# Gravitational Compton effect and photoproduction of gravitons by electrons

**N. A. Voronov**
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The differential cross sections for scattering and photoproduction of gravitons by electrons are calculated in the Born approximation. A technique is developed which is necessary for determining on the basis of the pole graphs the contact diagram of an arbitrary process involving gravitons with an accuracy to terms linear with respect to the graviton momentum. The method is applied to the process under consideration. Low energy theorems for the reactions are discussed.

## 1. INTRODUCTION

The scattering of a graviton by an electron and the transformation of a photon into a graviton when it collides with an electron (graviton photoproduction) were considered within the framework of the usual scheme of quantization of the gravitational field [1] in the lowest (Born) approximation in $\kappa$, where $\kappa = \sqrt{16\pi G}$ and G is the gravitational constant. Mironovskiĭ [2] has discussed the photoproduction of a graviton on an electron, but the expression he gave for the cross section does not agree with the formula obtained by us. The t-channels of the considered processes constitute annihilation of an electron-positron pair into two gravitons and into a graviton and photon. The Born approximation for the last two reactions was calculated earlier by Vladimirov [3]. We note that in [2,3] they used a different procedure for expanding the Lagrangian density in powers of $\kappa$.[1)] However, as was verified by us, both procedures lead to identical expressions for the matrix elements. An analogous statement was made in [4] for the scattering of a graviton by a scalar particle. The formulas obtained by us for the cross section, after going over to the t-channel, did not coincide with the analoguos results of Vladimirov.

In a number of papers, the condition of gauge invariance and certain assumptions concerning the analytic properties of the amplitude of a process in which a graviton takes part were used to prove the corresponding low-energy theorems. Jackiw [5] has discussed the low-energy theorem for the scattering of a graviton by a scalar particle and has shown that the result obtained by this approach is equivalent to the low-energy theorem proved with the aid of dispersion methods. Zakharov and Kobzarev [6] considered the radiation of a graviton upon collision of n particles with spins 0, 1/2, and k, where k is an arbitrary integer. It was shown that the gauge invariance makes it possible to reconstruct the amplitude of the process accurate to terms linear in the graviton momentum. In the present paper, the low energy theorems for the gravitational Compton effect and for the production of a graviton on an electron are discussed from this point of view.

We list briefly the necessary definitions and symbols[2)]. The scattering matrix S is connected with the invariant amplitude of the process A by the relation

$$S=1+i\frac{(2\pi)^4\delta\left(\sum_f p_f-\sum_i p_i\right)}{\prod_f (2p_{0f})^{1/2}\prod_i (2p_{0i})^{1/2}}A,$$

where $p_f$ and $p_i$ are the momenta of the final and initial states. In the case when a graviton takes part in the process, the amplitude A can be rewritten in the form

$$A=A_{mn}h_{mn}(q) \quad \text{or} \quad A=A_{ik}'h_{ik}^*(q'), \tag{1}$$

depending on whether the graviton is absorbed or emitted in this reaction. Here $h_{mn}(q)$ ($h_{ik}^*(q')$) is the polarization tensor of the entering (emerging) graviton with momentum $q_m$ ($q_i'$), having the following properties:

$$h_{mn}(q)=h_{nm}(q),\quad h_{mn}(q)q_n=0,\quad h_{mm}(q)=0,\quad h_{mn}(q)h_{mn}^*(q)=1$$

($h_{mn}^*(q)$ is the complex conjugate of the tensor $h_{mn}(q)$), $h_{mn}(q)$ determines the deviation of $h_{mn}(x)$ of the metric tensor $g_{mn}(x)$ from the flat tensor

$$g_{mn}(x)=\delta_{mn}+\varkappa h_{mn}(x) \tag{2}$$

(where $\delta_{mn}$ is the Feynman $\delta$-symbol), in accord with the expression

$$h_{mn}(x)=\sqrt{2}\sum_q\left(a_q e^{i(qx)}h_{mn}(q)\frac{1}{\sqrt{2q_0}}+a_q^+e^{-i(qx)}h_{mn}^*(q)\frac{1}{\sqrt{2q_0}}\right). \tag{3}$$

We shall henceforth use only the chiral polarization tensor $h_{mn}^\lambda(q)$. It is convenient here to introduce in its place the isotropic vector $\epsilon_m$ [7]:

$$h_{mn}^\lambda(q)=\varepsilon_m\varepsilon_n,\quad \varepsilon_m\varepsilon_m=0. \tag{4}$$

The tensor $A_{mn}$ (1) (as well as $A_{ik}'$) is symmetrical and, by virtue of gauge invariance of the amplitude A, it is transverse [8]:

$$A_{mn}=A_{nm},\quad A_{mn}q_n=0. \tag{5}$$

In all the expressions where the expansion (2) is used, we use the Feynman summation rule, whereas in the general-covariant expressions we use the ordinary rule. The tensor index following the comma denotes the partial derivative, and that following a semicolon denotes the covariant derivative with respect to the corresponding variable $x^i$.

The plan of the paper is as follows. The Born approximation for the scattering of a graviton by an electron is calculated in Sec. 2. Similar calculations are carried out in Sec. 3 for the photoproduction of a graviton. In Sec. 4 we verify the transversality of the amplitudes obtained in Secs. 2 and 3, and in Sec. 5 we discuss the low-energy theorems for the considered processes.

## 2. GRAVITATIONAL COMPTON EFFECT

The Lagrangian density of the interaction of the electron with the gravitational field is equal, as always, to the sum of two terms: the Lagrangian density $\mathscr{L}_g$ of the gravitational field, and the density $\mathscr{L}_e$ of the general-covariant Lagrangian of the electron [9]:

$$\mathscr{L}_{int} = \mathscr{L}_g + \mathscr{L}_e,$$
$$\mathscr{L}_g = \varkappa^{-2}\sqrt{-g}g^{ik}(\Gamma_{ik}{}^l\Gamma_{lm}{}^m - \Gamma_{il}{}^m\Gamma_{km}{}^l), \quad (6)$$
$$\mathscr{L}_e = \sqrt{-g}[{}^1/{}_2 i(\bar\psi\gamma^i\vec\nabla_i\psi - \bar\psi\overleftarrow{\nabla}_i\gamma^i\psi) - m\bar\psi\psi].$$

Here $g = \det\|g_{ik}\|$; $\gamma^i = \lambda^i_{(a)}\gamma^{(a)}$, where $\lambda^i_{(a)}$ is a certain reference, and $\gamma^{(a)}$ are ordinary Dirac matrices;

$$\vec\nabla_i\psi = \psi_{,i} - \Gamma_i\psi; \quad \bar\psi\overleftarrow{\nabla}_i = \bar\psi_{,i} + \bar\psi\Gamma_i,$$

where $\Gamma_i = {}^1/{}_4\gamma^{(a)}\gamma^{(b)}\lambda^k_{(a);i}\lambda_{(b)k}$. For our purposes it is convenient to choose the reference in such a way that it is uniquely connected with the metric tensor. This can be done as in[10], and the result is

$$\lambda_{(a)i} = \sqrt{g_{ai}},$$

the square root being taken to mean in the matrix sense.

Substituting further (2) in (6) and expanding in powers of $\kappa$, we have

$$\mathscr{L}_g = \mathscr{L}_g{}^0 + \varkappa\mathscr{L}_g{}^1 + \varkappa^2\mathscr{L}_g{}^2 + \ldots,$$
$$\mathscr{L}_g{}^0 = -{}^1/{}_4(2h_{ik,l}h_{il,k} - 2h_{,l}h_{il,i} + h_{,i}h_{,i} - h_{ik,l}h_{ik,l}),$$
$$\mathscr{L}_g{}^1 = {}^1/{}_4[(h_{mn} - {}^1/{}_2\delta_{mn}h)\delta_{ik}\delta_{jl} + \delta_{mn}\delta_{jl}h_{ik} \quad (7)$$
$$+ \delta_{ik}\delta_{mn}h_{jl}](2h_{km,j}h_{ln,i} - 2h_{jk,l}h_{mn,i}$$
$$+ h_{ik,n}h_{jl,m} - h_{jk,m}h_{il,n}),$$

$\mathscr{L}_g^2$ is a certain polynomial of fourth degree in $h_{ik}$, and $h = h_{ik}\delta_{ik}$.

For the Lagrangian density of the electron we have analogously

$$\mathscr{L}_e = \mathscr{L}_e{}^0 + \varkappa\mathscr{L}_e{}^1 + \varkappa^2\mathscr{L}_e{}^2 + \ldots,$$
$$\mathscr{L}_e{}^0 = {}^1/{}_2 i(\bar\psi\gamma_{(i)}\psi_{,i} - \bar\psi_{,i}\gamma_{(i)}\psi) - m\bar\psi\psi,$$
$$\mathscr{L}_e{}^1 = {}^1/{}_2 h\mathscr{L}_e{}^0 - {}^1/{}_4 ih_{ia}(\bar\psi\gamma_{(a)}\psi_{,i} - \bar\psi_{,i}\gamma_{(a)}\psi),$$
$$\mathscr{L}_e{}^2 = {}^1/{}_8(h^2 - 2h_{ik}h_{ik})\mathscr{L}_e{}^0 + {}^1/{}_{16}i(3h_{ik}h_{ka} - 2hh_{ia}) \quad (8)$$
$$\times(\bar\psi\gamma_{(a)}\psi_{,i} - \bar\psi_{,i}\gamma_{(a)}\psi) + {}^1/{}_{16}i(h_{ik}h_{ai,k} - h_{ai}h_{ik,k})\bar\psi\gamma_{(a)}\psi$$
$$+ {}^1/{}_{32}i(h_{ci}h_{bi,a} - h_{bi}h_{ci,a})\bar\psi\gamma_{(a)}\gamma_{(b)}\gamma_{(c)}\psi,$$

where $\gamma^{(0)} = \gamma_{(0)}$, $\gamma^{(\alpha)} = -\gamma_{(\alpha)}$ ($\alpha = 1, 2, 3$).

The scattering matrix can be represented in the form

$$S = T\exp\left[i\int\mathscr{L}_{int}(x)\,dx\right],$$

and that part of the S matrix which is responsible for the process considered by us is equal to

$$S^2 = i\varkappa^2\int dx[:\mathscr{L}_g{}^2(x): + :\mathscr{L}_e{}^2(x):]$$
$$- {}^1/{}_2\varkappa^2 T\int dx\,dy[:\mathscr{L}_g{}^1(x): + :\mathscr{L}_e{}^1(x):][:\mathscr{L}_g{}^1(y): + :\mathscr{L}_e{}^1(y):]. \quad (9)$$

Applying Wick's theorem to (9), we obtain the amplitude for the scattering of a graviton by an electron in the form

$$A^c = A_{\mathrm{I}}{}^c + A_{\mathrm{II}}{}^c + A_{\mathrm{III}}^c \quad (10)$$

in accordance with the three types of diagrams I, II, and III (see Fig. 1), which they represent:

$$A_{\mathrm{I}}{}^c = \varkappa^2\frac{(\varepsilon_1 p_2)(\varepsilon_2{}^*p_1)}{2(p_1q_2)}\bar u(p_2)\hat\varepsilon_1\left[(\varepsilon_2{}^*p_1) - \frac{1}{2}\hat q_2\hat\varepsilon_2{}^*\right]u(p_1)$$
$$- \varkappa^2\frac{(\varepsilon_1 p_1)(\varepsilon_2{}^*p_2)}{2(p_1q_1)}\bar u(p_2)\left[(\varepsilon_2{}^*p_2) + \frac{1}{2}\hat\varepsilon_2{}^*\hat q_2\right]\hat\varepsilon_1 u(p_1), \quad (10a)$$

$$A_{\mathrm{II}}{}^c = \frac{\varkappa^2}{2(q_1q_2)}\bar u(p_2)\Big\{\hat q_1(\varepsilon_1\varepsilon_2{}^*)^2(p_1q_1) - (\varepsilon_1\varepsilon_2{}^*)[\hat q\;((\varepsilon_2{}^*q_1)(\varepsilon_1p_1)$$
$$+ (\varepsilon_1q_2)(\varepsilon_2{}^*p_1)) + \hat\varepsilon_1(\varepsilon_2{}^*q_1)(p_1q_1) + \hat\varepsilon_2{}^*(\varepsilon_1q_2)(p_1q_2)]$$
$$+ [\hat\varepsilon_1(\varepsilon_2{}^*q_1) + \hat\varepsilon_2{}^*(\varepsilon_1q_2)][(\varepsilon_1p_1)(\varepsilon_2{}^*q_1) + (\varepsilon_1q_2)(\varepsilon_2{}^*p_1)]$$
$$- (\varepsilon_1\varepsilon_2{}^*)(q_1q_2)\left[\hat\varepsilon_1(\varepsilon_2{}^*p_1) + \hat\varepsilon_2{}^*(\varepsilon_1p_1) + \frac{1}{2}\hat q_1(\varepsilon_1\varepsilon_2{}^*)\right]\Big\}u(p_1), \quad (10b)$$

$$A_{\mathrm{III}}^c = {}^1/{}_2\varkappa^2(\varepsilon_1\varepsilon_2{}^*)\bar u(p_2)[2\hat\varepsilon_1(\varepsilon_2{}^*p_1) + \hat\varepsilon_1(\varepsilon_2{}^*q_1) + \varepsilon_2{}^*(\varepsilon_1p_1)$$
$$- {}^1/{}_2\hat q_1(\varepsilon_1\varepsilon_2{}^*) + {}^1/{}_2\hat\varepsilon_1\hat\varepsilon_2{}^*\hat q_2]u(p_1), \quad (10c)$$

where $\epsilon_{1m}\epsilon_{1n}$ and $q_{1m}$ are the initial polarization and momentum of the graviton, $\epsilon^*_{2i}\epsilon^*_{2k}$ and $q_{2i}$ are the final polarization and the momentum of the graviton, $p_{1i}$ and $p_{2i}$ are the initial and final momenta of the electron. The amplitude $A^c$ is gauge-invariant; a proof of this statement is given in Sec. 4.

It is most convenient to calculate the cross section with the aid of helicity amplitudes. To construct them, we consider the two vectors[3)]:

$$\varepsilon_{1i} = [(u - s)(q_1 + q_2)_i$$
$$- t(p_1 + p_2)_i]/2[t(su - m^4)]^{1/2},$$
$$\varepsilon_{2i} = 2\varepsilon_{ijkl}p_{1j}q_{2k}q_{1l}$$
$$/[t(su - m^4)]^{1/2}; \quad (11)$$
$$s = (p_1 + q_1)^2,$$
$$t = (p_1 - p_2)^2 = (q_1 - q_2)^2,$$
$$u = (p_1 - q_2)^2.$$

$\epsilon_{1i}$ and $\epsilon_{2i}$ have the following properties[4)][11]:

$$(\varepsilon_\mu, \varepsilon_\nu) = \delta_{\mu\nu},$$
$$(\varepsilon_1, p_\mu) = -((su - m^4)/t)^{1/2},$$
$$(\varepsilon_2, p_\mu) = 0, \quad (\varepsilon_\mu, q_\nu) = 0$$
$$(\mu, \nu = 1, 2).$$

The last equation enables us to construct from $\epsilon_{1i}$ and $\epsilon_{2i}$ the polarization vectors $\epsilon_i^{(\pm)}$ for both the initial and the final graviton. We note, however, that $\epsilon_{1i}$ is symmetrical and $\epsilon_{2i}$ is antisymmetrical relative to permutation of the momenta $q_{1i}$ and $q_{2i}$. Therefore, if we define the polarization vectors for the initial graviton by the equation

$$\varepsilon_k^{(\pm)} = (\varepsilon_{1k} \pm i\varepsilon_{2k})/\sqrt{2}, \quad (12)$$

then the analogous vectors for the final gravitons are[5)]:

$$\varepsilon_k^{(\pm)} = (\varepsilon_{1k} \mp i\varepsilon_{2k})/\sqrt{2}. \quad (13)$$

We introduce the notation

$$S_{+-}(q_1, q_2, p_1) = \frac{1}{(su - m^4)}\frac{mt}{(s - m^2)(u - m^2)}\bar u(p_2)[(su - m^4)$$
$$+ 2i(\hat q_2 + m)\varepsilon_{ijkl}\gamma_{(i)}p_{1j}q_{2k}q_{1l}]u(p_1), \quad (14)$$
$$S_{++}(q_1, q_2, p_1)$$
$$= \frac{1}{(s - m^2)(u - m^2)}\bar u(p_2)[(s - u)\hat q_1 + tm - 2i\varepsilon_{ijkl}\gamma_{(i)}p_{1j}q_{2k}q_{1l}]u(p_1).$$

We then have for the helicity amplitudes of the graviton-electron scattering

$$A_{+-}^c = \varkappa^2\frac{m^2}{4}S_{+-}(q_1, q_2, p_1),$$
$$A_{++}^c = -\varkappa^2\frac{(su - m^4)}{4t}S_{++}(q_1, q_2, p_1). \quad (15)$$

We confine ourselves to the scattering of only unpolarized particles. Therefore in the calculation of $|A_{+-}^c|^2$ and $|A_{++}^c|^2$ we average over the initial electron polarizations and sum over the final ones. As a result we get

$$|A_{+-}^c|^2 = \varkappa^4\frac{(su - m^4)^3}{8t^2(s - m^2)^2(u - m^2)^2}[2(su - m^4) - t^2],$$
$$|A_{++}^c|^2 = \varkappa^4\frac{m^6t^2}{8(s - m^2)^2(u - m^2)^2}(2m^2 - t). \quad (16)$$

In accordance with the general rules (see[11]), the dif-

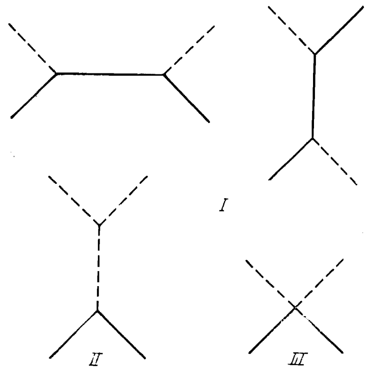


FIG. 1. Gravitational Compton effect on an electron. The dashed line represents the graviton.

ferential cross section is expressed in terms of the moduli of the helicity amplitudes in accordance with the formula

$$d\sigma^c = \frac{1}{16\pi}\frac{dt}{(s-m^2)^2}(|A^c_{+-}|^2+|A^c_{++}|^2).$$

In this expression, we have already summed over the final graviton polarization and averaged over the initial ones. Substituting here (16), we obtain ultimately

$$d\sigma^c = \frac{\varkappa^4}{128\pi}\frac{dt}{t^2(s-m^2)^4(u-m^2)^2}\{(su-m^4)^3[2(su-m^4)-t^2]+m^6t^4(2m^2-t)\}. \tag{17}$$

Let us express, finally, this cross section in the laboratory system ($p_1 = (m, 0, 0, 0)$)

$$d\sigma^c = \frac{G^2\,do}{[1+\omega_1(1-\cos\vartheta)/m]^3}\Big[m^2\Big(\mathrm{ctg}^4\frac{\vartheta}{2}\cos^4\frac{\vartheta}{2}+\sin^4\frac{\vartheta}{2}\Big) + 2m\omega_1\Big(\mathrm{ctg}^2\frac{\vartheta}{2}\cos^6\frac{\vartheta}{2}+\sin^6\frac{\vartheta}{2}\Big)+2\omega_1^2\Big(\cos^6\frac{\vartheta}{2}+\sin^6\frac{\vartheta}{2}\Big)\Big], \tag{18}$$

where $\vartheta$ is the graviton scattering angle and $\omega_1$ is the energy of the initial graviton.

At low energies of the initial graviton $\omega_1 \ll m$, Eq. (18) goes over into the expression

$$d\sigma^c = G^2m^2\Big(\mathrm{ctg}^4\frac{\vartheta}{2}\cos^4\frac{\vartheta}{2}+\sin^4\frac{\vartheta}{2}\Big)\,do, \tag{19}$$

where expression (19) coincides with the nonrelativistic formula for the cross section for the scattering of a graviton by a scalar particle. Westervelt[12] derived (19) classically, assuming that the scattering of the graviton is the result of the deviation of the isotropic geodesics in the field of an immobile pointlike mass.

At high initial-graviton energies ($\omega_1 \gg m$), fairly simple expressions are obtained in two limiting cases:

$$d\sigma^c = G^2m^2\frac{do}{\sin^4(\vartheta/2)} \qquad \vartheta \ll \sqrt{\frac{m}{\omega_1}}, \tag{20}$$

$$d\sigma^c = \frac{G^2m^2}{4}\frac{m}{\omega_1}\Big(1+\mathrm{ctg}^6\frac{\vartheta}{2}\Big)\,do, \qquad \vartheta \gg \sqrt{\frac{m}{\omega_1}}. \tag{21}$$

We proceed now to consider two-graviton annihilation of an electron-positron pair. Using the invariant form of the cross section (17) and the cross-invariance of the amplitude $A^c$ (10), we can write down immediately a formula for the differential annihilation cross section

$$d\sigma^a = \frac{\varkappa^4}{128\pi} \times \frac{ds}{(s-m^2)^2(u-m^2)^2(t-4m^2)t^3}\{m^6t^4(2m^2-t)+(su-m^4)^3[2(su-m^4)-t^2]\}. \tag{22}$$

The kinematic invariants in this channel take on the values

$$s=(p_- - q_1)^2, \quad t=(p_- + p_+)^2=(q_1+q_2)^2, \quad u=(p_- - q_2)^2,$$

where $p_{-i}$ and $p_{+i}$ are the momenta of the electron and positron, while $q_{1i}$ and $q_{2i}$ are the momenta of the outgoing gravitons. To compare the obtained expression for the cross section (22) with the analogous result of[3], we change over to the c.m.s., in which the annihilation cross section is expressed. We obtain

$$d\sigma^a = \frac{G^2}{4}\frac{do}{\varepsilon p}\frac{m^6(\varepsilon^2+p^2)+p^6\sin^6\theta(\varepsilon^2+m^2+p^2\cos^2\theta)}{(m^2+p^2\sin^2\theta)^2}, \tag{23}$$

where $\theta$ is the angle between $\mathbf{p}_-$ and $\mathbf{q}_1$, $p = |\mathbf{p}_-|$, and $\epsilon$ is the energy of one of the gravitons. Comparing (23) with the corresponding Vladimirov formula, we see that they do not agree. In the nonrelativistic approximation ($p = mv \ll m$) we have

$$d\sigma^a = \frac{G^2m^2}{4}\frac{do}{v}. \tag{24}$$

In the relativistic approximation, just as for electron-graviton scattering, we confine ourselves to only two limiting cases:

$$d\sigma^a = \frac{G^2m^2}{2}\,do, \qquad \theta \ll \frac{m}{p}, \tag{25}$$

$$d\sigma^a = \frac{G^2}{4}p^2\sin^2\theta(1+\cos^2\theta)\,do, \qquad \theta \gg \frac{m}{p}. \tag{26}$$

## 3. GRAVITON PHOTOPRODUCTION

For the photoproduction of a graviton we have

$$\mathscr{L}_{int} = \mathscr{L}_g + \mathscr{L}_\gamma + \mathscr{L}_e(A),$$

where $\mathscr{L}_g$ is the Lagrangian density of the gravitational field (6),

$$\mathscr{L}_e(A) = \sqrt{-g}\{{}^1/_2 i[\bar\psi\gamma^i(\vec\nabla_i - ieA_i)\psi - \bar\psi(\overleftarrow{\nabla}_i + ieA_i)\gamma^i\psi] - m\bar\psi\psi\}, \tag{27}$$

and $\mathscr{L}_\gamma$ is the density of the Lagrangian of the electromagnetic field in Riemann space:

$$\mathscr{L}_\gamma = -\frac{\sqrt{-g}}{16\pi}g^{ij}g^{kl}F_{ik}F_{jl}. \tag{28}$$

Since the graviton photoproduction in the Born approximation is an effect of first order in $\kappa$, it suffices to take into account in $\mathscr{L}_{int}$ terms up to those linear in $\kappa$ inclusive. We have

$$\begin{gathered}\mathscr{L}_e(A) = \mathscr{L}_e^0 + e\bar\psi\gamma_{(i)}\psi A_i + \varkappa\mathscr{L}_e^1 + {}^1/_2\varkappa e(h\delta_{ia}-h_{ia})\bar\psi\gamma_{(a)}\psi A_i + \ldots,\\ \mathscr{L}_\gamma = \mathscr{L}_\gamma^0 + \varkappa\mathscr{L}_\gamma^1 + \ldots,\\ \mathscr{L}_\gamma^0 = -\frac{1}{16\pi}F_{ik}F_{ik},\\ \mathscr{L}_\gamma^1 = \frac{1}{8\pi}F_{ij}F_{kj}h_{ik} - \frac{1}{2}h\mathscr{L}_\gamma^0.\end{gathered} \tag{29}$$

The calculations that follow are similar to those in Sec. 2. Without stopping to dwell on them, we present immediately the amplitude $A^p$ for the photoproduction of a graviton, an amplitude that can also be represented in the form of three terms, each of which corresponds to a definite type of diagram shown in Fig. 2:

$$A_I^p = \frac{\sqrt{2\pi}\,e\varkappa}{(p_1k)}\bar u(p_2)\Big[(p_2\varepsilon^*) + \frac{1}{2}\hat\varepsilon^*\hat q\Big](p_2\varepsilon^*)\hat e u(p_1) - \frac{\sqrt{2\pi}\,e\varkappa}{(p_1q)}\bar u(p_2)\hat e(p_1\varepsilon^*)\Big[(p_1\varepsilon^*) - \frac{1}{2}\hat q\hat\varepsilon^*\Big]u(p_1), \tag{30a}$$

$$A_{II}^p = -\frac{\sqrt{2\pi}\,e\varkappa}{(kq)}\bar u(p_2)[\hat e(\varepsilon^*k)^2 + \hat\varepsilon^*(eq)(\varepsilon^*k) - \hat k(\varepsilon^*k)(e\varepsilon^*)$$

$$-\hat{\varepsilon}^{*}(e\varepsilon^{*})(kq)]u(p_1), \tag{30b}$$

$$A_{\rm III}{}^{p}=-\sqrt{2\pi}e\varkappa\bar{u}(p_2)\times\hat{\varepsilon}^{*}(e\varepsilon^{*})u(p_1), \tag{30c}$$

where $k_i$ is the photon momentum, $e_i$ is its polarization vector, and $\epsilon_i^*\epsilon_k^*$ and $q_i$ are the graviton polarization tensor and momentum.

The helicity amplitudes for the polarization vectors (12) and (13) take the following form in the notation of (14)[6]:

$$A_{+-}{}^{p}=-\sqrt{4\pi}e\varkappa((su-m^4)/t)^{1/2}S_{+-}(k,q,p_1),$$
$$A_{++}{}^{p}=-\sqrt{4\pi}e\varkappa((su-m^4)/t)^{1/2}S_{++}(k,q,p_1), \tag{31}$$

and their moduli are

$$|A_{+-}^{p}|^2=\frac{2\pi e^2\varkappa^2}{(s-m^2)^2(u-m^2)^2}m^2t(su-m^4)(2m^2-t),$$
$$|A_{++}^{p}|^2=\frac{2\pi e^2\varkappa^2}{(s-m^2)^2(u-m^2)^2}\frac{(su-m^4)^2}{t}[2(su-m^4)-t^2]. \tag{32}$$

From this we get the differential scattering cross section for this process

$$d\sigma^p=\frac{e^2\varkappa^2}{2}\frac{dt}{t}\frac{su-m^4}{(s-m^2)^2}\Big[\Big(\frac{m^2}{s-m^2}+\frac{m^2}{u-m^2}\Big)^2+\Big(\frac{m^2}{s-m^2}+\frac{m^2}{u-m^2}\Big)-\frac{1}{4}\Big(\frac{s-m^2}{u-m^2}+\frac{u-m^2}{s-m^2}\Big)\Big]. \tag{33}$$

In the laboratory system we have

$$d\sigma^p=\frac{Ge^2}{m^2}\frac{\cos^2(\vartheta/2)\,do}{[1+\omega(1-\cos\vartheta)/m]^3}\Big[m^2\Big(\mathrm{ctg}^2\frac{\vartheta}{2}\cos^2\frac{\vartheta}{2}+\sin^2\frac{\vartheta}{2}\Big)+2m\omega\Big(\cos^4\frac{\vartheta}{2}+\sin^4\frac{\vartheta}{2}\Big)+2\omega^2\sin^2\frac{\vartheta}{2}\Big], \tag{34}$$

where $\vartheta$ is the angle between **k** and **q**, and $\omega$ is the photon energy. Formula (34) differs from Mironovskiĭ's result[2]. At low photon energies ($\omega \ll m$), Eq. (34) takes the form

$$d\sigma^p=Ge^2\,\mathrm{ctg}^2\frac{\vartheta}{2}\Big(\cos^4\frac{\vartheta}{2}+\sin^4\frac{\vartheta}{2}\Big)do. \tag{35}$$

In[2], the cross section for this case is equal to zero. The reason is that no account has been taken of that part of the amplitude (corresponding to diagram II of Fig. 2), which determines the behavior of the cross section in the nonrelativistic region.

At photon energies that are large in comparison with the electron mass, there are two limiting cases, when the expression for the cross section becomes simple:

$$d\sigma^p=Ge^2\frac{do}{\sin^2(\vartheta/2)},\qquad \vartheta\ll\sqrt{\frac{m}{\omega}}, \tag{36}$$

$$d\sigma^p=\frac{Ge^2}{4}\frac{m}{\omega}\frac{\mathrm{ctg}^2(\vartheta/2)}{\sin^2(\vartheta/2)}do,\qquad \vartheta\gg\sqrt{\frac{m}{\omega}}. \tag{37}$$

In the t-channel, i.e., for the process of graviton-photon annihilation of an electron-positron pair, the s-channel cross section (33) goes over into the expression

$$d\sigma^a=\frac{e^2\varkappa^2}{2}\frac{ds}{t^2}\frac{su-m^4}{t-4m^2}\Big[\Big(\frac{m^2}{s-m^2}+\frac{m^2}{u-m^2}\Big)^2+\Big(\frac{m^2}{s-m^2}+\frac{m^2}{u-m^2}\Big)-\frac{1}{4}\Big(\frac{s-m^2}{u-m^2}+\frac{u-m^2}{s-m^2}\Big)\Big]. \tag{38}$$

Finally, in the c.m.s. (of the t channel)

$$d\sigma^a=\frac{Ge^2}{4}\frac{p}{\varepsilon}\frac{\sin^2\theta\,do}{m^2+p^2\sin^2\theta}\Big[\varepsilon^2+p^2(1+\sin^2\theta)-\frac{2p^4\sin^4\theta}{m^2+p^2\sin^2\theta}\Big], \tag{39}$$

where $\theta$ is the angle between $\mathbf{p}_-$ and **k**, $p = |\mathbf{p}_-|$, and $\epsilon$ is the photon energy. This formula does not agree with the formula obtained in[3] for the cross section of a similar process.

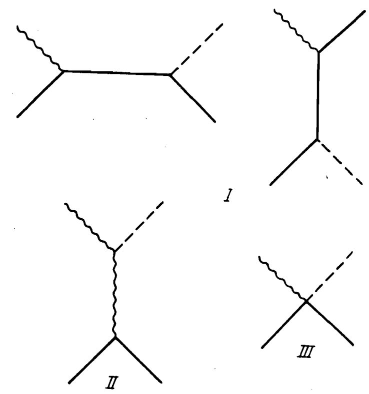


FIG. 2. Photoproduction of a graviton on an electron: the wavy line represents the photon, and the dashed line the graviton.

We consider the limiting cases for formula (39). In the nonrelativistic approximation, when $p = mv \ll m$, we have

$$d\sigma^a=\frac{Ge^2}{4}v\sin^2\theta\,do. \tag{40}$$

In the relativistic approximation ($p \gg m$)

$$d\sigma^a=\frac{Ge^2}{2}\frac{p^2}{m^2}\sin^2\theta\,do,\qquad \theta\ll\frac{m}{p}, \tag{41}$$

$$d\sigma^a=\frac{Ge^2}{4}(1+\cos^2\theta)do,\qquad \theta\gg\frac{m}{p}. \tag{42}$$

## 4. RECONSTRUCTION OF THE CONTACT PART OF THE AMPLITUDE

The gauge invariance means that the amplitude is not altered by the transformation

$$\varepsilon_i\varepsilon_k\to\varepsilon_i\varepsilon_k+\varepsilon_iq_k+\varepsilon_kq_i - \text{for the graviton}, \tag{43}$$

$$e_i\to e_i+k_i - \text{for the photon}. \tag{44}$$

It can be verified by direct calculation that formulas (1) and (30) are invariant relative to the transformations (43) and (44). For more complicated reactions, however (e.g., with participation of hadrons), we can no longer obtain an expression for the contact diagrams in all cases. In such cases it becomes necessary to reconstruct the contact part of the amplitude from the pole part, using gauge invariance. This question was considered in[6], and also in[5].

In the present section we develop the technique necessary for an unambiguous reconstruction of contact diagrams, accurate to terms linear in the graviton momentum, and apply it to processes considered in Secs. 2 and 3. The agreement between the formulas obtained in this manner for the contact part, on the one hand, with formulas (10c) and (30c) calculated in the Lagrangian formalism, on the other, proves the gauge invariance of the amplitudes, and demonstrates the effectiveness of the method of reconstruction in the given approximation.

We start the reconstruction of the contact diagram from pole diagrams with the simplest case—the photoproduction of a graviton. We express the amplitude of this process in the form

$$A^p=A^p_{m,ik}e_m\varepsilon_i^*\varepsilon_k^*.$$

The contribution to the divergence of the amplitude from the pole diagrams is then

$$q_k(A^p_{m,ik})_{\rm pole}e_m=\sqrt{2\pi}e\varkappa\bar{u}(p_2)[-q_i\hat{e}+{}^1/_2(eq)\gamma_{(i)}+{}^1/_2\hat{q}e_i]u(p_1). \tag{45}$$

It is easy to obtain from this the contact part, since each term in (45) contains $q_i$ raised to the first power:

$$A^p_{cont} = -\sqrt{2\pi}\, e\varkappa \bar{u}(p_2)\hat{\varepsilon}^*(e\varepsilon^*)u(p_1),$$

which coincides with (30c), obtained by the Gupta scheme. We note that $(A^p_{m,\,ik})_{cont}e_m$ is automatically symmetrical. As shown by Zakharov and Kobzarev[6], this is ensured by the momentum conservation law in scattering. It can also be shown that $A^p_{m,\,ik}\epsilon^*_i\epsilon^*_k$ is transverse in the photon momentum.

We proceed to consider the scattering of a graviton by an electron. We introduce the definition

$$A^c = A^c_{mn,ik}\varepsilon_{1m}\varepsilon_{1n}\varepsilon_{2i}^*\varepsilon_{2k}^*.$$

For the divergence of the pole part of the amplitude, corresponding to diagrams I and II of Fig. 2, we obtain the expression

$$\begin{gathered} q_{2k}(A_{ik}{}^c)_{pole} = {}^1/_4\varkappa^2\bar{u}(p_2)\,[{}^3/_2 q_{2i}\hat{\varepsilon}_1(\varepsilon_1,\ p_1+p_2) \\ -{}^1/_2(\varepsilon_1 p_1)(\gamma_{(i)}\hat{q}_2+\hat{q}_2\gamma_{(i)})\hat{\varepsilon}_1-(p_1+p_2)_i\hat{\varepsilon}_1(\varepsilon_1 q_2) \\ -\varepsilon_{1i}\hat{\varepsilon}_1(p_1+p_2,q_2)-\varepsilon_{1i}\hat{q}_2(\varepsilon_1 p_1)-\gamma_{(i)}(\varepsilon_1 p_1)(\varepsilon_1 q_2) \\ +\varepsilon_{1i}\hat{q}_2(\varepsilon_1 q_2)+{}^1/_2(\varepsilon_1 q_2)\hat{\varepsilon}_1\hat{q}_2\gamma_{(i)}]u(p_1); \\ A_{ik}{}^c = A^c_{mn,ik}\varepsilon_{1m}\varepsilon_{1n}. \end{gathered} \tag{46}$$

The reconstruction of the contact part of the amplitude from the divergence (46) is not trivial. We need a certain statement, which we now proceed to prove.

Let the contribution to the divergence from the pole diagrams be of the form

$$K_{ikl}q_k q_l; \tag{47}$$

then the corresponding contact part is determined from the formula

$$K_{ik} = -{}^1/_2 q_l(K_{ikl}+K_{kil}-K_{lik}-K_{lki}+K_{kli}+K_{ilk}). \tag{48}$$

In fact, $K_{ik}$ is symmetrical and has the correct divergence. In addition, (48) takes automatically into account the fact that $q_k$ and $q_l$ are on a par in expression (47):

$$K_{ik} = {}^1/_2(K_{lik}+K_{lki})q_l - {}^1/_2(K_{ikl}+K_{ilk})q_l - {}^1/_2(K_{kil}+K_{kli})q_l.$$

We shall prove the uniqueness of formula (48). Let $\bar{K}_{ik}$ and $K_{ik}$ be two different contact terms. Then their difference is symmetrical and transverse:

$$\bar{K}_{ik}-K_{ik}=T_{ik},\qquad T_{ik}q_k=0.$$

It follows from (47) that

$$T_{ik} = T_{l,\,ik}q_l,$$

where $T_{l,\,ik}$ is a certain linear combination of $K_{ikl}$, having the following properties:

$$T_{l,\,ik} = T_{l,\,ki} = -T_{k,\,il}.$$

The series of equations given below proves that these properties of $T_{l,\,ik}$ are contradictory:

$$T_{l,\,ik} = -T_{k,\,il} = T_{i,\,kl} = -T_{l,\,ki} = -T_{l,\,ik},$$

i.e., $T_{l,\,ik} \equiv 0$, which proves the uniqueness of (48).

Application of (48) yields

$$\begin{gathered} \varepsilon_{1i}\hat{q}_2(\varepsilon_1 q_2) \to -\varepsilon_{1i}\varepsilon_{1k}\hat{q}_2, \\ (\varepsilon_1 q_2)\hat{\varepsilon}_1\hat{q}_2\gamma_{(i)} \to \hat{\varepsilon}_1(q_{2i}\varepsilon_{1k}+q_{2k}\varepsilon_{1i})-\hat{\varepsilon}_1\hat{q}_2(\varepsilon_{1i}\gamma_{(k)}+\varepsilon_{1k}\gamma_{(i)}). \end{gathered} \tag{49}$$

The final expression for the contact part of the amplitude coincides with formula (10c) for the non-pole diagram obtained in Sec. 2. This means that $A^c_{ik}$ is transverse in the momentum of the outgoing graviton. From the obvious equality

$$A^c_{mn,ik}(q_1,q_2) = A^c_{ik,mn}(-q_2,-q_1)$$

and from the preceding result follows the transversality of $A^c_{min,ik}\epsilon^*_{2i}\epsilon^*_{2k}$ with respect to the momentum of the incoming graviton.

## 5. LOW-ENERGY THEOREMS

We assume now that the divergence of the amplitude acquires from the pole diagrams terms of the following (third) order in $q_i$:

$$K_{iklm}q_k q_l q_m.$$

Similar expressions can appear also in the divergences of pole diagrams if $(A_{mn})_{pole}$ (1) depends in a complicated manner on $q_i$ (for example, a graviton is emitted following hadron scattering), and we expand the amplitude in a series in $q_i$. In this case the corresponding contact part is no longer uniquely determined by its divergence and by the symmetry condition. We can write down several different formulas that satisfy these requirements, for example

$$\begin{gathered} K_{1ik} = -{}^1/_2 q_l q_m(K_{iklm}+K_{kilm}-K_{likm}-K_{lkim}+K_{klim}+K_{ilkm}), \\ K_{2ik} = -{}^1/_2 q_l q_m(K_{iklm}+K_{kilm}-K_{milk}-K_{mkli}+K_{kmli}+K_{imlk}). \end{gathered}$$

Subtracting one from the other, we have

$$(K_1-K_2)_{ik} = -q_l q_m(K_{il[km]}+K_{li[mk]}-K_{kl[mi]}-K_{lk[im]})$$

(the square brackets denote antisymmetrization with respect to the indices). The amplitude corresponding to this expression can be rewritten in the form

$$2R_{iklm}K_{ik[lm]} = 2R_{iklm}K_{iklm}, \tag{50}$$

where $R_{iklm}$ is the Riemann tensor, calculated with accuracy $O(q^2)$. It is clear from the construction that (50) is gauge-invariant regardless of the form of the tensor $K_{iklm}$[6]. The same result can be obtained also from other considerations. We stipulate that the expression $K_{ik}\epsilon_i\epsilon_k$ be automatically gauge-invariant. The simplest form of the tensor $K_{ik}$, satisfying this condition, is

$$K_{ik} = K_{[im][kn]}q_m q_n,$$

where $K_{ikmn}$ is arbitrary. Then

$$K_{ik}\varepsilon_i\varepsilon_k = K_{[im][kn]}\varepsilon_i q_m\varepsilon_k q_n = {}^1/_4 K_{imkn}F_{im}(q)F_{kn}(q),$$

where $F_{im}(q) = (\epsilon_i q_m - \epsilon_m q_i)$. It is easy to verify that

$$R_{iklm}(q) = -{}^1/_2 F_{ik}(q)F_{lm}(q) + o(q^2), \tag{51}$$

i.e., the final result coincides with (50).

We have found that the uncertainty arising when the contact term is reconstructed from the divergence of the pole diagrams takes the form $K_{iklm}R_{iklm}$, where $K_{iklm}$ is arbitrary. The presence of another graviton or a photon in the reaction enables us to refine the form of the tensor $K_{iklm}$ (50). Let us discuss first the photoproduction of a graviton. The requirement of transversality with respect to the photon momentum leads to the following form of the residual term (50):

$$\begin{gathered} F_{ab}(k)R_{iklm}(q)K_{ab,\,iklm}, \\ F_{am}(k)R_{iklm}(q)K_{a,\,ikl}, \\ \dots\dots\dots\dots \\ F_{ab}(k)R_{abik}(q)K_{ik}. \end{gathered} \tag{52}$$

We have thus obtained that the nonsingular residual term, which is automatically transverse in the momenta of the graviton $q_i$ and of the photon $k_i$, is quadratic in $q_i$ and is linear in $k_i$. This means that the Born approximation (30a)—(30c) describes the photoproduction of the graviton with accuracy $O(q^2)O(k)$.

For the gravitational Compton effect, the residual term (50) is equal to one of the following expressions:

$$\begin{gathered} R_{iklm}(q_1)R_{iklm}(q_2)K, \\ R_{iklm}(q_1)R_{ikln}(q_2)K_{m,\,n}, \\ \dots\dots\dots\dots \\ R_{abcd}(q_1)R_{iklm}(q_2)K_{abcd,\,iklm}. \end{gathered} \tag{53}$$

Repeating next Jackiw's arguments [5], which are based on the assumption of cross-symmetry of the residual term in the momentum conservation law, we find that for the scattering of a graviton by an electron, Eq. (53) is of the order of $q_1^4$. It follows therefore that the Born approximation (10) describes with accuracy $O(q_1^4)$ the scattering of a graviton by an electron. A similar statement was derived in [13] with the aid of dispersion methods. The classical cross section for the scattering of a graviton by an electron (19) corresponds to the part of the amplitude that does not depend on the graviton energy.

We can formally make the arbitrary functions $K_{\dots}$ in (52) and (53) singular (singularity of type $1/(kq)$ or $1/(q_1q_2)$), i.e., we introduce the uncertainty (52) or (53), where $K_{\dots}$ is already nonsingular into the vertex of the emission of the a graviton by particles with spin $s \geq 1$ [6]. However, the accuracy of our predictions concerning the Born approximation is then lowered, to $O(q)$ for photoproduction and to $O(q_1^2)$ for the Compton effect.

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1) It was assumed, following [1], that $\sqrt{-g}\, g^{ik} = \delta^{ik} - \kappa y^{ik}$, unlike (2), where the deviation of the metric tensor from a flat one is considered.

2) The notation used here coincides, in the main, with the notation of [7], where the scattering of a graviton by a scalar particle was calculated.

3) We assume that $\sqrt{x^2} = |x|$.

4) Here $\delta_{\mu\nu}$ is the Feynman symbols, and therefore $(\epsilon_1, \epsilon_1) = (\epsilon_2, \epsilon_2) = 1$; $(\epsilon_1, \epsilon_2) = 0$.

5) If we calculate the helicity amplitudes for the scattering of a graviton by a scalar particle with polarization vectors (12) and (13), then we obtain the following relation with the results of [7].

$$A^c_{+-} = -A_{+-}, \qquad A^c_{++} = -A_{++},$$

where $A_{+-}$ and $A_{++}$ are expressions from [7].

6) It is interesting that the helicity amplitudes of photon-electron elastic scattering are also expressed in terms of the functions (14):

$$A_{+-}^{c\gamma} = 4\pi e^2 S_{+-}(k_1, k_2, p_1), \qquad A_{++}^{c\gamma} = 4\pi e^2 S_{++}(k_1, k_2, p_1),$$

where $k_{1i}$ ($k_{2i}$) is the momentum of the initial (final) photon.


1. S. N. Gupta, Proc. Phys. Soc., London, **A65**, 161, 608, 1952; Phys. Rev. **96**, 1683, 1954.
2. V. N. Mironovskiĭ, Zh. Eksp. Teor. Fiz. **48**, 358 (1965) [Sov. Phys.-JETP **21**, 236 (1965)].
3. Yu. S. Vladimirov, Zh. Eksp. Teor. Fiz. **45**, 251 (1963) [Sov. Phys.-JETP **18**, 178 (1964)].
4. B. M. Barker, M. S. Bhatia and S. N. Gupta, Phys. Rev. **182**, 1387, 1969.
5. R. Jackiw, Phys. Rev. **168**, 1623, 1968.
6. I. Yu. Kobsarev and V. I. Zakharov, Ann. Phys. **60**, 448, 1970.
7. D. Gross and R. Jackiw, Phys. Rev. **166**, 1287, 1968.
8. S. Weinberg, Phys. Rev. **134**, B882, 1964.
9. V. Fok and D. Ivanenko, Zs. Phys. **30**, 648 (1929). V. A. Fok, Zs. Phys. **57**, 261 (1929). E. Cartan, Leçons sur la Theorie de Spineurs, Hermann, Paris, 1938.
10. V. I. Ogievetskiĭ and I. V. Polubarinov, Zh. Eksp. Teor. Teor. Fiz. **48**, 1625 (1965) [Sov. Phys.-JETP **21**, 1093 (1965)].
11. V. B. Berestetskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, Relyativistskaya kvantovaya teoriya (Relativistic Quantum Theory), Nauka (1968), Chap. 1.
12. P. J. Westervelt, Phys. Rev. **D3**, 2319, 1971.
13. R. M. Muradyan and M. P. Chaveleishvili, Teor. Mat. Fiz. **8**, 16 (1971).


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