PERIODIC STRUCTURES OF A KINETIC NATURE IN CONDUCTING MEDIA WITH A STATIONARY FLUX

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Submitted January 19, 1971

Zh. Eksp. Teor. Fiz. 61, 1133–1143 (September, 1971)

A conducting medium with a stationary flux due to an external electric field, light pressure, or a sound beam may, at a certain critical value of the flux, become unstable with respect to formation of a periodic distribution of the magnetic and electric fields, charge density, lattice displacements, and temperature. All the quantities mentioned depend periodically on the coordinates that are transverse to the flux direction. In this case two types of transitions to the inhomogeneous state are possible. For a "soft" transition the amplitude of the inhomogeneous part is proportional to the square root of the excess over critical. For a "hard" transition, which is similar to first-order phase transitions, the amplitude changes discontinuously from zero to a finite value. Some possibilities of observing the phenomena are mentioned.

1. INTRODUCTION

ASSUME that in a conducting medium (metal, semimetal, and in some cases, semiconductor) there is some stationary flux, for example a flux produced by radiation pressure or a sound flux. It can also be simply an electric current produced by an external electric field. The field producing the flux, and the flux itself, will be regarded as homogeneous in both the longitudinal and transverse directions. It then turns out that under a rather wide range of conditions, at fluxes exceeding a certain critical value, the conductor goes over into a new state, the character of which depends on the geometry of the conductor.

If the dimensions of the body in the flux direction exceed its transverse dimensions, then growing transverse oscillations are produced in the body. They were investigated in a number of papers: ferromagnetic waves in [1], acoustomagnetic in [2], optimagnetic in [3] and galvanomagnetic waves, which produce microwave radiation in semiconductors and radiation at lower frequencies in semimetals, in [4].

On the other hand, if the dimensions of the body in the flux direction are smaller than its transverse dimensions (the body is a plate or disk, and the flux is perpendicular to its plane), then the indicated fluxes produce in the body a different transition, which we shall consider in the present paper. A stationary magnetic field is produced in the body both in the longitudinal and in the transverse direction, and varies periodically in the direction transverse to the flux. The onset of a spatially-periodic magnetic field is accompanied by the appearance of an electric field, a charge distribution, a crystal-lattice displacement distribution and a temperature distribution, all of which are periodic in space. In a conductor of cylindrical form, in which the flux is directed along the axis, there may be produced an R structure, i.e., a radial distribution characterized by a Bessel function, and also a φ structure, i.e., a periodic azimuthal distribution of the indicated quantities.

The structures may be produced via phase transitions of either first or second order (i.e., either jumpwise or smoothly). In the latter case, near critical values of the flux, the amplitude of all the quantities forming the periodic or quasiperiodic structure is proportional to the square root of the excess above the critical value. A quasithermodynamic analysis of the transition to the structure, based on the principle that the derivative of the entropy be extremal with respect to time, also results in proportionality to the square root of the excess above critical value.

We consider transitions to the structure in four cases.

A. The crystal carries a flux of electromagnetic waves (we shall call this flux optical) or a flux of transverse acoustic oscillations, and a magnetic field whose direction, generally speaking, does not coincide with that of the flux. In this case the structure is produced via a second-order phase transition.

B. There is no external magnetic field, but account is taken of the temperature dependence of the coefficient of proportionality $\gamma = \gamma(T)$ between the flux density I and the electric field $\mathbf{E} = \gamma \mathbf{I}$ produced by the flux. In this case a first-order transition is produced if $\partial\gamma/\partial T > 0$.

C. The flux is an electric current and there is an external magnetic field. This case has a number of singularities connected with the fact that the current produces its own magnetic field. In this case a φ structure is impossible.

D. Electric current in an anisotropic medium in the absence of an external magnetic field. In an anisotropic medium, a structure is apparently possible in the presence of any flux, but the necessary parameters of the medium have not yet been determined experimentally, so that quantitative predictions are as yet impossible. The occurrence of a structure in this case is possible only in crystals of monoclinic and triclinic syngony.

In all the indicated cases, a common property of the geometries of the conductor is the fact that the flux must propagate in the direction of the smallest dimension.

At a certain critical flux I_c, a structure is produced in the direction of the largest transverse dimension; in this case the direction of the largest dimension spans a sinusoid half-period of the quantities indicated above. When the flux is further increased, the direction of the largest dimension spans a larger number of half-waves. At even larger fluxes, periodicity also appears in the smaller second transverse direction. A doubly periodic structure is the result. (In the case of a disk, an R structure is produced first, and then a φ structure.) With further increase of the flux, a transition to the vibrational state takes place. In the last section we indicate the conditions for observing these phenomena.

2. PERIODIC STRUCTURES IN A MAGNETIC FIELD FOR A FLUX OF ELECTROMAGNETIC WAVES OR TRANSVERSE SOUND

We consider in this section the structures in three cases: singly- and doubly-periodic structures in plates, R and φ structures in a round disk, and φ structures in a ring with small wall thickness.

The flux is directed along the smallest plate dimension (z axis), and an external magnetic field $\mathbf{H}_0$ is present. An electric field

$$\mathbf{E} = \rho\mathbf{j} + \rho_1[\mathbf{jH}] + \rho_2\mathbf{H}(\mathbf{jH}) + \gamma\mathbf{I} + \gamma_1[\mathbf{IH}] + \gamma_2\mathbf{H}(\mathbf{IH}). \quad (1)^*$$

is produced along the z axis. Here j is the current and ρ, ρ_1, and ρ_2 are the ohmic, Hall, and focusing resistances. I should be taken to mean the flux density of the acoustic or optical energy, while γ, γ_1, and γ_2 are the corresponding opto- or acousto-electric coefficients multiplied by the ratio $R_L(R_L + R_C)$ (R_C and R_L are, respectively, the crystal and load resistances). A distinguishing feature of the coefficients γ for the indicated fluxes is that when $\mathbf{I} \parallel \mathbf{H}_0$ the quantity $\gamma_{\parallel} = \gamma + \gamma_2 H^2$ depends on the magnetic field, as shown in [5,6]. In the case of a longitudinal sound or thermal flux connected with ∇T, such a dependence takes place in certain particular cases, for example in the scattering of electrons by paramagnetic impurities.

The case of a strong current will be considered separately, for in this case it is necessary to take into account the current's magnetic field. The flux should be almost independent of the coordinate z.

At a definite flux $I > I_c$, the fluctuations $\mathbf{E}'$ and $\mathbf{H}'$ will be unstable. This phenomenon can be explained qualitatively as follows. Assume that in the presence of $\mathbf{I} \parallel \mathbf{H}_0$ a fluctuation magnetic field H'_z, which depends on the coordinate transverse to the flux I (x axis), has set in. The presence of such a field denotes the occurrence of a fluctuation current along the y axis: $j'_y = (c/4\pi)\,\mathrm{curl}_y\,\mathbf{H}'$, together with a dissipation due to the current and cancelling out the fluctuation field. At the same time, an additional electric field is produced, equal to $IH_0H'\partial\gamma_{\parallel}/\partial H^2$, along the z axis and dependent on x. The curl of such a field differs from zero, and therefore a growing magnetic field H'_y is produced along the y axis, dependent on x, and also a current j'_z connected with H'_y. This current also leads to a weakening of the fluctuation field. But the presence of H'_y leads to the appearance of an electric field $E'_y(x)$, which depends on x. This field again produces a field $H'_z(x)$.

We thus have two factors: dissipation connected with the current and weakening the fluctuation field, and "antidissipation" due to the solenoidal electric field. The "antidissipative" process is determined by the equations

$$\frac{\partial H_y'}{\partial t} = c\frac{\partial\gamma_{\parallel}}{\partial H^2}IH_0\frac{\partial H_z'}{\partial x}, \qquad \frac{\partial H_z'}{\partial t} = -c\gamma_2 IH_0\frac{\partial H_y'}{\partial x},$$

whence

$$\frac{\partial^2 H_y'}{\partial t^2} = -c^2(IH_0)^2\gamma_2\frac{\partial\gamma_{\parallel}}{\partial H^2}\frac{\partial^2 H_y'}{\partial x^2}.$$

Putting $H' \propto e^{\Gamma t}\cos kx$, we obtain

$$\Gamma = \pm ickIH_0\sqrt{\gamma_2\partial\gamma_{\parallel}/\partial H^2}.$$

The damping of the fluctuation field is determined by the crystal resistance ρ, and, as usual, equals $c^2k^2\rho/4\pi$. Therefore when

$$I \geq I_{c1} = \frac{ck\rho}{4\pi H_0}\left(\frac{\partial\gamma_{\parallel}}{\partial H^2}\gamma_2\right)^{-1/2}$$

the field H' increases. In order for the increase to be aperiodic, it is necessary that the nondissipative terms in the expression for the electric field (i.e., the "Hall" field) not lead to the formation of an electric field, i.e., we must have $\mathrm{curl}\,[\mathbf{j}' \times \mathbf{H}_0] = \mathrm{curl}\,[\mathbf{I} \times \mathbf{H}'] = 0$. These equations are satisfied when $k_z = 0$. In this case a stationary inhomogeneous field distribution can set in.

We proceed from these qualitative considerations to a quantitative calculation. Putting

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{H}', \quad \mathbf{E} = \mathbf{E}_0 + \mathbf{E}', \quad E', H' \propto e^{ikx - i\omega t}$$

and linearizing (1) and Maxwell's equations with respect to $\mathbf{E}'$ and $\mathbf{H}'$, we obtain

$$\omega = -c\gamma_1(\mathbf{kI}) + \frac{c^2k_z^2\rho_1 H}{4\pi} - \frac{ic^2k^2\rho_0}{4\pi} + ick_z\gamma_2(\mathbf{IH}_0) + ick_x\left(\gamma_2\frac{\partial\gamma_{\parallel}}{\partial H^2}\right)^{1/2}(\mathbf{IH}_0), \quad (2)$$

where ρ_0 is the resistance at $H_0 = 0$.

The real part of the frequency is equal to zero (aperiodic solution) at $k_z = 0$. This condition can readily be realized in the case of an electric current; in the case of an optical or acoustic flux it requires, first, that the circuit be closed in the direction of the z axis, for otherwise the vanishing of the fluctuation current j'_z on the boundaries leads to a dependence of all quantities on z, and second, that the thickness of the plate be much smaller than the characteristic attenuation length of the flux. For an optical flux this is the thickness of the skin layer, and for an acoustic flux the sound damping length.

The small finite ratio of the plate thickness to the characteristic attenuation length of the flux leads to a small dependence of H'_z and H'_y on z and to the appearance of a small component H'_x. At $k_z = 0$ we have $\mathrm{Im}\,\omega > 0$ if

$$\frac{4\pi(\mathbf{IH}_0)}{c\rho}\left(\gamma_2\frac{\partial\gamma_{\parallel}}{\partial H^2}\right)^{1/2} \equiv k_0 > k_x.$$

We see that at $\mathbf{I} \perp \mathbf{H}_0$ we have $\mathrm{Im}\,\omega < 0$ and no structure is produced. On the crystal boundary $x = \pm L_x/2$ we have $j'_x = 0$; hence

$$k_x = \pi p_x/L_x, \quad p_x = 1, 2, 3.$$

*$[\mathbf{jH}] \equiv \mathbf{j} \times \mathbf{H}$; $(\mathbf{jH}) \equiv \mathbf{j} \cdot \mathbf{H}$.

Therefore an inhomogeneous magnetic-field distribution sets in when

$$I \geqslant I_c^x = \frac{c\rho}{4H_0}\left(\gamma_2 \frac{\partial \gamma_\parallel}{\partial H^2}\right)^{-1/2} \frac{p_x}{L_x}. \tag{3}$$

The dependence of H'_z on x leads to the appearance of j'_y. It is therefore necessary to close the circuit in the y direction. With increasing I, superposition of the excited modes will take place.

So far we have considered $H' = H'(x)$, i.e., an inhomogeneity that depends only on one of the coordinates transverse to I and H_0. On the other hand, if $k_0 > \pi(L_x^{-2} + L_y^{-2})^{1/2}$, i.e., $I > I_c^{x,y}$, then a magnetic field is produced proportional to $\exp[i(k_x x + k_y y)]$, or a doubly-periodic inhomogeneity. In this case H'_x, H'_y, $H'_z \neq 0$ and $H'_x/H'_y = -k_y/k_x$. Closed currents circulate inside the plates, so that there is no need for an external circuit in the x or y direction. The growth of H′ is limited by nonlinear effects, and a stationary structure of the magnetic field sets in. With further increase of the flux intensity, excitation of fluctuations with $k_z = \pi/L_z$ is possible, leading to the appearance of the oscillatory instability considered in [1-3].

Let us investigate the ratio H'_y/H'_z. In a singly-periodic structure, H'_z and H'_y are shifted 90° in phase: $H'_z/H'_y = i\sqrt{\rho_0/2\rho}$. In a weak magnetic field $H_0 < c/\mu$ and in a strong field at $\rho_1 \neq 0$ ($\mu_\mp$ is the carrier mobility) we have $H'_z/H'_y = i/\sqrt{2}$; in a strong magnetic field at $\rho_1 = 0$ and $n_- \neq n_+$ ($n_\mp$ are the carrier densities), this ratio decreases by a factor $\sqrt{n_+\mu_+/n_-\mu_-}$. Finally, at $\rho_1 = 0$ and $n_1 = n_+$ this ratio equals $i\mu_- H_0/c\sqrt{2}$.

We can consider analogously a disk with $R \gg L$ (L is the thickness) at H_0 and I parallel to its axis. The critical flux needed for the occurrence of a radially inhomogeneous stationary magnetic field is determined by the condition $J_0(k_0R) = 0$, where J_m is a Bessel function of order m. If the flux I is larger than the one for which the condition $\operatorname{Im}(k_0R) = 0$ is satisfied, then the inhomogeneous part depends not only on the radius but also on the azimuthal angle φ: $H' \propto \operatorname{Im}(k_0R)\cos m\varphi$. With increasing m, the flux necessary for the occurrence of a structure also increases. Such R and φ structures take place as before, if the flux propagating along the axis depends little on z. If $k_z \neq 0$, oscillatory instability sets in. The maximum m at which there is still no oscillatory regime is approximately equal to $\pi R/L$.

In a hollow cylinder in which the difference between the external and internal radii is $R_e - R_i \ll R_i$, an approximate stationary solution (accurate to $(R_e - R_i)/R_i$) at which $H' = H'(\varphi)$ is possible. To this end it is necessary to have

$$\frac{2}{c}\frac{R_i}{\rho}\left(\gamma_2 \frac{\partial \gamma_\parallel}{\partial H^2}\right)^{1/2} IH_0 \geqslant p.$$

The periodicity of the stationary magnetic field in space leads to periodic deformation of the crystal lattice, and to a potential electric field and a charge. The equation for the lattice displacements u is (N—density, s—speed of sound).

$$Ns^2 \frac{\partial^2 \mathbf{u}}{\partial x^2} + \frac{1}{c}[\mathbf{j}'\mathbf{H}] = 0.$$

Confining ourselves to the case of free surfaces of the crystal at $x = \pm L_x/2$, on which the normal components of the stress tensor are equal to zero, we find that $u = u_0 e^{ikx}$, where

$$u_0 = (\mathbf{H}_0\mathbf{H}')/4\pi N s^2 k.$$

The charge-density amplitude referred to the carrier charge density is of the order of

$$\left(\frac{\mu H_0}{c}\right)^2 ck\rho_0 \frac{\rho}{\rho_0}\frac{H'}{H_0} \ll \frac{H'}{H_0}.$$

We shall now show that $I_c(H_0)$ has a minimum at $H_0 \approx c/\mu$. As seen from (3), in a weak magnetic field H_0 we have $I_c \propto 1/H_0$; in this case

$$I_c = \frac{c\rho}{4H_0L_x}\left(\gamma_2 \frac{\partial \gamma_\parallel}{\partial H^2}\right)^{-1/2} \backsim \frac{1}{H_0}.$$

In a strong magnetic field at a light-flux frequency $\omega < \Omega$ (Ω is the Larmor frequency), we have

$$I_c \approx \frac{c\rho}{4H_0L_x}\left(\gamma_2 \frac{\partial \gamma_\parallel}{\partial H^2}\right)^{-1/2}\left(\frac{\mu H_0}{c}\right)^3 \backsim H_0^4,$$

and at $\omega = \Omega$

$$I_c \backsim H_0^2,$$

if the Hall resistance $\rho_1 \neq 0$. If $\rho_1 = 0$ but $n_- \neq n_+$, the slope of the I(H) curve increases by a factor $[(n_-\mu_-)/(n_+\mu_+)]^{1/2}$. On the other hand, if $\rho_1 = 0$ and $n_- = n_+$, we have

$$I_c \backsim H^5 \quad \text{at} \quad \omega < \Omega,$$
$$I_c \backsim H^3 \quad \text{at} \quad \omega = \Omega.$$

Consequently $I_c(H_0)$ is minimal at $\mu H_0/c \approx 1$.

3. FERROMAGNETIC CRYSTAL

In ferromagnetic metals at low temperatures, a structure is produced in which the magnetic moment varies periodically. In ferromagnetic metals, the frequency of the spin waves, even at small values of the wave vector, is higher than the relaxation frequency of the magnetic moment. If we write the equation of the magnetic moment [7] in the form

$$\frac{d\mathbf{M}}{dt} = g[\mathbf{M}\mathbf{H}^e] - \frac{\mathbf{M}}{\tau_1} + \frac{\mathbf{n}(\mathbf{n}\mathbf{M}) - \mathbf{M}}{\tau_2}, \tag{4}$$

then at $gM_0 \gg \tau_1^{-1}, \tau_2^{-1}$ we have

$$\begin{aligned} \mathbf{E}' = {} & \rho\mathbf{j}' + \rho_1[\mathbf{j}'\mathbf{H}] + \rho_2\mathbf{H}(\mathbf{j}'\mathbf{H}) + \rho_{1M}[\mathbf{j}'\mathbf{M}] + \rho_{2M}\mathbf{H}(\mathbf{j}'\mathbf{M}) \\ & + \rho_{3M}\mathbf{M}(\mathbf{j}'\mathbf{H}) + \rho_{4M}\mathbf{M}(\mathbf{j}'\mathbf{M}) + \gamma_1[\mathbf{I}\mathbf{H}'] + \gamma_2\mathbf{H}'(\mathbf{I}\mathbf{H}) \\ & + \gamma_2\mathbf{H}(\mathbf{I}\mathbf{H}') + 2\frac{\partial\gamma}{\partial H^2}\mathbf{I}(\mathbf{H}\mathbf{H}') + \gamma_{1M}[\mathbf{I}\mathbf{M}'] + \gamma_{2M}\mathbf{M}(\mathbf{I}\mathbf{H}') \\ & + \gamma_{3M}\mathbf{H}(\mathbf{I}\mathbf{M}') + \gamma_{2M}\mathbf{M}'(\mathbf{I}\mathbf{H}) + \gamma_{3M}\mathbf{H}'(\mathbf{I}\mathbf{M}) + \gamma_{4M}\mathbf{M}'(\mathbf{I}\mathbf{M}) + \gamma_{4M}\mathbf{M}(\mathbf{I}\mathbf{M}'). \end{aligned} \tag{5}$$

The coefficients $\rho_{1M}, \rho_{2M}, \gamma_{1M}, \ldots$, are larger than $\rho_1, \gamma_1, \ldots$ [8] We substitute (5) in Maxwell's equations and use (4). Then in the case $\mathbf{H}_0 \parallel \mathbf{M}_0$, which we consider here, we obtain a bicubic equation for k_x. This equation goes over at $gM_0 \gg \tau_1^{-1}, \tau_2^{-1}$ into a biquadratic equation

$$k^4 + \left\{\frac{H_0}{M_0} - 2\left(\frac{4\pi}{c}\right)^2 \gamma_2 \left[\gamma_{2M} + 2\gamma_{3M}\frac{M_0}{H_0} + 2\gamma_{4M}\frac{M_0^2}{H_0^2}\right] \times \frac{1}{\rho}\left[\rho_0 + 2\rho_{3M}\frac{M_0}{H_0} + 2\rho_{4M}\frac{M_0^2}{H_0^2}\right]^{-1}\right\}(IH_0)^2k^2$$

$$-2\left(\frac{4\pi}{c}\right)^2\gamma_2\left[\gamma_{2M}+2\gamma_{3M}\frac{M_0}{H_0}+2\gamma_{4M}\left(\frac{M_0}{H_0}\right)^2\right]$$
$$\times\left[\rho_0+\rho_{3M}\frac{M_0}{H_0}+\rho_{4M}\left(\frac{M_0}{H_0}\right)^2\right]^{-1}\frac{1}{\rho}(IH_0)^2=0.$$

If γ_2, γ_{3M}, and γ_{4M} have the same sign, then it is necessary to choose out of the two roots of the last equation the root in which the radical is proceeded by a plus sign. As above, allowance for the boundary conditions leads to the following criterion for the appearance of inhomogeneity:

$$\left(\frac{k_{0M}L_x}{\pi}\right)^2=\frac{k_0L_x}{\pi}\frac{\rho_0}{\gamma_2}\left(\gamma_2+2\gamma_{3M}\frac{M_0}{H_0}+2\gamma_{4M}\frac{M_0^2}{H_0^2}\right)$$
$$\times\left(\rho_0+2\rho_{3M}\frac{M_0}{H_0}+2\rho_{4M}\frac{M_0^2}{H_0^2}\right)^{-1}\geqslant n.$$

When $M_0 \ll H_0$ and $M_0 \gg H_0$ we have $k_{0M} \approx k_0$.

4. CASE OF STRONG DIRECT CURRENT

The case of an electric current (of density j_0) calls for a special study. We confine ourselves to a current flowing along the axis of a cylinder of radius R, placed in an external strong magnetic field H_0, with the current is so strong that its field $H_j(R) \gg H_0$. In this case an R structure is possible, but only if the Hall resistance is very small, i.e., at $n_1 = n_+$ and $\mu_\pm H/c \gg 1$ or else at $n_- < n_+$ and $\mu_+ H < c < \mu_- H$.

We linearize (1) at $I = 0$ and $j_0 \neq 0$ and take into account $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_j + \mathbf{H}'$. We obtain equations for H'_z and H_φ:

$$\text{rot}_\varphi \mathbf{E}'=0,\quad \text{rot}_z\mathbf{E}'=0.$$

Unlike the preceding cases, $\mathbf{H}'$ cannot depend on φ, i.e., a φ structure is impossible. Indeed, at $H' \propto \exp(im\varphi)$ we find[4] that

$$\text{Re}\,\omega=\frac{c^2m^2}{4\pi R^2}\rho_1 H_j(R),$$

i.e., a wave of the helicoidal type. But at $\mathbf{H}' = \mathbf{H}'(r)$ we have $H'_r = 0$. Eliminating H'_z, we can obtain

$$\frac{d^2}{dr^2}(rH_\varphi')+b(r)rH_\varphi'=0.$$

The possibility of a solution that is quasiperiodic in r is determined by the sign of b(r), which was determined for the limiting cases $H_\varphi \ll H_z$ and $H_\varphi \gg H_z$.

For $\rho_1 = 0$ and $H_\varphi \gg H_z$ we have

$$br^2=(\mu_-H_\varphi/c)^2\delta\gg 1$$

(δ is a number on the order of unity, determined only during the course of numerical integration). When $H_\varphi \ll H_z$ we have $b(r) < 0$ and no periodic structure is produced. Since the characteristic length of variation of $H_\varphi(r)$ and $H_z(r)$ is of the order of r, we can employ the WKB approximation. We obtain $rH'_\varphi \approx \sin\sqrt{br}$. The boundary condition $H'(R) = 0$ calls for $bR^2 = (\pi p/2)^2$, which can readily be satisfied when $\mu_- H_\varphi/c \gg 1$.

For flow of a strong current, it is of interest to consider anisotropic crystals in the absence of a homogeneous field. A structure can arise only in crystals having no resistivity tensor of third rank (i.e., in crystals with an inversion center), and furthermore only in crystals of monoclinic and triclinic syngony, (just as in [2]). In this case the calculation is very cumbersome, and it is convenient to perform it in the WKB approximation, assuming the length of the period of the structure to be small compared with the sample dimensions transverse to the current. If we introduce a quantity with the dimension of the square of the magnetic field $\overline{H}^2$, equal in order of magnitude to the ratio of the fourth-rank resistivity tensor ρ_{iklm} to the second-rank resistivity tensor ρ_{ik}, namely, $\overline{H}^2 = |\rho_{iklm}/\rho_{ik}|$, then we can state that a structure is produced at $H_j \gg \overline{H}$.

5. NONLINEAR THEORY

In the preceding sections we have shown that at definite values of the flux there arises in the crystal an instability with respect to formation of a periodic or quasi-periodic structure. In the linear theory it is impossible to estimate the amplitudes of the variations of the magnetic field and of the other quantities, and to demonstrate that when $I \gg I_c$ the deviations from H_0 have a periodic character.

In the singly-periodic case, in a weak magnetic field, it is convenient to write down the nonlinear system in the form

$$\frac{d}{dx}\frac{H_y}{H_0}=\frac{k_0}{\sqrt{2}}\frac{H_z^2-H_0^2}{H_0^2}\left[1-\left(\frac{\mu H}{c}\right)^2\delta\right],\tag{6}$$

$$-\frac{d}{dx}\frac{H_z}{H_0}=\frac{k_0}{\sqrt{2}}\frac{H_zH_y}{H_0^2}\left[1-\left(\frac{\mu H}{c}\right)^2\delta\right],\tag{7}$$

where δ is a number on the order of unity and appears when the kinetic coefficients are expanded in a series in $(\mu H/c)^2$.

It is seen from (6) and (7) that H_y reaches an extremum H_y^e at $H_z = H_0$, i.e., on the boundaries of the crystal; H_z is extremal at $H_y = 0$. (It follows from (7) that $H_z \neq 0$ for all values of x. Indeed, in the opposite case, the derivative of any order would be $d^pH_z/dx^p = 0$, and consequently $H_z \equiv 0$, contradicting the boundary conditions.) Near the extrema we have

$$H_y^2 \propto (x_{ey}^2-x^2),\quad H_z^2-(H_z^e)^2\propto(x-x_{ez})^2,$$

where x_{ey} and x_{ez} are the points at which H_y and H_z reach the extremal values H_y^e and H_z^e. It follows from (6) and (7) that

$$H_z^2+H_y^2-H_0^2\ln\frac{H_z^2}{H_0^2}=H_1^2,\tag{8}$$

where H_1^2 = const. Since $H_z = H_0$ and $H_y = H_y^e$ on the boundary of the crystal, we get $H_1^2 = H_0^2 + (H_y^e)^2$.

Let us show that $|H_y^e| \ll H_0$. Indeed, near the boundary we have $H_z = H_0 + h$, where $h \ll H_0$. Linearizing with respect to h, we get $h = C\cos k_0x$, $H_y = C\sin k_0x$, where C = const and consequently $|H_y| \ll H_0$. But then, using the relation $H_1^2 = H_z^2 - H_0^2\ln(H_z^2/H_0^2)$, we get $|H_z - H_0| \ll H_0$. Thus, the deviations from a homogeneous distribution are always small (an analysis shows that these deviations are stable against excitation of higher modes). From (8) and (6) we get

$$\int_H^{H_0}\frac{dH_z^2}{H_z^2}\left[H_1^2-H_z^2+H_0^2\ln\frac{H_z^2}{H_0^2}\right]^{-1/2}=\begin{cases}k_0(x-1/2),\ x>0\\ -k_0(x+1/2),\ x<0\end{cases}.\tag{9}$$

The condition that the current be continuous at x = 0 leads to an equation for the determination of H_z^e.

The left-hand side of (9) can be integrated only numerically. We therefore consider the case $I - I_c \ll I_c$. Putting $H_z = H_0 + h_z$, where $h_z \ll H_0$ and $H_y \ll H_0$, and iterating (8) (or the system (6)–(7)), we get

$$H_z = H_0 + C\cos k_0 x, \quad H_y = C\sin k_0 x,$$
$$C = H_0\left(\frac{I-I_c}{3I_c}\right)^{1/2}\left[1+\delta\left(\frac{I-I_c}{I_c}\right)^{1/2}\left(\frac{\mu H_0}{c}\right)^2 \mathrm{sign}(H_0 h)\right]^{-1/2}.$$

Thus we arrive at the conclusion that the growth of the random fluctuation of the magnetic field H_z depends on its sign. At h_z antiparallel to H_0, the amplitude is larger than at $h_z \parallel H_0$. This difference, however, is of the order of

$$\left(\frac{\mu H_0}{c}\right)^2\left[\frac{I-I_c}{I_c}\right]^{1/2} \ll 1.$$

The reasoning is perfectly analogous for a strong field. Just as in a weak field, H_y is extremal when $H_z = H_0$ and H_z is extremal when $H_y = 0$. Iteration leads to the same results as above:

$$H_z - H_0 \approx H_0\sqrt{\frac{I-I_c}{I_c}},$$

and if H_0 and h_z are antiparallel, the latter is larger than when they are parallel. The difference of the amplitudes is of the order of

$$\left(\frac{c}{\mu H_0}\right)^2\left[\frac{I-I_c}{I_c}\right]^{1/2} \ll 1.$$

Thus, in these cases the transition to the structure is of second order, i.e., the excitation regime is "soft." (We assume in this section that the results remain unchanged in the case of a disk or of a doubly-periodic structure.)

6. OCCURRENCE OF A STRUCTURE UNDER NONISOTHERMAL FLUCTUATIONS

In addition to the foregoing mechanism whereby structures are produced, an entirely different mechanism, which does not require the presence of a magnetic field, is also possible. We already know that a flux of density I produces an electric field $\mathbf{E} = \gamma \mathbf{I}$ in a conductor. Periodic and quasiperiodic structures are produced if the coefficient γ increases with temperature. Since $T = T_0 + T'$, where $T_0 = \text{const}$ and $T' \ll T_0$, the expression for the fluctuation field takes the form (in place of (1))

$$\mathbf{E}' = \alpha\nabla T' + \gamma_1[\mathbf{IH}'] + \frac{c\rho}{4\pi}\,\mathrm{rot}\,\mathbf{H}' + \frac{\partial\gamma}{\partial T}\mathbf{I}T'$$

(α is the thermoelectric coefficient). We substitute (8) in Maxwell's equation and use the heat-conduction equation

$$-\mathrm{div}\left\{\varkappa\nabla T' + \frac{\partial\beta}{\partial T}\mathbf{I}T' + \beta_1[\mathbf{IH}']\right\} = \frac{c\gamma}{4\pi}\mathbf{I}\,\mathrm{rot}\,\mathbf{H}'$$

(Here κ is the thermal conductivity and β, β_1, and β_2 are the coefficients in the expression for the heat flux $\mathbf{q} - \varphi\mathbf{j}$, which is proportional to $\mathbf{I}$:

$$\mathbf{q} - \varphi\mathbf{j} = \beta\mathbf{I} + \beta_1[\mathbf{IH}] + \beta_2\mathbf{H}(\mathbf{IH}),$$

φ is the electric-field potential.) At $I = I_z$, putting H', $T' \propto \exp(ikx)$, we find that a stationary inhomogeneous distribution of the field and of the temperature is possible if

$$k_{0T}^2 \equiv \frac{I^2}{\rho}\frac{\partial\gamma}{\partial T}\left[\frac{\gamma T}{\varkappa} + \frac{4\pi}{c}\frac{\beta_1}{\varkappa}\right] > \left(\frac{\pi p}{L_x}\right)^2.$$

The coefficient of I^2 should be larger than zero. Then $H' = H_y'$ is shifted in phase by $\pi/2$ relative to T', and $\mu H'/c \approx T'/T$.

Just as when $H_0 \neq 0$, we can obtain a structure in which the magnetic field is proportional to $\exp[i(k_x x + k_y y)]$ if $k_{0T}^2 > k_x^2 + k_y^2$, or an inhomogeneous distribution of the magnetic field in a cylindrical sample if $\mathrm{Im}(k_{0T}R) = 0$. Qualitatively, the occurrence of an inhomogeneous distribution can be understood as follows. If the temperature in a medium with $I \neq 0$ has deviated from equilibrium by $T'(x)$, then an additional electric field $E_z' = IT'\partial\gamma/\partial T$ is produced. The curl of this field differs from zero, and leads to a field increment $H_y'(x)$ and to an associated current $j_z'(x) = (c/4\pi)\,\mathrm{curl}\,\mathbf{H}'$. Unlike in the preceding case, the presence of H_y' does not lead to the appearance of $E_y'(x)$, but H_y' changes the heat flux, since a flux component proportional to $\mathbf{I}\times\mathbf{H}'$ and directed along the x axis is produced. Since the divergence of this flux differs from zero, a new value of T' is established. When $\partial\gamma/\partial T > 0$, this mechanism increases T', provided the dissipative processes (resistance and thermal conductivity) are sufficiently small. Just as above, the growth is aperiodic when $k_z = 0$.

When $H_0 \neq 0$, allowance for $\partial\gamma/\partial T \neq 0$ leads to a simultaneous occurrence of H_z', H_y', and T', all of which are periodic in x if

$$k_{0TH}^2 = k_0^2\left[1 + \left(\frac{c}{4\pi}\right)^2\rho\frac{\partial\gamma}{\partial T}\left(\frac{\gamma T}{\varkappa} + \frac{4\pi\beta_1}{c\varkappa}\right)\frac{I^2}{(IH_0)^2}\right.$$
$$\left.\times\left(\gamma_2\frac{\partial\gamma_\parallel}{\partial H^2}\right)^{-1}\right] > \left(\frac{\pi p}{L_x}\right)^2.$$

The expression in the square brackets can be either larger or smaller than unity, depending on the sign of $\partial\gamma/\partial T$. In the former case the term $\partial\gamma/\partial T \neq 0$ leads to smaller I_c than in Sec. 2, and in the latter to larger I_c. Estimates show that the contribution of the term proportional to $\partial\gamma/\partial T$ is significant when $H_0 \ll c/\mu$ and $H_0 \gg c/\mu$. In analogy with Sec. 5, nonlinear theory shows that the excitation regime is soft and the amplitude of the magnetic field is

$$\frac{\varkappa}{\beta_1}\left(\frac{I-I_c}{I_c}\right)^{1/2}\left(1 - \frac{5}{3}\frac{\partial\ln\beta_1}{\partial\ln T}\Big/\frac{\partial\ln\gamma_1}{\partial\ln T}\right)^{-1/2},$$

if the radicand in the denominator is positive. If this expression is negative, then the excitation regime is hard.

7. QUASITHERMODYNAMIC THEORY

In the case of a soft regime, the transition to the structure can be regarded as the analog of a second-order phase transition. Starting from the principle of extremal entropy production, it can be shown that the amplitude of the inhomogeneous part of the magnetic field (and of other quantities connected with the magnetic field) is proportional to the square root of the excess above critical value. In accordance with the electrodynamic calculation presented above, we assume that the critical fluxes at which H'_z and H'_y are different from zero coincide. Then, expanding the time derivative of the entropy S in a series with respect to H'_z and H'_y (S_0 is the entropy at $H' = 0$), we obtain

$$\frac{\partial S}{\partial t}-\frac{\partial S_0}{\partial t}=a_zH_z'^2+a_yH_y'^2+\frac{b_z}{2}H_z'^4+\frac{b_y}{2}H_y'^4+cH_z'^2H_y'^2.$$

(Just as in [9], we confine ourselves to the case when a quasiphase transition is possible on an entire curve, and not at an individual point; then the expansion contains no terms with odd powers of H'.) The quantities a_z and a_y vanish at $I = I_c$, i.e., $a_{z,y} = \alpha_{z,y}(I - I_c)$. An extremum of $\partial S/\partial t$ is reached when

$$H_z'(a_z+b_zH_z'^2+cH_y'^2)=0,$$
$$H_y'(a_y+b_yH_y'^2+cH_z'^2)=0,$$

From this we get H'_z, $H'_y \neq 0$ and

$$H_z'^2=\frac{b_za_y-a_zc}{b_zb_y-c^2}\propto I-I_c,$$
$$H_y'^2=\frac{b_ya_z-a_yc}{b_zb_y-c^2}\propto I-I_c,$$

i.e., H'_z and H'_y are indeed proportional to the square of the excess above critical value.

8. CONDITIONS FOR OBSERVING PERIODIC STRUCTURES

The above-described distribution of the magnetic and electric fields, currents, charge, temperature, and lattice displacements can be effected in metals, semimetals, and semiconductors. For passage of electromagnetic waves, the thickness of the plate or disk should be smaller than or of the order of the dimensions of the skin layer. For bismuth, antimony, or metals at helium temperatures and at transverse crystal dimensions ≈ 1 cm the intensity of the light wave (outside the crystal) is $\approx 10^4$ W/cm^2 if the frequency of the incident wave is $\approx 10^8$ sec^{-1}. The thickness of the plate (disk) is then $\approx 10^{-2}$ cm. At a higher frequency, it is necessary to decrease the thickness, and the critical intensity decreases like the square root of the frequency. For InSb at nitrogen temperature, the critical intensity amounts to 10^6 W/cm^2. In the case of a strong current, quasiperiodic R structures, which are realizable at $H_\varphi > H_z > c/\mu_\pm$, can be obtained in bismuth or antimony. For example, such a structure was realized in the experiments of Bartelink[10] where, however, the case $L \gg R$ was investigated, which led to an oscillatory instability, in accord with the statements made above.

A hard transition regime is realizable with an acoustic flux, since the acoustoelectric field is proportional to the damping of the sound, which increases with temperature, and therefore $\partial\gamma/\partial T > 0$. The intensity of the sound flux is ≈ 1 W/cm^2 in semimetals at helium temperatures. With an optical flux, satisfaction of the inequality $\partial\gamma/\partial T > 0$ is possible only for carrier scattering by charged impurities.

The authors are grateful to B. L. Gel'mont and V. I. Perel' for valuable remarks.

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Translated by J. G. Adashko
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ERRATA

Articles by V. L. Lyuboshitz and M. I. Podgoretskiĭ

These papers, dealing with identity in quantum mechanics and published in JETP in 1968–1971, contain a number of misprints and errors, which in some cases are dangerous. Most of them are either obvious or do not play a significant role. We list here some of the most important corrections.

In the article published in Sov. Phys.–JETP 28, 469 (1968), formula (9) should read:

$$|C\rangle = A_1^*|A\rangle + A_2^*|B\rangle, \quad |D\rangle = A_1'^*|A\rangle + A_2'^*|B\rangle.$$

On p. 474 we consider an inappropriate example with hypothetical particles having identical quantum numbers but different masses. In view of the uncertainty of the employed concepts and in view of its hypothetical character, this example does not admit of a unique interpretation.

At the end of the article, on p. 475, there is an inaccurate remark concerning the intermediate statistical equilibrium; this remark is valid only if the internal states of all the system particles are the same at the initial instant, i.e., if they correspond to the same superposition.

In the paper published in Sov. Phys.–JETP 30, 100 (1969), in formula (12), the quantity R_{AB} should be replaced by R_{AB}^*. The expression ahead of formula (17) should take the form

$$\ldots = \frac{R_{AB}^*}{{}^1/_2(\Gamma_A + \Gamma_B) + i(m_B - m_A)} = \langle A|B\rangle.$$

In the paper published in Sov. Phys.–JETP 30, 91 (1970), on p. 92, col. 2 (4th line from the bottom), read:

"Formulas (21) and (24) are in this case no longer valid, but Eq. (22) and the first relation in (23) remain true."

In the article published in Sov. Phys.–JETP 33, 5 (1971), in formulas (13), the minus sign has been left out from the arguments of the exponentials.

Formula (14) should read:

$$\frac{d\sigma(\theta)}{d\Omega} = |f(\theta)|^2 + |f(\pi-\theta)|^2 \pm 2\,\mathrm{Re}\, f(\theta) f^*(\pi-\theta) \Big[|\langle C_0|D_0\rangle|^2 + 4\,\mathrm{Im}\,\alpha_0\gamma_0^*\delta_0\beta_0^* \exp\left(-\frac{i(m_A-m_B)(\tau_1-\tau_2)}{2}\right) \sin\frac{(m_A-m_B)(\tau_1-\tau_2)}{2} \Big].$$

Formula (15) should read:

$$\frac{d\sigma(\theta)}{d\Omega} = |f(\theta) \pm f(\pi-\theta)|^2 \mp 8|\alpha_0\beta_0|^2 \times \sin^2\frac{(m_A-m_B)(\tau_1-\tau_2)}{2}\,\mathrm{Re}\, f(\theta) f^*(\pi-\theta).$$