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# *Absorption of the Energy of an Electromagnetic Field in a Randomly Inhomogeneous Plasma*

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The problem of resonance absorption of the energy of an electromagnetic field in a randomly inhomogeneous plasma is considered. In an isotropic plasma, absorption is related to excitation of plasma oscillations at points at which the dielectric constant vanishes in a random manner. In a magnetoactive plasma there exist surfaces on which plasma oscillations arise. Since the plasma oscillations decay rapidly, the effective losses due to excitation of plasma oscillations are equal to the heat evolved in the plasma in the resonance regions or at resonance points. Expressions for the quasistatic part of the effective dielectric tensor are derived for a magnetoactive plasma which are valid at resonance frequencies and permit one to correctly take into account the energy of the excited plasma oscillations. A number of examples are considered which illustrate the general relations. Energy losses are calculated for a one-dimensional, randomly inhomogeneous plasma in a quasi-static field, for a three-dimensional inhomogeneous plasma in the field of a metallic spheriodal dipole (self-consistent solution) and for a magnetoactive plasma in the field of a plane capacitor.

## 1. INTRODUCTION

IT is well known that different types of waves that propagate independently in the geometrical-optics approximation can interact in a certain sense in regions where the conditions for the geometrical-optics approximation are violated. This interaction is particularly effective at points where the refractive indices of these waves are close in magnitude[1,2]. An interaction of this type between an electromagnetic wave and a plasma wave leads to a noticeable absorption of the former in the resonant regions of the plasma. In the resonant region the electromagnetic wave excites plasma waves (oscillations), which are effectively absorbed. On the other hand, an important role is played in the formation of radio emission from a plasma by the inverse process, the transformation of plasma waves into electromagnetic ones[3,4].

The initial investigations of these problems were closely related with the method of geometrical optics. The mathematical analysis was based on the phase-integral method[1-4]. Subsequent studies considered the problem of interaction in the resonant regions in a more general form. The review[5] contains references to most investigations of this question. Most of these papers deal with one-dimensional problems concerning an inhomogeneous plasma. Yet one case of volume interaction of waves in the plasma can be investigated in sufficient depth. We have in mind a randomly inhomogeneous plasma.

The investigation of a regularly inhomogeneous plasma with a concentration that depends on one coordinate shows that the heat absorbed by the heated plasma can be calculated by two methods. First, it is possible to calculate directly the amplitude of the plasma waves, the energy of which determines the absorbed heat. Second, we can calculate, by using the cold-plasma formulas, the limit of the integral that expresses the amount of absorbed heat as the number of collisions $\nu_{\mathrm{eff}}$ tends to zero. As $\nu_{\mathrm{eff}} \to 0$, the quantity $\operatorname{Im} \epsilon(\omega, \mathbf{k})$ also tends to zero. The heat, however, can remain finite, owing to the infinite increase of the field at the resonant points, or else because of the increase of the value of $\lim \int |\mathrm{E}|^2 dV$ as $\nu_{\mathrm{eff}} \to 0$[5-7]. The values of the absorbed energy calculated in this manner coincide. A similar picture is also obtained for a randomly inhomogeneous plasma at frequencies close to resonance. At the points where $\epsilon(\omega, \mathbf{r})$ vanishes (isotropic plasma), plasma oscillations are produced and their intensity determines the absorbed heat. This heat can be calculated if one knows the effective dielectric constant of the randomly inhomogeneous plasma.

## 2. ISOTROPIC PLASMA

Let us assume that the random function $\epsilon(\omega, \mathbf{r})$ can vanish in random fashion at individual points of the plasma volume. We assume statistical homogeneity of the process $\epsilon(\omega, \mathbf{r})$.

An important role is played in the electrodynamics of a randomly inhomogeneous medium by the effective dielectric constant $\epsilon_{ij}^{\mathrm{eff}}(\omega, \mathbf{k})$. If $\epsilon_{ij}^{\mathrm{eff}}(\omega, \mathbf{k})$ of the randomly inhomogeneous medium is known, then many problems concerning the inhomogeneous medium reduce to corresponding problems in a homogeneous absorbing medium[8]. In the concrete calculations it is usually necessary to employ an approximation wherein the first term of the mass operator is retained. This approximation is expressed by the formula[8]

$$\varepsilon^{l}_{\mathrm{eff}}(\omega, k) = \varepsilon_0 \frac{1 + {}^2/_3 \xi^{l}_{\mathrm{eff}}(\omega, k)}{1 - {}^1/_3 \xi^{l}_{\mathrm{eff}}(\omega, k)}, \quad \varepsilon^{tr}_{\mathrm{eff}}(\omega, k) = \varepsilon_0 \frac{1 + {}^2/_3 \xi^{tr}_{\mathrm{eff}}(\omega, k)}{1 - {}^1/_3 \xi^{tr}_{\mathrm{eff}}(\omega, k)},$$

$$\xi^{l}_{\mathrm{eff}}(\omega, k) = -2\langle \xi^2 \rangle q(p, p_0),$$

$$\xi^{tr}_{\mathrm{eff}}(\omega, k) = \langle \xi^2 \rangle \frac{p_0^2}{p} \int_0^\infty \Gamma_\xi(x) \exp(ip_0 x) \sin px \, dx + \langle \xi^2 \rangle q(p, p_0), \quad (1)$$

$$q(p, p_0) = \sqrt{\frac{\pi p}{2}} \int_0^\infty \Gamma_\xi(x) \frac{1}{\sqrt{x}} \exp(ip_0 x) \left[ \left( \frac{1}{px} - i \frac{p_0}{p} \right) J_{5/2}(px) - \frac{p_0^2}{p^2} J_{3/2}(px) \right] dx.$$

Here $J_{n/2}$ is a Bessel function with half-integer index, $\epsilon^{l}_{\text{eff}}$ and $\epsilon^{\text{tr}}_{\text{eff}}$ are the effective longitudinal and transverse polarizabilities of the medium, $p = kl$, $p_0 = k_0\sqrt{\epsilon_0(\omega)}\,l$, $\Gamma_\xi(x)$ is the normalized correlation function of the random polarizability of the medium with correlation radius $l$, k is the wave number, and $k_0 = \omega/c$. The random polarizability $\xi(\omega, \mathbf{r})$ is connected with the dielectric constant in the following manner:

$$\xi(\omega, \mathbf{r}) = 3\frac{\varepsilon(\omega, \mathbf{r}) - \varepsilon_0(\omega)}{\varepsilon(\omega, \mathbf{r}) + 2\varepsilon_0(\omega)}. \quad (2)$$

The permittivity $\epsilon_0(\omega)$ is determined by the equation $\langle \xi \rangle = 0$.

Formulas (1) are valid if $|\epsilon^{\text{eff}}_{ij}(\omega, k)| \ll 1$. In many cases this condition can be rewritten in the form

$$\langle \xi^2 \rangle k_0 \sqrt{\varepsilon_0(\omega)}\, l \ll 1 \quad (3)$$

for large-scale inhomogeneities ($k_0\sqrt{\epsilon_0}\,l \gg 1$) and

$$\langle \xi^2 \rangle k_0^2 \varepsilon_0(\omega) l^2 \ll 1 \quad (4)$$

for small-scale inhomogeneities ($k_0\sqrt{\epsilon_0}\,l \ll 1$). It follows from (3) and (4) that in the case of large-scale inhomogeneities it is necessary to satisfy the inequality $\langle \xi^2 \rangle \ll 1$, which is valid only for small relative fluctuations of the dielectric constant. To the contrary, for small-scale fluctuations, formulas (1) also describe the average field correctly when $\langle \epsilon(\omega, \mathbf{r}) \rangle \approx 0$. The analysis of expression (1) for this case can be found in[9].

It is important for us here to distinguish between two cases, depending on whether $\epsilon(\omega, \mathbf{r})$ can assume a value zero with a finite probability. If $\epsilon(\omega, \mathbf{r})$ does not vanish, then the imaginary part of $\epsilon^{\text{eff}}_{ij}(\omega, \mathbf{k})$ is connected only with the process of the transfer of the energy of the regular component of the field (average field) to the random electromagnetic wave. The quantity $\text{Im}\,\epsilon^{\text{eff}}_{ij}$ coincides with $\text{Im}\,\xi^{\text{eff}}_{ij}(\omega, \mathbf{k})$ and is determined by the cross section of scattering of the electromagnetic wave into electromagnetic radiation.

If $\epsilon(\omega, \mathbf{r})$ vanishes, then another branch of plasma excitations, namely plasma oscillations, comes into play. This case differs mathematically from the preceding one in that the quantity $\epsilon_0(\omega)$ becomes complex[8,9]. The imaginary part of the quasistatic permittivity $\epsilon_0(\omega)$ is proportional to the density of the probability that the random dielectric constant can assume zero values, $W(\epsilon = 0)$. In this case $\text{Im}\,\epsilon^{\text{eff}}_{ij}$ consists of two quantities. The second of them has a quasistatic character and describes the process of wave scattering into quasistatic plasma oscillations. At a frequency close to resonance, this process prevails over ordinary scattering into electromagnetic waves[9]. Therefore the "effective heat"

$$Q^{\text{eff}} = \frac{\omega}{2\pi}\,\text{Im}\,\varepsilon^{\text{eff}}\,|\langle E \rangle|^2 \quad (5)$$

becomes in this case essentially the true heat (the plasma oscillations are strongly absorbed).

To illustrate the foregoing picture, we consider a number of examples.

### A. Plasma in a Homogeneous Quasistatic Field

We begin with the case of a one-dimensional randomly-inhomogeneous small-scale plasma placed in a quasistatic electric field $E^l$ whose direction coincides with the direction of the variation of $\epsilon(\omega, x)$. This problem has already been discussed in[9]. We therefore confine ourselves here only to those results which are needed to make the analysis complete.

Let the field outside the plasma layer (at infinity) be $E^l_0$. The solution of the quasistatic equations for the conditions under consideration will be

$$E_x^l = E_0^l / \varepsilon(\omega, x). \quad (6)$$

The average field in the layer is expressed by the formula

$$\langle E_x^l \rangle = E_0^l\, \mathrm{P}\int_{-\infty}^{\infty} \frac{W(\varepsilon)}{\varepsilon}\, d\varepsilon - i\pi W(0). \quad (7)$$

The symbol P denotes integration in the sense of principal value, and $W(\epsilon)$ is the distribution function of the random quantity $\epsilon$.

Formula (7) determines the quasistatic dielectric constant of the one-dimensional plasma:

$$\varepsilon_{\text{eff}}^{-1} = \mathrm{P}\int_{-\infty}^{\infty} \frac{W(\varepsilon)}{\varepsilon}\, d\varepsilon - i\pi W(0). \quad (8)$$

Of course, such a plasma will also radiate electromagnetic waves. However, since we assume the frequency to be close to resonant, the energy of the electromagnetic waves[9] will be low.

For a normal distribution of $\epsilon$ with variance of $\sigma^2_{\Delta\epsilon} = \langle \Delta\epsilon^2 \rangle$ and mean value $\langle \epsilon \rangle = 0$ (at the Langmuir frequency) we obtain the formula

$$Q^{\text{eff}} = \frac{\omega}{2} W(0)\, |E_0^l|^2 = \frac{\omega |E_0^l|^2}{2\sqrt{2\pi}\,\sigma_{\Delta\varepsilon}}. \quad (9)$$

This quantity can also be obtained by directly calculating the losses at the resonant point ($\epsilon = 0$). Formula (9) makes it possible to calculate the energy of plasma oscillations excited at points where $\epsilon = 0$. At the same time, $Q^{\text{eff}}$ is the true heat released in the plasma, since the plasma oscillations attenuate strongly.

### B. Ellipsoidal Dipole Immersed in a Randomly Inhomogeneous Plasma

In an inhomogeneous plasma with normally-distributed electron-density fluctuations we have at a frequency close to the Langmuir frequency[8]

$$\varepsilon_0(\omega) = 0.56\langle\varepsilon\rangle + 0.52 i \sigma_N / \langle N \rangle,$$
$$\sigma_N^2 = \langle (N - \langle N \rangle)^2 \rangle, \quad \varepsilon = 1 - 4\pi e^2 N(\mathbf{r}) / m\omega^2. \quad (10)$$

Formula (10) is valid when $\langle \epsilon \rangle \ll \sigma_N/\langle N \rangle$, $\sigma_N/\langle N \rangle \ll 1$.

Using the formulas obtained in[10] for the impedance of a thin metallic spheroidal dipole in a randomly inhomogeneous plasma, we can easily calculate the heat released in the space surrounding the spheroidal dipole (excited by an external electromotive force $K(z) = e\delta(z)e^{i\omega t}$ at the center of the dipole):

$$Q^{\text{eff}} = \frac{e^2 \omega L}{16 \ln(L/a_0)} \frac{\sigma_N}{\langle N \rangle}. \quad (11)$$

Unlike the preceding case, where we have considered the absorption of the energy of a given field (a field of given current), we have obtained here a self-consistent solution. In particular, here $Q^{\text{eff}} \to 0$ as $\sigma_N \to 0$, since the current in the antenna vanishes at the resonant frequency as $\sigma_N \to 0$. It is appropriate to recall here the analogous situation with the radiation of a dipole in an absorbing medium[11].

### 3. MAGNETOACTIVE PLASMA

The tensor of the effective dielectric constant of a magnetoactive plasma was determined in[8,12]:

$$\varepsilon_{ij}^{\text{eff}}(\omega, k) = \varepsilon_{ij}^{0}(\omega) + \xi_{ij}^{\text{eff}}(\omega, k),$$

$$\varepsilon_{ij}^{0} = \begin{vmatrix} \varepsilon_0 & -ig_0 & 0 \\ ig_0 & \varepsilon_0 & 0 \\ 0 & 0 & \eta_0 \end{vmatrix}, \quad \xi_{ij}^{\text{eff}} = \begin{vmatrix} \xi_{11}^{\text{eff}} & \xi_{12}^{\text{eff}} & 0 \\ \xi_{21}^{\text{eff}} & \xi_{22}^{\text{eff}} & 0 \\ 0 & 0 & \xi_{33}^{\text{eff}} \end{vmatrix}, \tag{12}$$

$$\xi_{11}^{\text{eff}} = \xi_{22}^{\text{eff}}, \quad \xi_{12}^{\text{eff}} = -\xi_{21}^{\text{eff}}.$$

Formula (12) is valid when $|\xi_{ij}^{\text{eff}}(\omega, \mathbf{k})| \ll 1$. The following expression was obtained for the tensor $\xi_{ij}^{\text{eff}}$ for the case of small-scale fluctuations of the electron density $(k_0 l \ll 1)$ and neglecting spatial dispersion $(k = 0)$:

$$\begin{aligned} \xi_{11}^{\text{eff}} &= B_1(k_0 l)^2 + iC_1(k_0 l)^3 + iC_1'(k_0 l)^3 + \ldots, \\ \xi_{33}^{\text{eff}} &= B_2(k_0 l)^2 + iC_2(k_0 l)^3 + iC_2'(k_0 l)^3 + \ldots, \\ \xi_{12}^{\text{eff}} &= B_3(k_0 l)^2 + iC_3(k_0 l)^3 + iC_3'(k_0 l)^3 + \ldots. \end{aligned} \tag{13}$$

The coefficients $B_i$ are calculated in final form. At $k_0 l \ll 1$, the contribution determined by the terms $B_i(k_0 l)^2$ to the real part of $\epsilon_{ij}^{\text{eff}}$ is small and can be neglected. At the same time, the small terms, $C_i(k_0 l)^3$ and $C_i'(k_0 l)^3$ are significant, since they determine the damping of the average field as a result of scattering into the extraordinary and ordinary waves, respectively.

The coefficients $C_i$ and $C_i'$ were obtained in[8,12] in the form of integrals. The expressions given in[8,12] are valid only at frequencies for which there is no plasma resonance. At the plasma-resonance frequencies, $C_i$ and $C_i'$ diverge. This divergence is a consequence of the use of an approximation that is valid when $k_0 n_1(\theta) l \ll k_0 n_2(\theta) l \ll 1$ ($n_1$ and $n_2$ are the refractive indices of the extraordinary and ordinary waves propagating at an angle $\theta$ to the magnetic field). A more accurate calculation gives correct final expressions for $C_i$ and $C_i'$ in the entire frequency range, with the exception of the singular points $\omega = 0$, $\omega_H$, $\omega_0$, $\omega_p$ and near these points. Thus, at $k_0 l \ll 1$ the expressions (13) are also valid at the plasma-resonance frequencies, with the exception of the singular points. We shall not write out $C_i$ here, since we are interested only in the losses connected with $\operatorname{Im} \epsilon_{ij}^{0}(\omega)$.

The tensor $\epsilon_{ij}^{0}(\omega)$ was calculated in[12] for all frequencies with the exception of the singular points:

$$\begin{aligned} \varepsilon_0 &= \varepsilon - \frac{1}{2}\frac{\sigma_N^2}{\langle N\rangle^2}\frac{1}{\eta-\varepsilon}\frac{v^2(1+u)}{(1-u)^2}\left[\frac{\eta}{\varepsilon}\sqrt{\frac{\varepsilon}{\eta-\varepsilon}}\operatorname{arctg}\sqrt{\frac{\eta-\varepsilon}{\varepsilon}} - 1\right], \\ \eta_0 &= \eta - \frac{\sigma_N^2 v^2}{\langle N\rangle^2}\frac{1}{\eta-\varepsilon}\left[1 - \sqrt{\frac{\varepsilon}{\eta-\varepsilon}}\operatorname{arctg}\sqrt{\frac{\eta-\varepsilon}{\varepsilon}}\right], \\ g_0 &= g + \frac{\sigma_N^2}{\langle N\rangle^2}\frac{1}{\eta-\varepsilon}\frac{v^2\sqrt{u}}{(1-u)^2}\left[\frac{\eta}{\varepsilon}\sqrt{\frac{\varepsilon}{\eta-\varepsilon}}\operatorname{arctg}\sqrt{\frac{\eta-\varepsilon}{\varepsilon}} - 1\right]. \end{aligned} \tag{14}$$

Here $\epsilon$, $\eta$, and g are the average values of the components of the random tensor $\epsilon_{ij}(\omega, \mathbf{r})$; $v = \omega_0^2/\omega^2$, $u = \omega_H^2/\omega^2$, $\sigma_N^2 = \langle (N - \langle N \rangle)^2 \rangle$.

The conditions for the applicability of these expressions can be written in the form

$$|\varepsilon_0 - \varepsilon| \ll 1, \quad |\eta_0 - \eta| \ll 1, \quad |g_0 - g| \ll 1, \tag{15}$$

It is easily seen that these inequalities are indeed violated near the singular points $\omega = 0$, $\omega_0$, $\omega_H$, $\omega_p = \sqrt{\omega_0^2 + \omega_H^2}$.

We shall need in what follows an analysis of expression (14) at the plasma-resonance frequencies, which can be defined by the condition $(\eta - \epsilon)/\epsilon < -1$). Under this condition, the quantities $\epsilon_0$, $\eta_0$, and $g_0$ become complex, thus indicating that additional damping appears, not connected with scattering into ordinary and extraordinary waves. This damping describes the scattering of the energy of the average field into quasistatic plasma oscillations. Naturally, this damping is described by the imaginary part of the quasistatic tensor $\epsilon_{ij}^{0}(\omega)$. It is important to note the following. The functions (14) are multiple-valued at $(\eta - \epsilon)/\epsilon < -1$. The resonance conditions determine two frequency bands at which plasma resonance is possible (Fig. 1).

In the frequency band adjacent to $\omega = 0 (\eta - \epsilon < 0)$, the tensor $\epsilon_{ij}^{0}(\omega)$ is described by one branch of these functions. For the frequencies adjacent to $\omega = \omega_p$ $(\eta - \epsilon > 0)$ it is necessary to use the other branch. The choice of the branches can be justified by introducing vanishing collisions.

We introduce the notation

$$a = \frac{\eta - \varepsilon}{\varepsilon}, \quad f = \frac{\operatorname{arctg}\sqrt{a}}{\sqrt{a}}.$$

When $a < -1$ (resonance condition) we have

$$f = -\frac{1}{2\sqrt{|a|}}\ln\frac{\sqrt{|a|}-1}{\sqrt{|a|}+1} \mp i\frac{\pi}{2\sqrt{|a|}}.$$

Thus, at the resonant frequencies we have

$$\begin{aligned} \varepsilon_0 &= \varepsilon + \frac{1}{2}\frac{\sigma_N^2}{\langle N\rangle^2}\frac{1}{\eta-\varepsilon}\frac{v^2(1+u)}{(1-u)^2}\left[1 - \left|\frac{\eta}{\varepsilon}\right|\frac{1}{2\sqrt{|a|}}\right. \\ &\quad \times \left.\ln\frac{\sqrt{|a|}-1}{\sqrt{|a|}+1} \mp i\frac{\pi}{2\sqrt{|a|}}\left|\frac{\eta}{\varepsilon}\right|\right], \\ \eta_0 &= \eta - \frac{\sigma_N^2}{\langle N\rangle^2}\frac{v^2}{\eta-\varepsilon}\left[1 + \frac{1}{2\sqrt{|a|}}\ln\frac{\sqrt{|a|}-1}{\sqrt{|a|}+1} \pm i\frac{\pi}{2\sqrt{|a|}}\right], \\ g_0 &= g - \frac{\sigma_N^2}{\langle N\rangle^2}\frac{v^2\sqrt{u}}{(\eta-\varepsilon)(1-u)^2}\left[1 - \left|\frac{\eta}{\varepsilon}\right|\frac{1}{2\sqrt{|a|}}\right. \\ &\quad \times \left.\ln\frac{\sqrt{|a|}-1}{\sqrt{|a|}+1} \mp i\frac{\pi}{2\sqrt{|a|}}\left|\frac{\eta}{\varepsilon}\right|\right]. \end{aligned} \tag{16}$$

The upper sign must be taken at $\epsilon < 0, \eta > 0$ $(\eta - \epsilon > 0)$, and the lower one at $\epsilon > 0, \eta < 0$ $(\eta - \epsilon < 0)$.

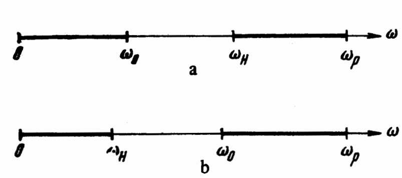


FIG. 1. Frequency bands at which plasma resonance is possible: a) $\omega_H > \omega_0$, b) $\omega_H < \omega_0$.

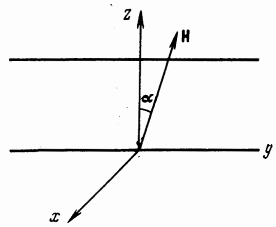


FIG. 2

By way of an example, we consider the following simple problem. Let a plasma in a parallel-plate infinite capacitor be placed in a magnetic field directed at an angle $\alpha$ to the normal to the capacitor plates (Fig. 2). In a uniform plasma[7] there is no loss at any frequency. The impedance z of the capacitor has only a reactive component, which becomes infinite at $\alpha = \theta$, where $\theta$ is the resonance angle determined by the resonance condition $\epsilon_{33} = \epsilon \sin^2\alpha + \eta \cos^2\alpha = 0$. We now take into account the inhomogeneity of the electron density. At a specified plate potential $\varphi = \varphi_0 \exp(i\omega t)$ + c.c., the current through the capacitor is a random quantity. The effective dielectric constant makes it possible to determine the average field, the average current, and the effective impedance of the capacitor. For the case of small-scale inhomogeneities ($k_0 l \ll 1$, $l/d \ll 1$, d is the distance between plates), we can neglect the formation of the scattered ordinary and extraordinary waves at the plasma-resonance frequencies, and take into account only the imaginary part of the tensor $\epsilon^0_{ij}(\omega)$. The average field in the capacitor is uniform:

$$\langle E_z \rangle = \varphi_0 / d.$$

The effective heat released in the plasma is

$$Q^{\text{eff}} = \frac{\omega}{2\pi} \operatorname{Im} \varepsilon_{33}^{\text{eff}} \left| \frac{\varphi_0}{d} \right|^2, \qquad (17)$$

where

$$\varepsilon_{33}^{\text{eff}} = \varepsilon^{\text{eff}} \sin^2\alpha + \eta^{\text{eff}} \cos^2\alpha,$$

$$\operatorname{Im} \varepsilon_{33}^{\text{eff}} \approx \operatorname{Im} \varepsilon_{33}^0(\omega) = \frac{1}{2} \frac{\sigma_N^2}{\langle N \rangle^2} \times \frac{\pi v^2}{|\eta - \varepsilon| \sqrt{|a|}} \left[ \frac{1}{2} \left| \frac{\eta}{\varepsilon} \right| \frac{1+u}{(1-u)^2} \sin^2\alpha + \cos^2\alpha \right],$$

and $\epsilon^{\text{eff}}$ and $\eta^{\text{eff}}$ are defined by the formulas (12) and (16).

The effective impedance of the capacitor (determined relative to the average current) is

$$z^{\text{eff}} = \frac{\varphi_0}{\langle I \rangle} = \frac{\operatorname{Im} \varepsilon_{33}^0 d}{2\pi\omega |\varepsilon_{33}^0|^2} - i \frac{\operatorname{Re} \varepsilon_{33}^0 d}{2\pi\omega |\varepsilon_{33}^0|^2}. \qquad (18)$$

The loss to the formation of the plasma oscillations leads to the appearance of an active part of the impedance and to a finite value of the impedance at $\alpha = \theta$ ($\theta$ is the resonance angle for a uniform plasma described by the average tensor). In the regions off resonance, it is useful to take into account the scattering into ordinary and extraordinary waves.

## 4. CONCLUSION

We have shown that the effects of formation and absorption of plasma oscillations at resonant points and regions of a randomly inhomogeneous plasma can be taken into account and calculated with the aid of an effective dielectric constant. It is important that correct results are obtained from the cold-plasma formulas, so that the calculations become simpler. Heating of a randomly inhomogeneous plasma can be calculated with the aid of an effective heat (heat released in the plasma upon absorption of an average field), which is the true heat at resonant frequencies in a plasma with small-scale inhomogeneities. The foregoing results make it possible to calculate the absorption only in a field of sources with finite dimensions $L \gg l$, $r_D$ ($r_D$ is the Debye radius). Under this condition we can neglect the spatial dispersion connected either with the thermal motion of the electrons or with the macroscopic inhomogeneity of the medium.

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