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# OSCILLATIONS OF A NONDEGENERATE PLASMA IN QUANTIZED MAGNETIC FIELDS

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The magnetohydrodynamics equations of a plasma in quantized fields are considered. The stress tensor is calculated and the possibility of a hydrodynamic description of the plasma oscillations is discussed. Oscillations of a nondegenerate plasma are considered by means of the dielectric tensor calculated by the Green's function technique.

THE behavior of a substance in strong magnetic fields is a matter of considerable interest, especially in connection with problems of astrophysics[1–4]. In addition, some phenomena in quantizing magnetic fields can be observed under laboratory conditions in the study of semiconductors and semimetals. We shall discuss the behavior of an equilibrium nondegenerate plasma in a quantizing magnetic field, such that $\hbar\Omega_e \gg T_e$, where $\Omega_e$ is the cyclotron frequency of the electron. For a low-temperature plasma ($T_e \ll I$, where I is the atomic ionization potential), the magnetic fields are bounded from above by the condition $\hbar\Omega_e \ll I$, for in the opposite case the medium is weakly ionized[4] and the characteristic plasma phenomena are absent. The corresponding temperatures and fields are $\sim 10^3$ °K and $\sim 10^8$ Oe. At higher temperatures, $T_e \gg I$, the condition $\hbar\Omega_e \gg T_e$ can be realized in the plasma primarily for substances with small values of Z (Z is the nuclear charge) and, for example, at thermonuclear temperatures $\sim 10^4$ eV, require fields $H \geq 10^{12}$ Oe. We assume that the ions are not quantized, $\hbar\Omega_e \ll T_i$; for simplicity, moreover, we neglect collisions between particles. Under these conditions the distinguishing features of the quantizing field manifest themselves in the change in the stress tensor of the electron component, and this leads to singularities in the spatial dispersion of such a plasma. In this paper we present the magnetohydrodynamic equations of a plasma in a quantizing field, calculate the dielectric tensor, and discuss the plasma oscillations in the magnetohydrodynamic region ($\omega$, $kv_{Ti}$, $kv_{Te} \ll \Omega_i$) as well as near the electron cyclotron frequency harmonics.

## 1. EQUATIONS OF PLASMA HYDRODYNAMICS IN A QUANTIZING MAGNETIC FIELD

We consider the hydrodynamic equations of a collisionless charged liquid in an electromagnetic field. The Hamiltonian of a free particle in the second quantization representation has the form

$$\hat{\mathscr{H}}(t)=\frac{1}{2m}\int\psi^+\left(\hat{\mathbf{p}}-\frac{e}{c}\mathbf{A}\right)^2\psi\,d\mathbf{r}+e\int\varphi\psi^+\psi\,d\mathbf{r}-\mu\int\mathbf{H}\psi^+\boldsymbol{\sigma}\psi\,d\mathbf{r}, \tag{1}$$

where $\psi(\mathbf{r}, t)$ and $\psi^+(\mathbf{r}, t)$ are the Heisenberg field operators; $\varphi$ is the electric field potential, $\mathbf{A}$ is the vector potential of the magnetic field $\mathbf{H}$, $\boldsymbol{\sigma}$ is the spin operator in units of $\hbar/2$, $\mu$ is the magnetic moment of the particle, $\mathbf{p} = ih\nabla$, and the remaining symbols are standard (the spin indices are not written).

The particle-flux operator $\hat{\mathbf{j}}$ is defined as

$$\hat{\mathbf{j}}=\frac{i\hbar}{2m}((\nabla\psi^+)\psi-\psi^+\nabla\psi)-\frac{e}{mc}\mathbf{A}\psi^+\psi, \tag{2}$$

the particle density is $\hat{\rho} = m\psi^+\psi$. These quantities are related by the continuity equation:

$$\partial\rho/\partial t+\operatorname{div}\mathbf{j}=0.$$

Commuting the particle flux operator $\hat{\mathbf{j}}$ with the Hamiltonian (1) and averaging the resulting identity over the local-equilibrium ensemble, we obtain

$$m\frac{\partial j_i}{\partial t}=\frac{e}{c}[\mathbf{jH}]_i-\frac{\partial T_{ik}{}^0}{\partial r_k}-en\frac{\partial\varphi}{\partial r_i}+(\mathbf{M}\nabla)H_i+[\mathbf{M}\operatorname{rot}\mathbf{H}]_i, \tag{3*}$$

where

$$T_{ik}{}^0=-\frac{1}{2m}\left\langle\left(\hat{p}_i+\frac{e}{c}A_i\right)\psi^+\left(\hat{p}_k-\frac{e}{c}A_k\right)\psi+\left(\hat{p}_k+\frac{e}{c}A_k\right)\psi^+\left(\hat{p}_i-\frac{e}{c}A_i\right)\psi\right\rangle+\frac{1}{4m}\hat{p}_i\hat{p}_k\langle\psi^+\psi\rangle, \tag{4}$$

$$\mathbf{M}=\mu\langle\psi^+\hat{\boldsymbol{\sigma}}\psi\rangle=n\mu\boldsymbol{\sigma};\ n=\langle\psi^+\psi\rangle,\ \mathbf{j}=\langle\hat{\mathbf{j}}\rangle.$$

For the quantities $T^0_{ik}$ and $\mathbf{M}$ one should write, generally speaking, separate equations of motion. Thus, for the quantity $n\sigma$ we obtain in analogous fashion

$$n\frac{\partial\boldsymbol{\sigma}}{\partial t}+n(\mathbf{v}\nabla)\boldsymbol{\sigma}=-\frac{2\mu}{\hbar^2}n[\mathbf{B}\boldsymbol{\sigma}]. \tag{5}$$

We shall henceforth use a gauge in which $\varphi = 0$. We calculate the stress tensor $T^0_{ik}$ for the case of a constant homogeneous magnetic field $\mathbf{H}_0 = (0, 0, H)$. (The vector potential is chosen to be of the form $\mathbf{A}_0 = (0, Hx, 0)$). To do it we express $T^0_{ik}$ in terms of the temperature Green's function for charged particles, $G(\mathbf{r}, \mathbf{r}', \tau, \tau')$

$$T_{ik}{}^0=-\frac{1}{2m}\lim_{\substack{\mathbf{r}'\to\mathbf{r}\\ \tau'\to\tau+0}}\left\{\left[\left(\hat{p}_i'+\frac{e}{c}A_i(\mathbf{r}',\tau')\right)\left(\hat{p}_k-\frac{e}{c}A_k(\mathbf{r},\tau)\right)+\left(\hat{p}_k'+\frac{e}{c}A_k(\mathbf{r}',\tau')\right)\left(\hat{p}_i-\frac{e}{c}A_i(\mathbf{r},\tau)\right)\right]G(\mathbf{r},\mathbf{r}',\tau,\tau')\right\}+\frac{1}{4m}\hat{p}_i\hat{p}_kG(\mathbf{r},\mathbf{r},\tau\ \tau+0), \tag{6}$$

where $G(\mathbf{r}, \mathbf{r}', \tau, \tau')$ is defined as[5]:

$$G(\mathbf{r},\mathbf{r}',\tau,\tau')=-\langle T_\tau\psi(\mathbf{r},\tau)\psi^+(\mathbf{r}',\tau')\rangle. \tag{7}$$

For noninteracting particles of type $\alpha$ in a constant magnetic field, the temperature Green's function takes the form[5]

*$[\mathbf{jH}] \equiv \mathbf{j}\times\mathbf{H}$.

$$G_\alpha^0(\mathbf{r},\mathbf{r}',\omega_m)=\sum_q \frac{\psi_q(\mathbf{r})\psi_q^*(\mathbf{r}')}{\xi_q-i\omega_m},\qquad \omega_m=(2m+1)T,\ \xi_q=\mu_\alpha-\varepsilon_q, \tag{8}$$

$\mu$ is the chemical potential, $\epsilon_q$ and $\psi_q$ are the eigenvalue and eigenfunction of a particle in the state characterized by the aggregate of quantum numbers $q = (p_y, p_z, n, \sigma)$:

$$\varepsilon_q=\frac{|e|\hbar H}{mc}\left(n+\frac{1}{2}\right)+\frac{p_z^2\hbar^2}{2m}-\mu\sigma H, \tag{9}$$

$$\psi_q(\mathbf{r})=\frac{1}{2\pi}\exp\left[i(p_y y+p_z z)\right]\varphi_n\left(x+\frac{p_y c\hbar}{eH}\right)\qquad n=0,1,2,\ldots;$$

and $\varphi_n$ is the normalized wave function of an oscillator with frequency $\Omega = |e|H/mc$.

In the case of an homogeneous and nondegenerate gas of charged particles, the stress tensor $T_{ik}^0$ is easily calculated:

$$T_{ik}^0=\begin{pmatrix}\frac{n\hbar\Omega}{2}\operatorname{cth}\frac{\hbar\Omega}{2T} & 0 & 0\\ 0 & \frac{n\hbar\Omega}{2}\operatorname{cth}\frac{\hbar\Omega}{2T} & 0\\ 0 & 0 & nT\end{pmatrix}. \tag{10}$$

For ultrastrong magnetic fields such that $\hbar\Omega_e/T_e \gg 1$, the transverse electron pressure is much higher than the longitudinal pressure:

$$\tau=T_{xx}^0/T_{zz}^0=\hbar\Omega_e/2T_e. \tag{11}$$

The difference between the transverse components of $T_{ik}^0$ and the longitudinal ones is connected with the fact that the energy (pressure) of the transverse motion in the limit $\hbar\Omega_e \gg T_e$ is due to the electrons at the zero Landau level and equals $n\hbar\Omega_e/2$, whereas for longitudinal motion the pressure is $\sim nT_e$ is not even dependent on the magnetic field. However, such results contradict the results of[1], where the energy-momentum tensor of an electron gas in a quantizing magnetic field was calculated and the following was obtained for a nonrelativistic, nondegenerate gas:

$$\tau^*=\frac{2\hbar\Omega_e}{T_e}\exp\left(-\frac{\hbar\Omega_e}{T_e}\right). \tag{12}$$

The relativistic energy-momentum tensor $T_{\mu\nu}$ was calculated in[1]:

$$T_{\mu\nu}=\frac{\hbar c}{2}\left(\bar\Psi\gamma_\nu\frac{\partial\Psi}{\partial x_\mu}-\frac{\partial\bar\Psi}{\partial x_\mu}\gamma_\nu\Psi\right), \tag{13}$$

$$\gamma=\left\{\begin{pmatrix}0 & -i\sigma\\ i\sigma & 0\end{pmatrix},\begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix}\right\}\qquad x=(\mathbf{r},ict),\quad \mu,\nu=1,2,3,4.$$

Here $\psi = \begin{pmatrix}\psi\\ \chi\end{pmatrix}$ is the 4-component operator wave function. The tensor (13) is nonsymmetric and depends on the electromagnetic field gauge. The gauge-invariant tensor $T_{\mu\nu}^{\mathrm{inv}}$ is of the form

$$T_{\mu\nu}^{\mathrm{inv}}=T_{\mu\nu}-ieA_\mu\bar\Psi\gamma_\nu\Psi=T_{\mu\nu}-\frac{1}{c}A_\mu j_\nu, \tag{14}$$

$$A=(\mathbf{A},i\varphi),\ j=(\mathbf{j},ic\rho).$$

The tensor $T_{\mu\nu}^{\mathrm{inv}}$ coincides with the one obtained in[1] only for an equilibrium state in which the electric current is equal to zero, and satisfies the equation[6]

$$\frac{\partial T_{\mu\nu}^{\mathrm{inv}}}{\partial x_\nu}=\frac{1}{c}F_{\mu\nu}j_\nu,\qquad F_{\mu\nu}=\partial A_\nu/\partial x_\mu-\partial A_\mu/\partial x_\nu. \tag{15}$$

As shown in[6] the four-dimensional divergence of the antisymmetrical part of (14) is equal to zero by virtue of the equations of motion. We calculate $T_{\mu\nu}^{\mathrm{inv}}$ in the nonrelativistic limit ($v/c \ll 1$), using the expansion of the bispinor $\Psi = \begin{pmatrix}\varphi\\ \chi\end{pmatrix}$ in powers of $1/c$[7]:

$$\chi=\frac{\sigma}{2mc}\left(\hat{\mathbf{p}}-\frac{e}{c}\mathbf{A}\right)\psi. \tag{16}$$

Substituting (16) in (14), we get for the stress tensor:

$$T_{ik}=T_{ik}^0-\frac{e\hbar}{2mc}\varepsilon_{kjl}F_{ij}\langle\psi^+\sigma_l\psi\rangle, \tag{17}$$

where $\epsilon_{kjl}$ is the completely antisymmetric tensor ($\epsilon_{123} = 1$); $F_{ij}$ is the three-dimensional part of the tensor $F_{\mu\nu}$. We omit here terms for which $\partial T_{ik}/\partial x_k = 0$.

If we take the nonrelativistic limit in (15), we obtain for the particle flux density an equation that differs in form from (3):

$$m\frac{\partial j_i}{\partial t}=-\frac{\partial T_{ik}}{\partial r_k}+emE_i+\frac{1}{c}[\mathbf{j}_{\mathrm{el}}\mathbf{H}]_i. \tag{18}$$

In the right-hand side of (18), the total electric current $\mathbf{j}_{\mathrm{el}}$ differs from $e\mathbf{j}$ if the particles have spin[8]:

$$\mathbf{j}_{\mathrm{el}}=e\mathbf{j}+\mu c\,\mathrm{rot}\,\langle\psi^+\sigma\psi\rangle. \tag{19}$$

If we substitute the values of $\mathbf{j}_{\mathrm{el}}$ and $T_{ik}$ from (19) and (17) in (18) and simplify, we again get Eq. (3). Since $T_{ik}$ in[1] actually coincides in the nonrelativistic limit with (17), we easily obtain

$$T_{ik}=\begin{pmatrix}2n\hbar\Omega e^{-\hbar\Omega/T} & 0 & 0\\ 0 & 2n\hbar\Omega e^{-\hbar\Omega/T} & 0\\ 0 & 0 & nT\end{pmatrix}. \tag{20}$$

For $T_{ik}$, the ratio of the transverse and longitudinal components agrees with (12). This tensor differs from $T_{ik}^0$ is directly corrected with the kinetic pressure, and the spin contribution is explicitly separated.

## 2. PLASMA OSCILLATION HYDRODYNAMIC ANALYSIS

We consider the small oscillations of a plasma in an ultrastrong magnetic field. We seek the solution with all quantities proportional to $\exp i(\mathbf{k}\cdot\mathbf{r} - \omega t)$.

We assume that in the presence of a wave the stress tensor $T_{ik}^0$ retains its local equilibrium form in the coordinate system where the constant magnetic field is directed along the z axis. We express the values of $\mathbf{j}$ and $\boldsymbol{\sigma}$ found from the linearized equations (3) and (5), respectively, in the form $E_i\sigma_{ik} = j_{\mathrm{el},i}$, where $\sigma_{ik}$ is the electric conductivity tensor. However, the Onsager symmetry relations are then violated[9]. Thus, even without allowance for the spin, we have (here and below $k_y = 0$, $k_x = k_\perp$)

$$\sigma_{xz}=\frac{ik_\perp k_z}{4\omega\pi}\left[\frac{\omega_{0i}^2}{2D_i^2}v_{Ti}^2+\frac{\omega_{0e}^2}{D_e^2}u_x^2\right],$$

$$\sigma_{zx}=\frac{ik_\perp k_z}{4\omega\pi}\sum_\alpha\frac{\omega_{0\alpha}^2}{2D_\alpha^2}v_{T\alpha}^2,$$

$$D_i^2=\omega^2-\Omega_i^2-\frac{k^2}{2}v_{Ti}^2+\frac{k_z^2}{2}v_{Ti}^2\frac{\Omega_i^2}{\omega^2},$$

$$D_e^2=\omega^2-\Omega_e^2-k_\perp^2u_x^2-\frac{k_z^2}{2}v_{Te}^2+\frac{k_z^2}{2}v_{Te}^2\frac{\Omega_e^2}{\omega^2}, \tag{21}$$

$\omega_{0\alpha}$ is the plasma particle frequency for particles of sort $\alpha$

$$v_{T\alpha}^2=2T_\alpha/m_\alpha, \quad u_x^2=\hbar\Omega_e/2m_e.$$

It can be shown that the contribution made to $\sigma_{ik}$ by the presence of spin satisfies Onsager's principle automatically, and thus the summary electric conductivity tensor does not satisfy this principle. It is known that collisionless hydrodynamics leads to wrong laws for the dispersion, say, of Langmuir waves, even in the absence of a magnetic field[10]. The quantization of the electron state in a magnetic field is equivalent to some degree, in the sense of the stress tensor, to a distribution with two distinct temperatures $T_\parallel$ and $T_\perp$. Of course, the equilibrium state in a magnetic field is described by one temperature and is stable, in distinction from the case with $T_\parallel \neq T_\perp$[11], but the electron stress tensor components $T^0_{xx} \sim n_e\hbar\Omega_e/2$ and $T^0_{zz} \sim n_eT_e$ are the same for $T_\perp = \hbar\Omega_e/2$ and $T_\parallel = T_e$. It can be shown that, considering a stress tensor of the form

$$T_{ik}^0=T_\parallel^0\tau_i\tau_k+T_\perp^0(\delta_{ik}-\tau_i\tau_k), \tag{22}$$

where $\tau$ is a unit vector in the direction of the instantaneous value of the magnetic field $T^0_\parallel = n_eT$, and $T^0_\perp = n_eT$, we obtain for the electron component of tensor $\sigma_{ik}$ in the "isothermal" case of $T_\parallel$ = const and $T_\perp$ = const[1)]:

$$\sigma_{xz}^{(e)}\sim-\frac{ik_\perp k_z\omega_{0e}^2}{4\pi\omega D_e^2}\left[-\frac{2T_\perp}{m_e}+\frac{T_\parallel}{m_e}+O(k_\perp^2)\right], \tag{23}$$

$$\sigma_{zx}^{(e)}\sim-\frac{ik_\perp k_z\omega_{0e}^2}{4\pi\omega D_e^2}\left[-\frac{T_\parallel}{m_e}+O(k_\perp^2)\right].$$

When $T_\parallel = T_\perp$, the symmetry of the conductivity tensor is restored.

## 3. PLASMA OSCILLATIONS. KINETIC ANALYSIS

We calculate the dielectric tensor $\epsilon_{ik}$ in a coordinate system in which the external magnetic field is directed along the z axis, using a Green's function. The approach is analogous to the kinetic treatment of the problem. To this end, we use the known definitions[5]:

$$\varepsilon_{ik}=\delta_{ik}+4\pi\omega^{-1}i\sigma_{ik}, \quad E_i\sigma_{ik}=j_i,$$
$$j_i=\sum_\alpha\left\{\lim_{\substack{r'\to r\\ \tau'\to\tau+0}}\left[-\frac{ie_\alpha\hbar}{2m_\alpha}\left(\frac{\partial}{\partial r_i}-\frac{\partial}{\partial r_i'}\right)-\frac{e_\alpha^2A_i}{m_\alpha c}\right]G(\mathbf{r},\mathbf{r}',\tau,\tau')\right.$$
$$\left.+\mu_\alpha c\varepsilon_{ijk}\frac{\partial}{\partial r_j}\sigma_kG_\alpha(\mathbf{r},\mathbf{r},\tau,\tau+0)\right\}, \quad G=G^{(0)}+G^{(1)}, \tag{24}$$

where $G^{(1)}$ is the correction to the Green's function of non-interacting particles in a constant magnetic field, and is linear in the perturbing potential $\mathbf{A}_1\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)$. It is calculated by the usual method[5]:

$$G^{(1)}(\mathbf{r},\mathbf{r}',\tau,\tau')=\sum_{\sigma_1}\int_0^{1/T}d\tau_1\int_{-\infty}^{\infty}d\mathbf{r}_1G^{(0)}(\mathbf{r},\mathbf{r}_1,\tau,\tau_1)\cdot \tag{25}$$
$$\exp(i\mathbf{k}\mathbf{r}_1-i\omega\tau_1)\hat\Sigma(\mathbf{r}_1,\sigma_1)G^{(0)}(\mathbf{r}_1,\mathbf{r}',\tau_1,\tau'),$$

$$\hat\Sigma(\mathbf{r},\sigma)=-\frac{e}{2mc}(2\mathbf{A}_1\mathbf{p}+\hbar\mathbf{k}\mathbf{A}_1)+\frac{e^2\mathbf{A}_0\mathbf{A}_1}{mc^2}-\mu\sigma\mathbf{H}_1. \tag{26}$$

It is expedient to integrate with respect to $p_y$ in expression (8) for $G^0(\mathbf{r}, \mathbf{r}', \omega_m)$[12]:

$$G^{(0)}(\mathbf{r},\mathbf{r}',\omega_m)=\frac{\eta}{(2\pi)^2\hbar}\exp\left[-i\eta\frac{(x+x')(y-y')}{2}\right] \tag{27}$$
$$\times\sum_{n=0}^{\infty}\sum_\sigma\int_{-\infty}^{\infty}dp_z\exp\{ip_z(z-z')\}\exp\left[-\eta\frac{(x-x')^2+(y-y')^2}{4}\right]$$
$$\times L_n\left(\eta\frac{(x-x')^2+(y-y')^2}{2}\right)\frac{1}{\xi_q+i\omega_m},$$

where $\eta = |e|H/hc$ is the square of the reciprocal magnetic length, and $L_n(x)$ is a Laguerre polynomial.

Summing over the frequencies $\omega_m$ and changing to the retarded response, we obtain

$$\lim_{t'\to t+0}G^{(1)}(\mathbf{r},\mathbf{r}',t,t')=-\frac{\eta^2}{(2\pi)^3\hbar^2}\sum_{n,n'=0}^{\infty}\sum_\sigma\int dx_1\,dp_z\,dy_1\cdot$$
$$\times\exp\left\{-i\eta\frac{(x+x_1)(y-y_1)}{2}-\eta\frac{(x-x_1)^2+(y-y_1)^2}{4}\right\}\cdot$$
$$\times L_n\left(\eta\frac{(x-x_1)^2+(y-y_1)^2}{2}\right)\exp(ik_\perp x_1)\hat\Sigma(\mathbf{r}_1,\sigma)\cdot$$
$$\times\exp\left\{-i\eta\frac{(x_1+x')(y_1-y')}{2}-\eta\frac{(x_1-x')^2+(y_1-y')^2}{4}\right\}$$
$$\times L_{n'}\left(\eta\frac{(x_1-x')^2+(y_1-y')^2}{2}\right)\exp(-i\omega t)\exp(ik_zz'+ip_z(z-z'))$$
$$\times\frac{n(\varepsilon^\sigma_{p_z-k_z,n'})-n(\varepsilon^\sigma_{p_z,n})}{\varepsilon^\sigma_{n,p_z}-\varepsilon^\sigma_{n',p_z-k_z}-\hbar\omega}. \tag{28}$$

Since we are considering a nondegenerate plasma, we have for $n(\epsilon_q)$ the Boltzmann expression $n(\epsilon_q) = \exp(\xi_q/T)$.

Substituting (28) in (24) and performing all the necessary operations, which are omitted here, we represent the resulting expression in the form of an expansion in powers of the small parameter $k_\perp^2/2\eta$. The ionic contribution to the dielectric tensor $\epsilon_{ik}$ is described by the usual formulas[10]. The final result is[2)]

$$\varepsilon_{ik}=\begin{pmatrix}\varepsilon_1 & ig & \xi\\ -ig & \varepsilon_2 & if\\ \xi & -if & \eta\end{pmatrix},$$

$$\varepsilon_1=1-\frac{\omega_{0e}^2}{\omega^2}-\frac{\omega_{0e}^2\Omega_e}{2\omega^2}\left[\frac{Z_-^{(e)}}{\omega-\Omega_e}-\frac{Z_+^{(e)}}{\omega+\Omega_e}\right]-\frac{\omega_{0i}^2}{2\omega}\left[\frac{Z_-^{(i)}}{\omega-\Omega_i}+\frac{Z_+^{(i)}}{\omega+\Omega_i}\right]$$
$$+\frac{\omega_{0e}^2}{\omega^2}\frac{\hbar k_\perp^2}{4m_e}\left[\frac{Z_-^{(e)}}{\omega-\Omega_e}-\frac{Z_+^{(e)}}{\omega+\Omega_e}-\frac{2Z_-^{(e)}}{\omega-2\Omega_e}+\frac{2Z_+^{(e)}}{\omega+2\Omega_e}\right].$$

$$g=\frac{\omega_{0e}^2\Omega_e}{2\omega^2}\left[\frac{Z_-^{(e)}}{\omega-\Omega_e}+\frac{Z_+^{(e)}}{\omega+\Omega_e}\right]-\frac{\omega_{0i}^2}{2\omega}\left[\frac{Z_-^{(i)}}{\omega-\Omega_i}-\frac{Z_+^{(i)}}{\omega+\Omega_i}\right];$$

$$\xi=\frac{-\omega_{0e}^2k_\perp}{2\omega^2k_z}[Z_+^{(e)}+Z_-^{(e)}-2]-\frac{\omega_{0i}^2k_\perp}{2\omega\Omega_ik_z}[Z_-^{(i)}-Z_+^{(i)}];$$

$$\varepsilon_2=\varepsilon_1-\frac{\omega_{0i}^2}{\omega^2\Omega_i^2}\frac{2k_\perp^2T_i}{m_i}Z^{(i)};$$

$$f=\frac{\omega_{0e}^2k_\perp}{2\omega^2k_z}[Z_+^{(e)}-Z_-^{(e)}]-\frac{\omega_{0i}^2k_\perp}{\omega\Omega_ik_z}\left[Z^{(i)}-\frac{1}{2}\{Z_-^{(i)}+Z_+^{(i)}\}\right];$$

$$\eta=1-\sum_\alpha\frac{\omega_{0\alpha}^2m_\alpha}{T_\alpha k_z^2}(Z^{(\alpha)}-1), \quad Z_\pm^{(\alpha)}=Z\left(\frac{\omega\pm\Omega_\alpha}{k_zv_{T\alpha}}\right),$$

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1) Allowance for the variation in the quantity $T_\perp \sim \Omega \sim |H|$ does not alter our assertion that the conductivity tensor is not symmetrical.

2) The calculation of the corrections ($\sim \hbar\Omega/T$) to the transverse classical tensor $\epsilon_{ik}$, assumed here to be small, is carried out in [13]. We are considering the limit $\hbar\Omega_e/T_e \gg 1$.

$$Z_{\pm}^{(e)} = Z\left(\frac{\omega \pm 2\Omega_e}{k_z v_{Te}}\right); \quad Z^{(\alpha)} = Z\left(\frac{\omega}{k_z v_{T\alpha}}\right), \quad Z(z) = X(z) - iY(z),$$

$$X(z) = 2ze^{-z^2}\int_0^z e^{t^2}\,dt, \quad Y(z) = \sqrt{\pi}\, ze^{-z^2}. \tag{29}$$

Expressions (29) for the tensor $\epsilon_{ik}$ are valid for particles far from the cyclotron harmonics.

We note that for spinless particles the dielectric tensor (29) will have, along with some changes in the numerical coefficients, quantum corrections to $\epsilon_2$ and f, of the form

$$\varepsilon_2^{\text{quant}} = \frac{\omega_{0\alpha}^2}{\omega^2}\frac{\hbar^2 k_\perp^2}{4m_\alpha T_\alpha}[1 - Z^{(\alpha)}], \tag{30}$$

$$f^{\text{quant}} = \frac{\omega_{0\alpha}^2 \hbar k_\perp}{2\omega T_\alpha k_z}[Z^{(\alpha)} - 1].$$

The spatial dispersion of such a quantum plasma of spinless particles will differ from zero even when T = 0, which does not happen in non-quantizing magnetic fields. The absence of similar quantum terms in $\epsilon_{ik}$ for electrons with spin is apparently a consequence of cancellation of the Landau ground level energy by the spin energy (cf. the analogous cancellation of spin and kinetic parts in the stress tensor $T_{ik}$ (17)).

In semiconductors and metals in which the effective mass $m_e^*$ differs from the mass $m_e$ of the electron in vacuum and, in addition, the Lande factor g appears in the spin terms, this cancellation does not take place. Then corrections of type (30) arise, but vanish when $m_e = m_e^*$ and g = 2:

$$\varepsilon_2 = \frac{\omega_{0e}^2 k_\perp^2 \hbar^2}{\omega_2 4 m_e T_e}\left(1 - Z\left(\frac{\omega}{k_z v_{Te}^*}\right)\right)\left[\frac{m_e^2}{m_e^{*2}} - \frac{m_e}{m_e^*}g + \frac{g^2}{4}\right],$$

$$f = \frac{\omega_{0e}^2 \hbar k_\perp}{2\omega T_e k_z}\left(Z\left(\frac{\omega}{k_z v_{Te}^*}\right) - 1\right)\left[\frac{m_e}{m_e^*} - \frac{g}{2}\right].$$

The modes of all the possible oscillations can be found by solving a general dispersion equation of the form[10].

$$\text{Det} \equiv |k^2\delta_{ik} - k_i k_k - c^{-2}\omega^2\varepsilon_{ik}(\omega, \mathbf{k})| = 0. \tag{31}$$

The slow magnetohydrodynamic wave (its spectrum is found from the condition $\eta = 0$) remains unchanged and, as before, is weakly damped when the condition $T_e \gg T_i$ is fulfilled[10]. The Alfven wave can be found from the relation $k_z^2 c^2 = \epsilon_1 \omega^2$, and for obvious reasons, differs in no way from the usual case. The Alfven velocity of a hydrogen plasma $c_A^2 = c^2(1 + \omega_{0i}^2/\Omega_i^2)$ becomes of the order of the speed of light when $n_i/H^2 \ll 10^2$ (H is in oersteds, and $n_i$ is the proton concentration in $cm^3$).

The dispersion relation for a fast magnetohydrodynamic wave is found from the equation

$$k^2c^2/\omega^2 = \varepsilon_2 - f^2/\eta. \tag{32}$$

the spectrum of that wave is given by ($k_z v_{Te} \ll \omega \ll \Omega_i$; $\omega \ll \omega_{0e}$)

$$\omega^2 = k^2 c_A^2 + \frac{\omega_{0i}^2}{\Omega_i^2}\frac{c_A^2}{c^2}k_\perp^2 v_{Ti}^2 - i\sqrt{\pi}\frac{\omega_{0e}^2 k k_\perp^2 c_A^3 v_{Ti}}{\Omega_e^2 k_z c^2}\left[\frac{m_i}{m_e}\mathscr{E}_i + \frac{v_{Ti}^3}{2v_{Te}^3}\mathscr{E}_e\right],$$

$$\mathscr{E}_\alpha = \exp\{-k^2 c_A^2 / k_z^2 v_{T\alpha}^2\}. \tag{33}$$

As distinct from the classical case ($\hbar\Omega_e \ll T_e$), the term $\sim (T_e/T_i)k_\perp^2 v_{Ti}^2$ is absent from the expression (33)[10]. For spinless particles the corresponding spectrum takes the form

$$\omega^2 = k^2 c_A^2 + \frac{k_\perp^2 \hbar}{m\Omega_e}\omega_{0e}^2\frac{c_A^2}{c^2} - i\sqrt{\pi}\frac{\omega_{0e}^2\hbar}{\Omega_e T_e}\frac{k c_A^3 v_{Ti}^2 k_\perp^2}{k_z c^2 v_{Te}}\left[\mathscr{E}_e + \frac{v_{Te}^3}{2v_{Ti}^3}\mathscr{E}_i\right].$$

The spectra of the high-frequency oscillations are not, in the main, different from those of the oscillations that exist at non-quantizing values of the magnetic field. Significant changes occur in the expressions for the dispersion near the cyclotron resonances. To find their spectrum it is necessary to add to the dielectric tensor (29) resonant terms containing the parameter $k_\perp^2/2\eta$ raised to smallest possible power (for the given $l$-th resonance). We have for propagation transverse to the magnetic field

$$\Delta\eta_{\text{res}} = \frac{\omega_{0e}^2}{\omega^2}\frac{1}{l!}\frac{T_e}{\hbar(l\Omega_e - \omega)}\left(\frac{k_\perp^2}{2\eta}\right)^l, \quad l = 1, 2, 3, \ldots, \tag{34}$$

$$\Delta\varepsilon_{1\,\text{res}} = \Delta\varepsilon_{2\,\text{res}} = -\Delta g_{\text{res}} = \frac{\omega_{0e}^2}{\omega^2}\left(\frac{k_\perp^2}{2\eta}\right)^{l'-1}\cdot$$

$$\times\frac{\Omega_e}{(l'\Omega_e - \omega)}\frac{l'}{2(l'-1)!}, \quad l' = 2, 3, \ldots,$$

$$\frac{\Delta\varepsilon_{1\,\text{res}}}{5} = \frac{\Delta\varepsilon_{1\,\text{res}}}{37} = -\frac{\Delta g_{\text{res}}}{15} = \frac{\omega_{0e}^2}{4\omega^2}\left(\frac{k_\perp^2}{2\eta}\right)^2\cdot$$

$$\times\frac{\Omega_e}{\Omega_e - \omega}\exp\left(-\frac{\hbar\Omega_e}{T_e}\right), \quad l' = 1.$$

The resonant terms proportional to $k_\perp^0$ and $k_\perp^2$ cancel out in the dispersion equation for the fundamental resonance, and it is therefore necessary to retain in $\epsilon_{ik}$ resonant terms proportional to $(k_\perp^2/2\eta)^2$. Then the terms in (28) with n = 2, n′ = 1, and higher do not cancel.

The move propagating in the vicinity of the $l$-th resonance with an electric field vector parallel to the constant magnetic field (an ordinary wave) takes the form (dispersion equation $\eta = k^2c^2/\omega^2$)

$$\omega_l = l\Omega_e + \frac{T_e}{\hbar}\frac{1}{l!}\left(\frac{k_\perp^2}{2\eta}\right)^l\frac{1}{(l\Omega_e/\omega_{0e})^2 - 1}. \tag{35}$$

For other than ordinary waves, i.e., a plane-polarized wave perpendicular to the constant magnetic field, the answers will be different for the fundamental and for the higher resonances (dispersion equation $k^2c^2/\omega^2 = \epsilon_2 - g^2/\epsilon_1$):

$$\omega = \Omega_e\left[1 - 3\left(\frac{k_\perp^2}{2\eta}\right)^2\exp\left(-\frac{\hbar\Omega_e}{T_e}\right)\right], \tag{36}$$

$$\omega_{l'} = l'\Omega_e + \left(\frac{k_\perp^2}{2\eta}\right)^{l'-1}\frac{\omega_{0e}^2}{\Omega_e}\frac{[l'(l'+1) - \omega_{0e}^2/\Omega_e^2]}{[l'^2(l'^2-1) + \omega_{0e}^4/\Omega_e^4](l'-2)!}. \tag{37}$$

The difference from the classical case[14] is that the role of the Larmor radius $R_2$ is assumed by the magnetic length $1/\sqrt{\eta}$. Propagation of cyclotron waves with $\omega_l > l\Omega_e$ becomes possible, which happens if $l\Omega_e > \omega_{0e}$ (a condition easily satisfied in a plasma in strong fields). The opposite situation, i.e., the existence of oscillations propagating in metals only at $\omega_l < l\Omega_e$ in the long-wave region, was discussed in[14].

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