GRAVITATIONAL RADIATION IN THE SCALAR-TENSOR GRAVITATION THEORY

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1. The problem of Dirac's large parameter\(^{(1)}\) and Mach's problem, which were acutely felt but not solved by Einstein\(^{(2)}\), were the stimulus for proposed changes in the general theory of relativity (GTR), consisting in the introduction of a scalar field \(\varphi(\textbf{r}, \textit{t})\) related to the gravitational 'constant' \(k\), which thus becomes dependent on the coordinates and time. Such a scalar-tensor theory of gravitation (STTG) was proposed by \(\text{Jordan}\)\(^{(3)}\) and later by Brans and \(\text{Dicke}\). The Lagrangian density in this theory is\(^{(4)}\)

\[
\mathcal{L} = \varphi \frac{\partial \varphi}{\partial \textit{t}} - \frac{16\pi}{c^4} \Lambda\varphi
\]

Here \(\Lambda\) is the Lagrangian density of material sources, \(R\) is the curvature scalar, \(\delta = 6\)\(^{(5)}\) is the coupling constant of the scalar and tensor fields; the comma denotes partial differentiation and the semicolon denotes covariant differentiation.

The scalar \(\varphi(\textbf{r}, \textit{t}) \sim k(\textbf{r}, \textit{t})\) was denoted in\(^{L3}\) and\(^{L5}\) by \(\varphi'\nu\) with \(\nu = +1\) and \(\nu = -1\), respectively. However, this distinction does not affect the field equations which follow from the above Lagrangian and neither does it manifest itself in the equations of motion. \(\text{Finkelstein}\)\(^{(6)}\) has proposed a modification of this theory, which we will not discuss, however.

In all its proposed versions the STTG does not solve in fact the problem of Mach's principle, since its field equations admit 'anti-Mach' solutions, for which the field \(\varphi(\textbf{r}, \textit{t})\) is not fully determined by the distribution of the source tensor \(T_{\text{ik}}(\textbf{r}, \textit{t})\). More complete solutions of this problem have been proposed in another direction by Al'tshuler\(^{(7)}\), Linden-Bell\(^{(8)}\), and Sclama\(^{(9)}\). However, the STTG presents interest independently of the Mach problem, namely as a theory of gravitational interaction with a variable parameter \(k(\textbf{r}, \textit{t})\).

The above Lagrangian density leads to the following field equations:

\[
\varphi G_{\alpha} + g_{\alpha} \Box \varphi - \varphi G_{\alpha} + \frac{8\pi}{(3 + 2\delta)} \om{T_{\text{ik}}(\textbf{r}, \textit{t})} = 8\pi \gamma T_{\alpha} = 0
\]

Here \(T_{\text{ik}}\) is the energy-momentum tensor of the field sources and \(T\) is its trace. At the same time the following equation of GTR remains valid:

\[
\gamma T_{\alpha} = 0.
\]

2. The purpose of the present paper is to show that the STTG leads to gravitational radiation, which under certain circumstances may surpass by many orders the gravitational radiation in the GTR. Scalar radiation can occur even for the case of a spherically symmetric system. Therefore, if the scalar field \(\varphi(\textbf{r}, \textit{t})\) exists, the metagalaxy may contain many more scalar gravitons than tensor gravitons, and these scalar gravitons could play an essential role in the process of formation of the basic elements of structure of the metagalaxy, the galactic clusters. We shall consider the radiation as a small perturbation of the pseudoeuclidian metric. We set

\[
\varphi = \varphi(1 + \gamma), \quad \gamma = \varphi(1 + \psi).
\]

where\(^{(5)}\)

\[
\varphi^{-1} = \frac{3 + 2\delta}{4 + 2\delta}, \quad k = \gamma k, \quad \gamma = 1.
\]

We shall consider \(T_{\text{ik}}\) to be small of first order.

We select the coordinate system in such a way, that the following conditions be satisfied\(^{(5)}\) (these differ
somewhat from the Landau-Lifshitz relations\textsuperscript{102):}
\begin{equation}
[h, \frac{1}{2} b^2 (h + 2q)] = 0.
\end{equation}

As was shown by Brans and Dick\textsuperscript{42}, the following wave equations hold
\begin{equation}
\left(\Delta - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) h_{\mu\nu} - \frac{1}{2} \frac{\partial}{\partial t} \frac{\partial}{\partial t} (h + 2q) = \frac{16\pi}{c^4} \gamma \kappa \Phi_{\mu\nu}.
\end{equation}

Let us consider far from the source system a plane wave propagating into the $x^1$ direction. According to Landau-Lifshitz\textsuperscript{101}, in addition to the conditions of type (2), the choice of the coordinate system allows one to impose four more conditions, e.g.,
\begin{equation}
h_{w0} = h_{a0} = h_{a1} = 0.
\end{equation}

Since both the quantities $h_{ik}$ and $\psi$ depend only on the variable $x^1 - ct$, it follows from (2) and (4) that all components of $h_{ik}$, except $h_{23}, h_{33}$, and $h_{33}$, vanish, and also that the following relation holds:
\begin{equation}
h_{11} + h_{22} = 2q.
\end{equation}

One can now calculate the components of the pseudotensor.

The pseudotensor
\[\mathbf{r}^{(a)} = k \mathbf{q}^{(a)} + \mathbf{h}^{(a)},\]
where $\mathbf{h}^{(a)}$ is the pseudotensor from\textsuperscript{102} and $\mathbf{q}^{(a)}$ is an additional pseudotensor, related to the presence of the scalar field $\varphi(t, \mathbf{r})$, and has the form\textsuperscript{11}
\[\mathbf{q}^{(a)} = \left(c^2 - 1/16\pi\right) \varphi^2 \left[2(5 - 1) \varphi^2 + (2 - 6) \varphi^{(a)} \varphi^{(a)}\right] + \left(c^2 / 8\pi\right) \varphi^2 \left[2(5 - 1) \varphi^2 - 2(5 - 1) \varphi^{(a)} \varphi^{(a)}\right] + \varphi^2 \varphi^2 \left[\varphi^2 + \varphi^2 \varphi^2\right] - \varphi^2 \varphi^2 \left[\varphi^2 + \varphi^2 \varphi^2\right] \varphi^2 - \varphi^2 \varphi^2 \left[\varphi^2 + \varphi^2 \varphi^2\right] \varphi^2.
\]

Substituting the above-mentioned expressions for $h_{ik}$ from (5), we obtain
\[\mathbf{r}^{(a)} = \frac{c^2}{16\pi} \int [h_{ii} + \frac{1}{4} (h_{11} + h_{22}) + (3 + 2\varphi^2)] \varphi^2.
\]

In the sequel the dot above a letter denotes differentiation with respect to time.

3. We compute the quantities $h_{23}, h_{33}, h_{33}$ and $\psi$ from (3). According to\textsuperscript{102} one may neglect the retardation in the first equation of (3), and thus
\begin{equation}
h_{\alpha - \varepsilon}^{\alpha - \varepsilon} \left(\frac{1}{c} - \varphi + \frac{1}{4} \frac{\partial}{\partial t} (h_{11} + h_{22}) + (3 + 2\varphi^2) \varphi^2\right) = - \frac{\varepsilon}{c} \int T_{\alpha} (t - \frac{\varepsilon}{c}) dV,
\end{equation}

where the dimensions of the radiating system have been assumed small compared to the distance $r_0$ from the selected origin to the point of observation. In the sequel the variable $t - r_0/c$ will be omitted. From (2) and the first equation of the system (3) it is easy to derive an equation like in the GTR
\[T_{\alpha} = 0.
\]

Then, by analogy with Landau-Lifshitz\textsuperscript{101}, we obtain from (7)
\begin{equation}
h_{11} = - \frac{2\varepsilon}{c} \int \varphi^2 (r) dV = - \frac{2\varepsilon}{c} \frac{\partial}{\partial t} D_{11},
\end{equation}
\begin{equation}
h_{22} = - \frac{2\varepsilon}{c} \int \varphi^2 (r) dV = - \frac{2\varepsilon}{c} \frac{\partial}{\partial t} D_{22},
\end{equation}

The solution of the second equation of the system (3) is
\[\psi (r, t) = \left(2\varepsilon/c^2(3 + 2\varphi^2)\right) \int T (t - r_0/c) (t - r_0/c) dV.
\]

We are grateful to A. M. Finkel'shteyn for indicating to us the form of the symmetric pseudotensor $\mathbf{q}^{(a)}$.

The energy losses due to the usual (tensorial) gravitational radiation coincide with the equations of Landau-Lifshitz\textsuperscript{102} and the energy loss per unit time due to scalar radiation is:
\begin{equation}
- \frac{dE}{dt} = \frac{ky}{4\pi c^3 (3 + 2\varphi^2)} \int \frac{\partial}{\partial t} T (t') dV dt.
\end{equation}

Here $t' = t - \frac{r_0 - r}{c}$, $dV$ is the element of solid angle on a sphere of radius $r_0$. One cannot neglect the retardation in the argument of $T$. Effecting in $T$ an expansion in powers of the variable $r$ which enters into the retardation, and neglecting $r$ compared to $r_0$ in the denominator, we obtain
\begin{equation}
\psi (r, t) = \int \frac{2\varepsilon}{c^2 (3 + 2\varphi^2)} \int \frac{\partial}{\partial t} r (t - \frac{r_0}{c}) m \frac{m}{c} + \frac{1}{2} \frac{\partial}{\partial r} r (t - \frac{r_0}{c}) \left(\frac{m}{c}\right)^2 dV
\end{equation}

where $n = r_0/c$.

4. The energy lost by the material sources during the emission of the scalar field surpasses in some cases by several orders of magnitude the energy lost through tensorial radiation. One of these cases is the turbulence of a hot plasma interacting with electromagnetic radiation, an effect, which according to Ozernof and Chechin\textsuperscript{111} occurred in the pregalactic stage of expansion of the metagalaxy and was the mechanism responsible for the formation of its present structure.

Considering the scalar radiation of such turbulent vortices one may retain the first term under the integral in (11). Then we obtain
\[\psi (r, t) = \int \frac{2\varepsilon}{c^2 (3 + 2\varphi^2)} \int \frac{\partial}{\partial t} (t - \frac{r_0}{c}) \left(\frac{m}{c}\right)^2 dV.
\]

The integral here is proportional to the radius-vector of the center of inertia of the system, and its second derivative is the rate of change of the total momentum as a function of time, $dP/dt$. If the system is closed and its total momentum does not change with time, the derivative vanishes. However, when the system interacts with electromagnetic radiation, equation (11) will contain only the energy-momentum tensor of the plasma, since for the electromagnetic field the trace of the tensor vanishes. If this is so, $dP/dt \neq 0$ and the emission of scalar radiation is possible already in this approximation. According to the estimates of Ozernof and Chechin\textsuperscript{111} the velocity of vortex motions is $v \approx 10^9 - 3 \times 10^9$ cm/sec during the epoch $t_0 \approx 3 \times 10^{13}$ sec, adjacent to the epoch of hydrogen recombination, when the interaction of the plasma with radiation is very strongly diminished. In the period preceding the recombination the mean density is $\rho \approx 10^{-29}$ g/cm$^3$. If one considers the turbulent vortices enveloping a mass exceeding by three orders the mass of the Galaxy $M = 10^{14} M_{\odot}$, $\approx 10^{17}$ g, the dimensions of the vortex are $r \approx (M/\rho)^{1/3} \approx 10^{23}$ cm, and the characteristic time is $t = r/v = 10^{23}/10^{14} \approx 10^{44}$ sec, then the power of scalar radiation of this vortex in the epoch $t_0$ is
\[\left(\frac{-dE}{dt}\right)_{t_0} = 10^{-12} M_{\odot} c \left(\frac{v}{c}\right)^4,
\]
\[3(4 + 2\varphi^2) \approx 10^5.\]

For $v \approx 10^6$ cm/sec this yields $(-dE/dt)_{t_0} \approx 3 \times 10^{40}$ erg/sec.

In the epoch preceding the instant $t_0$, $v = \text{const}$, $r \sim t^{1/2}$. Setting $\ln(t/t_0)$, $\approx 30$, we obtain for the en-
ergy during the epoch of coupling of nonrelativistic plasma to the radiation
\[ \Delta E \approx 10^{48} \text{ erg}, \]
which is by ten orders smaller than the energy of electromagnetic radiation in the volume of the vortex under consideration at time \( t_1 \). In the following epoch, i.e., after recombination, the turbulent motions are damped out and an estimate of the scalar radiation during that epoch does not essentially modify the value obtained above. If the mass of the gas comprised by the vortex is \( M \approx 10^{17} M_\odot \) and the velocity of the vortex motion is \( v \approx 3 \times 10^8 \text{ cm/sec}, \) then \( r \approx 10^{24} \text{ cm} \) and \( \Delta E \approx 3 \times 10^{49} \text{ erg}. \) We note that during that epoch of nonrelativistic plasma interacting with electromagnetic radiation, the ratio of the power of the scalar field to that of the tensor radiation in GTR is \( 2^{1/3} \) above. If the mass of the gas comprised by the vortex epoch does not essentially modify the value obtained during the epoch of coupling of nonrelativistic plasma, we use a simplified model of the star. We shall assume that the star remains homogeneous throughout its collapse and that its outer layer falls freely towards the center. This leads to the following order-of-magnitude relation:
\[ r \approx c/15 (kp) t_{\nu}. \]

The mass of such regions in the epoch discussed above is \( M \approx 10^{17} M_\odot \), if one takes into account the small decrease of the gravitational constant compared to its present value.

5. A spherically symmetric system is well known to be incapable of losing energy on account of tensorial-symmetric collapse of a star. The calculation which follows has only an estimation character, and therefore we use a simplified model of the star. We shall assume that the star remains homogeneous throughout its collapse and that its outer layer falls freely towards the center. This leads to the following order-of-magnitude relation:
\[ \rho(t) = \rho_0 (1 - t/t_\nu)^{-2}, \quad R = R_0 (1 - t/t_\nu)^{1/4}, \]
where \( \rho \) is the density and \( R \) is the radius of the star and \( \rho_0 \) and \( R_0 \) are their initial values; at the same time \( t_\nu \approx (kp)^{-1}. \)

Setting \( T = \rho c^2 \) we obtain from (10) and (11)
\[ \frac{dE}{dt} = \frac{1}{2\nu} kM M R_0^3 \left(1 - \frac{t}{t_\nu}\right)^{-2}, \]
and the total energy losses due to scalar radiation for the whole collapse time from radius \( R_0 \) to \( R_0 \ll R_0 \) is
\[ \Delta E = \frac{1}{4 + 2\nu} \left( \frac{R_0}{R_0} \right)^{1/2} kM^2 \frac{R_0}{R_0} \left( kM \right)^{-2} \]
\[ \text{This energy does not depend in our approximation on the initial radius } R_0. \]

In the case of a supernova eruption we set \( R_0 = 2 \times 10^6 \text{ cm}, \) \( M = 2 \times 10^{26} \text{ g}, \) which yields \( \Delta E \approx 10^{48} \text{ erg}. \) In some cases this energy is comparable to the mechanical energy of the expanding shell in a supernova flare.

6. We now consider the gravitational radiation of a spherically symmetric pulsating star. We shall see that for usual stars the damping time of the oscillations due to this radiation is much larger than the cosmological time. However, for pulsars and quasars the radiation may turn out to be substantially stronger. Unfortunately we do not have data for the appropriate estimates, and therefore consider an ordinary star, using two simplified models.

a) The total volume is subject to small harmonic oscillations, remaining homogeneous, so that the radius of the star is
\[ R = R_0 + r \cos \omega t, \quad r_0 \approx R_0. \]

Then the energy of oscillations of the star is
\[ E = \frac{1}{2} r_0^2 \omega^2 M \]
and the energy loss per unit time averaged over a period, due to gravitational radiation is
\[ -\frac{dE}{dt} = \frac{1}{20} \frac{k}{c^2 (4 + 26)} \omega^2 r_0^2 R_0^2. \]

Therefore the characteristic damping time of the oscillations is
\[ \tau = 6c^2 (4 + 26) \omega^2 r_0^2 R_0^2 \]
setting \( M = 10^{22} \text{ g}, \) \( \omega = 10^{-4} \text{ sec}, \) \( R_0 = 10^{11} \text{ cm}, \) we have \( \tau \approx 10^{16} \text{ sec}. \)

b) The star consists of a stationary interior part of radius \( R_0 \) and mass \( m_0 \) and a harmonically oscillating exterior part of radius:
\[ R = R_0 + r \cos \omega t \]
and mass \( m_0 \) with
\[ R_0 = (1 - \beta) R_0, \quad \beta \ll 1, \quad \omega \ll \beta R_0. \]

The energy of the oscillations of the star is
\[ E = \frac{1}{2} m_0 \omega^2 R_0^2. \]

Then the energy loss per unit time averaged over one period is
\[ -\frac{dE}{dt} = \frac{1}{70} \frac{k}{c^2 (4 + 26)} m_0^2 \omega^2 R_0^2. \]

Therefore the characteristic damping time of the oscillations is
\[ \tau = 12c^2 (4 + 26) \beta (kM \omega R_0^2) \]
Setting \( \beta = 0.1, \) \( m_0 = 10^{22} \text{ g}, \) \( R_0 = 10^{11} \text{ cm}, \) \( \omega = 10^{-4} \text{ sec}^{-1}, \) we get \( \tau \approx 10^{11} \text{ sec}. \)

In both models the characteristic times are very large.

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