p (see Fig. 1d) and downward at low pressures (see Fig. 1b). At the intermediate pressure p_{int} which, in our case, is 2 Torr, the signal has the form shown in Fig. 1c. Here one notices the absence of locking in the small neighborhood of x = 0, amounting to about 2—3 MHz.

The value of p_{int} depends appreciably only on $\zeta = n_2/\bar{n}$, i.e., the ratio of the amplitude of the second harmonic of spatial excitation to its mean value. This ratio is usually negative for lasers and depends on the disposition of the active medium within the cavity.[8] Our measurements show that p_{int} increases with decreasing ζ.

The absence of locking near x = 0 is also found to occur at low frequencies, below a certain definite value p* (Fig. 1a). In our case, p* is found to be about 1 Torr. When p < p*, the region in which there is no locking is substantial in size and increases with decreasing p. Thus, at a pressure of 0.8 Torr, below which three-mode generation is found to be difficult, this region extends over about 25 MHz.

At a pressure of 2.5 Torr (Fig. 1d) it is clear that the small beats are absent from the region of large detuning corresponding to nonlocking conditions for the other cases shown. This is connected with the increased competition between the modes as the pressure increases, which leads to the suppression of one of the side

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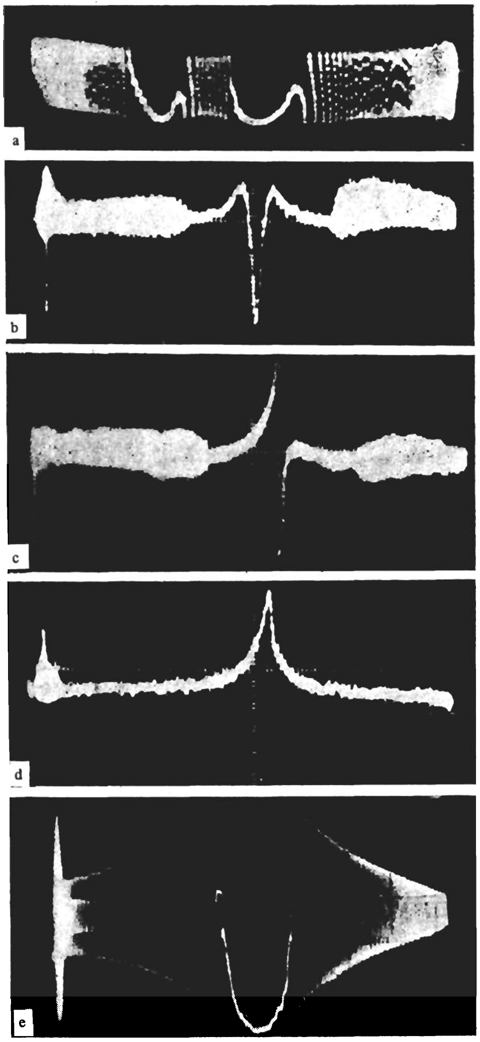

FIG. 1. Oscillogram showing small beats in the region of symmetric disposition of modes relative to the center of the amplified line for He:Ne^{20} = 7:1 : p = 0.8 Torr (a), p = 1.6 Torr (b), p = 2 Torr (c), p = 2.5 Torr (d), and for He:Ne^{20}:Ne^{22} = 14:1:1 p = 2 Torr (e). Total frequency scan is 200 MHz.

modes,[11] i.e., to two-mode generation. Here, the three-mode regime is realized only in a restricted range around x = 0 (for p = 2.5 Torr this interval amounts to about 45 MHz) where the modes are always in the locked state.

It is readily shown that the small beat signal is proportional to $E_2^2E_1E_3\cos\psi$, where E_j is the amplitude of the j-th mode and ψ is the relative phase angle. The field behavior in the neighborhood of x = 0 is largely due to the competition between the modes, which leads to a substantial change in E_1 and E_3, while E_2^2 varies relatively slowly.

It would appear that the main reason for the sharp resonance in the behavior of the small beats near the locking region is mode competition. This is confirmed by measurements using a laser with He : Ne^{20} : Ne^{22} = 14 : 1 : 1. It is well known[11] that, in this case, mode competition is absent and the mode intensities repeat the gain profile as the frequency is varied. This should lead to a smooth variation of $E_2^2E_1E_3$ in the locking region, which is, in fact, observed in the behavior of the small beats in Fig. 1e.

Since $E_2^2E_1E_3$ is always positive, and is a maximum at x = 0, the overturn of the resonance is connected only with the dependence of ψ on the pressure. The locking regime ($\dot\psi = 0$) and the absence of mode locking ($\dot\psi \neq 0$) must also be connected with the behavior of ψ.

THEORETICAL ANALYSIS

Rigorous analysis of the three-mode problem requires the solution of a set of six differential equations for the fields and frequencies of the generated modes. This has been done by Sayers and Allen[5] using the Lamb theory.[8] These calculations are based on a very large frequency increment and the most interesting region around x = 0 is not considered. Moreover, our experiments show that the character of the locking process cannot be explained without allowing for pressure-dependent factors.

We have therefore calculated the polarization of the medium, using third-order perturbation theory, for the case of three-mode generation with simultaneous allowance for depolarizing collisions and the trapping of resonance radiation, in a way similar to the approach adopted in the two-mode case in[10]. Neglecting trapping for the sake of simplicity, we shall now give a qualitative explanation of the experimental results on the basis of the analysis of the conditions for mode locking.

The relative phase angle is given by[8]

$$\dot\psi = \Sigma + A\sin\psi + B\cos\psi, \qquad (1)$$

where Σ represents the linear and nonlinear frequency shifts of the generated modes, and

$$A = 2E_1E_3\eta_{13} + E_2^2E_3E_1^{-1}\eta_{23} + E_2^2E_1E_3^{-1}\eta_{21},$$
$$B = 2E_1E_3\xi_{13} - E_2^2E_3E_1^{-1}\xi_{23} - E_2^2E_1E_3^{-1}\xi_{21}. \qquad (2)$$

The coefficients A and B are connected with the perturbation of the generated modes by combination tones, and the coefficients η_{ij} and ξ_{ij} are analogous to the Lamb coefficients.[8]

The locking regime ($\dot\psi = 0$) corresponds to the stationary solution of Eq. (1) which exists when

$$A^2 + B^2 \geqslant \Sigma^2; \qquad (3)$$

and it is then readily shown that

$$\cos\psi = -\frac{A\sqrt{A^2+B^2-\Sigma^2}+\Sigma B}{A^2+B^2}. \qquad (4)$$

Let us first explain the overturn of resonance as a function of pressure. It is sufficient to consider for this purpose the strictly symmetric mode disposition (x = 0). Symmetry considerations show that, in this case, we should have $E_1 = E_3$, $\eta_{23} = \eta_{21}$, $\Sigma = 0$, and B = 0. From Eqs. (2) and (4) we then have

$$A = 2[E_1^2\eta_{13} + E_2^2\eta_{23}], \qquad (5)$$

$$\cos\psi = -|A|/A \qquad (6)$$

For the sake of simplicity we shall confine our attention to the case where there is no competition between the middle and side modes. This is valid in our case of an inhomogeneously broadened line for x = 0 when the "holes" on the gain curve do not overlap for the middle and side modes.

Thus, for the intensity of the middle mode we can write $E_2^2 = E_0^2 \sim (\eta - 1)/2$ (where η is the relative excitation in the laser) just as in the case of single-mode generation. For the side modes, on the other hand, we have, assuming two-mode generation.

$$E_1^2=E_3^2\sim\frac{\eta-1}{2}\left(1+\frac{\gamma^2}{\gamma^2+\Delta^2}\right)^{-1}$$

where γ and Δ are the half-width of a homogeneous line and the mode separation, respectively.

When collisional broadening is taken into account, the coefficients η_{ij} assume the following form:

$$\eta_{23}\approx-D\frac{\gamma\gamma_a\gamma_b}{\Delta^2(\gamma_a+\gamma_b)},\qquad \eta_{13}\approx\eta_{23}4\zeta\frac{\Delta^2}{(\Delta^2+4\gamma^2)},\tag{7}$$

where D is determined by the spectral parameters of the transition. These expressions are obtained on the assumption that

$$ku\gg\Delta>\gamma\gg\gamma_a,\ \gamma_b.\tag{8}$$

Using Eqs. (7) and (8) and the expressions for the intensities, we have from Eq. (5)

$$A\approx-2DE_0^2\frac{\gamma\gamma_a\gamma_b}{\Delta^2(\gamma_a+\gamma_b)}a,\qquad a=1+4\zeta\frac{\Delta^2}{\Delta^2+4\gamma^2}.\tag{9}$$

In our experiments $\zeta = -0.36$ and, therefore, at low gas pressures ($\gamma \ll \Delta$) $a < 0$, $A > 0$, and $\psi = \pi$. For high pressures $a > 0$, $A < 0$, and $\psi = 0$, i.e., as p increases we first have locking with downward and then upward resonance.

The case $a = 0$, $A = 0$ is a singular point and corresponds to the intermediate pressure p_{int}. From Eq. (9) we have the half-width of the homogeneous line for p_{int}:

$$\gamma_{int}=\tfrac{1}{2}\Delta\sqrt{-4\zeta-1}.\tag{10}$$

In our case, γ_{int} = 126 MHz. Using the well-known expression for the half-width of the homogeneous line, γ(MHz) = 15 + 60p (Torr),[10] we find that $p_{int} \approx 1.85$ Torr. It also follows from Eq. (10) that p_{int} must increase with decreasing ζ.

All these results are in agreement with the experimental data described above.

Let us now consider the condition for mode locking as a function of detuning in the region of x = 0. For convenience, we shall transform from the coefficients A, B, and Σ to the new coefficients $A' = AE_1E_3$, $B' = BE_1E_3$, and $\Sigma' = \Sigma E_1E_3$. The expressions for these coefficients to within the first nonvanishing terms of the expansion in x are

$$\begin{aligned}A'&\simeq-2DE_0^4\frac{\gamma\gamma_a\gamma_b}{\Delta^2(\gamma_a+\gamma_b)}\left\{a+b\left(\frac{x}{x_0}\right)^2\right\},\\ B'&\simeq-2DE_0^4\frac{\gamma\gamma_a\gamma_b}{\Delta^2(\gamma_a+\gamma_b)}\frac{\Delta\gamma}{(\Delta^2+\gamma^2)}\frac{x}{x_0},\\ \Sigma'&\simeq DE_0^4\frac{\Delta^2}{\Delta^2+4\gamma^2}\frac{x}{\gamma},\end{aligned}\tag{11}$$

where $b = -4\zeta\Delta^2/(\Delta^2 + 4\gamma^2)$, x_0 is the detuning around x = 0, and the behavior E_1^2 and E_3^2 as functions of x may be assumed to be linear.

We note that when $x \neq 0$, we have $\Sigma B' < 0$. This means [see Eq. (4)] that the change in the relative phase angle ψ with detuning in the locking region may lead to a change in the sign of $\cos\psi$ for $A > 0$, whilst when $A < 0$ we have always $\cos\psi > 0$. In other words, for pressures $p < p_{int}$ ($A > 0$) the small beat signal in the locking region may change sign from minus to plus as the modes depart from symmetric disposition, and this is confirmed by experiment (Fig. 1b). On the other hand, when $p > p_{int}$ ($A < 0$) the small beat signal is positive in the locking region for all frequency differences (see Fig. 1e).

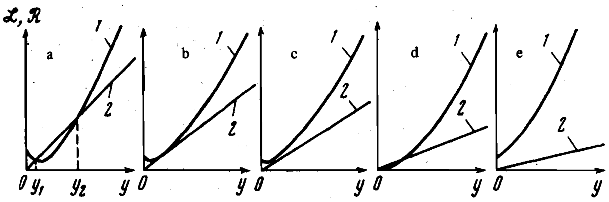

FIG. 2. Graphical solution for the locking condition ($\mathcal{L} \geq \mathcal{R}$) for gas pressures $p < p^*$ (a), $p = p^*$ (b), $p > p^*$ (c), $p = p_{int}$ (d), and $p > p_{int}$ (e). Curve 1—the function $\mathcal{L}$, 2—the function $\mathcal{R}$.

Using Eq. (11), we can rewrite condition (3) in the form

$$\left\{a+b\left(\frac{x}{x_0}\right)^2\right\}^2+\frac{\Delta^2\gamma^2}{(\Delta^2+\gamma^2)^2}\left(\frac{x}{x_0}\right)^2\geqslant\frac{\Delta^8(\gamma_a+\gamma_b)^2x_0^2}{4\gamma^4\gamma_a^2\gamma_b^2(\Delta^2+4\gamma^2)^2}\left(\frac{x}{x_0}\right)^2.\tag{12}$$

Figure 2 shows in a qualitative way the solution of this inequality which is written as $\mathcal{L} \geq \mathcal{R}$, where $y = (x/x_0)^2$.

The function $\mathcal{R}$ depends on the generation frequency shift and is a straight line whose slope decreases rapidly with increasing pressure.

The function $\mathcal{L}$ which governs the effect of the combination tone on the generation frequency is the sum of a parabola and a straight line. At low frequencies, when $a < 0$, it begins as a^2 (for y = 0) and passes through a minimum near y = a/b. As the pressure increases, this minimum descends and touches the origin. The behavior of $\mathcal{R}$ and $\mathcal{L}$ as functions of y provides a qualitative explanation of the nature of the locking process for symmetric mode disposition at different pressures.

At low pressures $p < p^*$ (Fig. 2a) there is no mode locking in the region y_1y_2, which is in agreement with experiment (Fig. 1a). The interval $0y_1$ where locking does occur is not observed in Fig. 1a because $0y_1$ is small and the apparatus has a finite resolution.

As p increases, the region y_1y_2 in which there is no mode locking becomes narrower and disappears altogether when $p = p^*$ (Fig. 2b). For $p \geq p^*$ (Figs. 2b and 2c) locking occurs for all points within the symmetric mode disposition. The resonance in the small beat signal is directed downward, since for x = 0 we have $a < 0$ and $\psi = \pi$, which is in agreement with experiment (Fig. 1b).

For $a = 0$ and $p = p_{int}$ the $\mathcal{L}$ curve emerges from the origin (Fig. 2d) and should lie below the $\mathcal{R}$ line in the neighborhood of y = 0, which again is in agreement with experiment (Fig. 1c).

At high pressures $p > p_{int}$ when $a > 0$ the behavior of $\mathcal{L}$ and $\mathcal{R}$ is as shown in Fig. 2e. Here, locking occurs at all points in the region of symmetric mode disposition with an upward resonance in the small beat signal ($a > 0$ and $\psi = 0$), which is in agreement with experiment (Fig. 1d).

INFLUENCE OF COMBINATION TONES ON MODE INTENSITIES

Figures 1b and 1d, which correspond to the locking regime in the region x = 0, exhibit a well-defined structure in this region with a width much less than γ. This is connected both with the change in $\psi(x)$ and in the mode

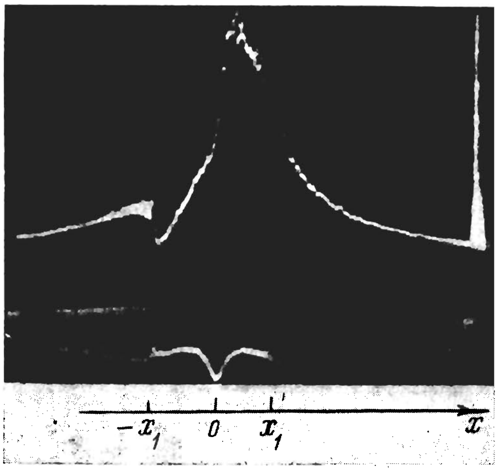

FIG. 3. Side mode intensity recorded with a Fabry-Perot interferometer as an optical filter (upper trace) and small beats signal as a function of deterring (lower trace) in the region of symmetric mode disposition. Total frequency scan 200 MHz.

intensity, Combination tones may have a substantial influence on the behavior of these quantities, and to elucidate it we investigated the mode intensities using the Fabry-Perot interferometer as a tuned optical filter by analogy with[11].

Figure 3 shows an oscillogram of the intensity of one of the side modes E_1^2 as a function of detuning in the region of symmetric deposition. At the same time, the lower beam of the oscillograph was used to record the small beat signal so that the presence or otherwise of locking could be exhibited.

It is clear that in the locking region $-x_1 < x < x_1$ the intensity is a nonlinear function of detuning and the slope dE_1^2/dx decreases with decreasing E_1^2. In the presence of locking the combination tone and the corresponding mode have the same frequency. Nevertheless, contributions due to the tone intensity $E_{T_1}^2$ and mode intensity $E_{M_1}^2$ can be formally separated on the basis of the different dependence on detuning. The intensity of the combination tone $E_{T_1}^2$ is proportional to the product of the intensities of modes which induce it, i.e., $E_{T_1}^2 \sim E_{M_2}^4 E_{M_3}^2$.[8] When the behavior of the mode intensities near $x = 0$ was taken into account,[10,11] it became clear that as $E_{M_1}^2$ decreases the contribution of $E_{T_1}^2$ to the resultant intensity E_1^2 should increase. Therefore, the slope dE_1^2/dx should decrease with decreasing mode intensity.

On the other hand, when there is no mode locking, the combination tone leads to oscillations in E_1^2 as a function of time, and this is clearly seen in Fig. 3 for $x \leq -x_1$. When $x \geq x_1$ the oscillations are weaker and this is connected with the lower value of $E_{M_2}^4 E_{M_3}^2$. The oscillation frequency is mainly dependent on x and varies between 10^3 and 10^5 Hz.

We note that similar measurements in the case of two-mode generation show a practically linear variation of mode intensity in the region of symmetric disposition and the absence of oscillations. This confirms the fact that the change in the slope dE_1^2/dx and the oscillations in the intensity E_1^2 with time under the three-mode conditions are connected with the presence of the combination tone which, in the two-mode case, lies outside the generation range and does not affect the characteristics of the laser radiation.

[1] P. W. Smith, Proc. IEEE 58, 1342 (1970).
[2] H. Statz, J. Appl. Phys. 38, 4648 (1967).
[3] D. G. Jones, M. D. Sayers, and L. Allen, J. Phys. **A2**, 95 (1969).
[4] A. Bambini and P. Burlamachi, IEEE J. Quantum Electron. 4, 101 (1968).
[5] M. D. Sayers and L. Allen, Phys. Rev. **A1**, 1730 (1970).
[6] P. L. Rubin, Preprint No. 153 (Preprint No. 153), FIAN, 1971.
[7] S. G. Zeiger, Zh. Tekh. Fiz. 38, 68 (1968) [Sov. Phys.-Tech. Phys. 13, 48 (1968)] Tezisy dokladov Vsesoyuznogo simpoziuma "Fizicheskie osnovy upravleniya chastotoĭ vynuzhdennogo izlucheniya," Kiev (Abstracts of Papers read at the All-Union Symposium on the Physical Foundations of the Control of the Frequency of Stimulated Radiation), Kiev, 1972.
[8] W. E. Lamb, Phys. Rev. **A134**, 1429 (1964).
[9] S. A. Gonchukov, O. E. Porodnikov, E. D. Protsenko, and V. A. Semchishen, ZhETF Pis. Red. 14, 235 (1971) [JETP Lett. 14, 155 (1971)].
[10] Yu. A. Vdovin, S. A. Gonchukov, M. A. Gubin, V. M. Ermachenko, A. N. Oraevskiĭ, and E. D. Protsenko, Preprint No. 116 (Preprint No. 116), FIAN, 1972.
[11] S. A. Gonchukov, I. O. Leĭpunskiĭ, E. D. Protsenko, and A. Yu. Rumyantsev, Opt. Spektrosk. 27, 813 (1969).

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