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VERIFICATION OF THE EQUIVALENCE OF INERTIAL AND GRAVITATIONAL MASS

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An experiment is described which establishes with 0.9×10^{-12} accuracy (95% confidence) that the ratio of the inertial to the gravitational mass is identical for aluminum and platinum. A torsion balance with characteristic oscillation period of 5 hours and 20 minutes and relaxation time exceeding 6×10^7 sec was used. Simulative effects which limited the achieved resolution were analyzed. The result 0.9×10^{-12} is close to the ratio of the strengths of the weak and strong interactions.

THE general theory of relativity is based on the fundamental experimental fact that the ratio of inertial to gravitational mass is identical for different bodies (the principle of equivalence). Einstein considered the precise confirmation of this fact to be more important than further tests of the advance of the perihelion of Mercury and the deflection of light in the gravitational field of the sun (see [1], for example). About 50 years ago Eötvös, Pekar, and Fekete established that the relative difference of the ratio of the inertial to the gravitational mass for different bodies does not exceed 3×10^{-9}.

During the period 1959–1964 the principle of equivalence was again tested by Dicke, Krotkov, and Roll.[2] A hypothetical difference between the accelerations of two bodies, made of gold and aluminum, respectively, in the gravitational field of the sun was measured. From measurements conducted during many months it was concluded that the ratio of the inertial and gravitational masses for these two bodies does not differ by more than 3×10^{-11} with 95% confidence.[2] An analysis of the experimental work in [2] shows that it is possible, in principle, to considerably improve the resolution by using a mechanical oscillatory system having a long relaxation time.[3] We shall here describe an experiment intended as a new test of the principle of equivalence.

1. MEASUREMENT TECHNIQUE

We retained the experimental scheme of Dicke, Krotkov, and Roll.[2] A torsion balance falling together with the earth in the gravitational field of the sun should be acted upon by a torsional mechanical moment that is proportional to a hypothetical difference between the accelerations of the materials comprising the balance (if the principle of equivalence is not fulfilled). Because of the earth's rotation this moment should vary sinusoidally with a 24-hour period (Fig. 1).

It was shown in [3] that if for the registration of a small periodic acceleration we use a mechanical oscillator having a long relaxation time, the minimum observable acceleration of a test mass m will be

$$F/m = \theta\sqrt{8\kappa T / m\hat{\tau}\tau^*}. \qquad (1)$$

Here $\hat{\tau}$ is the measurement time, τ^* is the relaxation time, and θ is the confidence coefficient of the observed acceleration. It is shown by (1) that when $\hat{\tau}$ remains

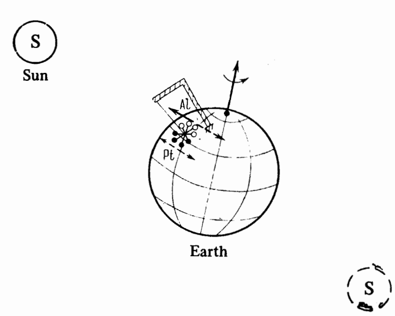

FIG. 1

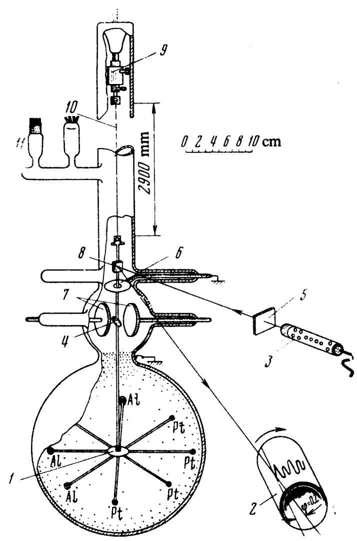

FIG. 2. Experimental scheme. 1—balance, 2—drum with photographic film, 3—laser, 4—conductive ellipsoid, 5—gray filter, 6—electrostatic shield, 7—condenser plates, 8—mirror, 9—rotational adjustment mechanism, 10—tungsten wire (5 X 10^{-4} cm diameter).

unchanged the sensitivity can be enhanced by increasing τ^*. If $\tau^* = 6 \times 10^7$ sec, $\hat{\tau} = 6 \times 10^5$ sec, and $m = 4$ g, an acceleration $\sim 1 \times 10^{-13}$ cm/sec^2 can be distinguished with 95% confidence against a background of Brownian fluctuations. To facilitate the detection of this small quantity the characteristic period τ_0 of the oscillator must be as close as possible to 24 hours. The relaxation time τ^* is determined under laboratory conditions by the pressure of residual gas around the oscillator:

$$\tau^{*}_{\text{gas}} \approx {}^1/_4 m S^{-1} \mu^{-1/2} n^{-1} (\varkappa T)^{-1/2} \qquad (2)$$

and by the friction in the wire:

$$\tau_{\text{wire}} \approx N \tau_0^2 \eta^{-1} \pi^{-2} \qquad (3)$$

where m is the mass of the oscillator, S is its total surface, μ is the mass of a gas molecule, n is the number of molecules per unit volume, η is the viscosity coefficient of the wire material, and N is its shear modulus. In our experiment $m \approx 4$ g, $S = 2$ cm^2, $\mu = 5.5 \times 10^{-23}$ g, $n \lesssim 3 \times 10^8$ cm^{-3} (vacuum chamber pressure $p \lesssim 1 \times 10^{-8}$ Torr), $N = 1.5 \times 10^{11}$ dyn/cm^2 (for tungsten), and $\eta = 1 \times 10^{10}$ poise. With these parameters we obtain

$$\tau^{*}_{\text{rad}} \approx 3 \cdot 10^9 \text{ sec}, \quad \tau^{*}_{\text{wire}} \approx 8 \cdot 10^9 \text{ sec}.$$

After three days the amplitude of the oscillations of the balance had changed by at most $\sim 3 \times 10^{-3}$ of their initial magnitude; from this result we derive the inequality $\tau^* > 6 \times 10^7$ sec. It was not possible to determine τ^* more accurately. The oscillatory period τ_0 was 1.92×10^4 sec (5 hours 20 minutes).

For the purpose of reducing the effects of variable local gradients in the gravitational field, the torsion balance was constructed in the form of an eight-pointed star (Figs. 1 and 2) with equal masses, four of aluminum and four of platinum, at the points. (In the experiment of [2] three weights were used—two of aluminum and one of gold.) In this case, if the principle of equivalence is not fulfilled the balance should oscillate with a period of 24 hours and an amplitude

$$\Delta\varphi = g\tilde{\Delta} / 3.07\ R\omega_0^2; \qquad (4)$$

here $g \approx 0.62$ cm/sec^2 is the acceleration of free fall on the sun,

$$\tilde{\Delta} = (m_{\text{Al}} / M_{\text{Al}} - m_{\text{Pt}} / M_{\text{Pt}}) \,/\, {}^1/_2 (m_{\text{Al}} / M_{\text{Al}} + m_{\text{Pt}} / M_{\text{Pt}}), \qquad (5)$$

where m and M are the inertial and gravitational masses of the bodies, R is the radius of the balance "beam," and $\omega_0 = 2\pi/\tau_0$ is the characteristic frequency of balance oscillations Eq. (4) holds true if $\tau_0 \ll 24$ hours). With $\tilde{\Delta} = 1 \times 10^{-12}$, $\tau_0 = 1.92 \times 10^4$ sec, and $R = 10$ cm we obtain $\Delta\varphi = 1.8 \times 10^{-7}$ rad.

The design of the apparatus is shown in Fig. 2. The radial "beam" of the balance, with 4×10^2 g-cm^2 moment of inertia, was made of Dural rods bearing aluminum and platinum weights at their ends (the total mass of the weights was 3.9 g), and was suspended by a tungsten wire 5×10^{-4} cm in diameter and 290 cm long. The total mass of the balance was 4.4 g. The wire had been annealed previously under tension in a vacuum; this operation enabled us to reduce the monotonic drift to $\sim 4 \times 10^{-6}$ rad/day. The balance was placed inside a glass vacuum chamber under a pressure $p \lesssim 1 \times 10^{-8}$ Torr, which did not vary during the course of the experiment. The lower, silvered, part of the vacuum chamber, where the "beam" was located, was grounded along with the balance. The upper part of the chamber contained the rotational adjustment mechanism which permitted us to change the equilibrium position of the balance. A conductive ellipsoid, located between the plates of an electric condenser, was attached rigidly to the stem of the balance. Pulses of electric voltage applied to the condenser plates permitted variation (slowing or building up) of the torsional oscillations. This was necessary because the damping time of the oscillations was at least two years. We were thus enabled to reduce the oscillation amplitude from a few radians to $\sim 1 \times 10^{-5}$ rad.

The oscillations of the balance were registered through the motion of a light spot on photographic film. The light source was a helium-neon laser. The laser beam was reflected from a mirror fastened on the balance to a drum bearing the film. The axis of the drum was inclined ≈ 0.2 rad with respect to the beam direction (Fig. 2), thus increasing the effective length l of the optical lever. (In our case $l \approx 5 \times 10^3$ cm.) Thus, in order to register a change $\Delta\varphi = 1 \times 10^{-7}$ rad in the amplitude of the oscillations, the resolution required on the film was 5×10^{-4} cm, which did not involve any special complications. The drum and film completed one revolution in seven days.

The upper part of Fig. 3 shows photocopies of two oscillation traces obtained with the described apparatus. It is easily seen that the system was "practically conservative" during the observation period of a few days.

The apparatus was placed in a passive foamed plastic thermostat (with $\approx 1.5 \times 10^{-2}$ stabilization coefficient) having an optical outlet for the recording system. The lower part of the vacuum chamber, where the balance was located, was shielded with a Permalloy magnetic shield. The experiment was performed in a basement room of Moscow State University that was thermally insulated very carefully (the temperature variations in the room during the course of a day did not exceed $\Delta T = 2 \times 10^{-2}$ °C).

The operator changed the film once every 2–4 days. The treatment of the film traces amounted to determining the magnitude of the harmonic having a daily period.

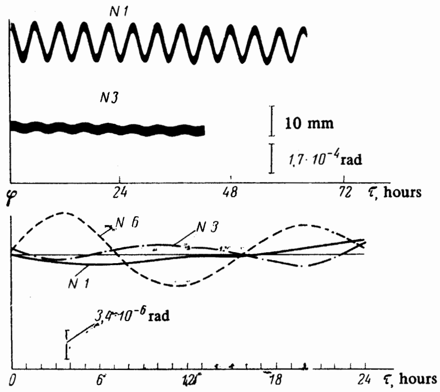

FIG. 3

The successive amplitudes of the characteristic oscillations were first obtained, and were used to plot a curve representing the average values for each half-period. In this way we excluded errors due to variation of the laser spot width on the film. A harmonic analysis then yielded the 24-hour harmonic. A monotonic drift of 4×10^{-6} rad/day was subtracted; this drift had been measured during one month.

Figure 3 shows the results obtained by a single operator for films Nos. 1, 3, and 6. Results obtained independently by three different operators were averaged. The average operator error was $\sim 3 \times 10^{-8}$ rad.

2. MEASUREMENTS AND SIMULATIVE EFFECTS

Our results are shown in the table, where the first column gives the number of the film and the second column gives the dates of the registration period. The third column gives the oscillation amplitudes $\Delta\varphi_i$ of the 24-hour harmonic (in radians); the average amplitude is $\overline{\Delta\varphi} = -0.55 \times 10^{-7}$ rad. The confidence interval $\pm 1.65 \times 10^{-7}$ rad was obtained by a statistical treatment of these results using Student's test at a 95% confidence level. Considering that $\tilde{\Delta} = 1 \times 10^{-12}$ corresponds to $\Delta\varphi = 1.8 \times 10^{-7}$ rad, we obtain

$$\tilde{\Delta} = (-0{,}3 \pm 0{,}9) \cdot 10^{-12} \tag{6}$$

with 95% confidence. The contribution to the confidence interval as a result of operator errors corresponds to $\tilde{\Delta} = 1 \times 10^{-13}$. It is easily calculated that with $\tau^* = 6 \times 10^7$ sec and measurement time $\hat{\tau} = 6 \times 10^5$ sec (7 days) we could achieve 5×10^{-13} resolution for $\tilde{\Delta}$ at the same confidence level. In our apparatus $\tau^* > 6 \times 10^7$ sec; therefore the attained level of resolution for (6) is not determined by the thermal fluctuations.

No.	$\Delta\hat{\tau}_i$	$\Delta\varphi_i \times 10^7$, rad
1	1.II—4.II *	+2.35
2	8.II—10.II	−0,40
3	10.II—12.II	−1,70
4	12.II ~ 13.II	+0,76 **
5	~ 14.II—15.II	−2,96 **
6	15.II—17.II	−0,10
7	19.II—21.II	−1,77

*All measurements were performed in 1971. Days are designated with Arabic numerals and months with Roman numerals.

**These values were obtained from the two halves of film bearing a continuous trace from 12.II to 15.II.

We shall now evaluate the principal effects that limited the sensitivity and that under our experimental conditions could simulate a nonzero value of $\tilde{\Delta}$.

A. Diurnal variations of the magnetic field in the laboratory induce a deflection of the balance with the amplitude

$$\Delta\varphi_{\text{mag}} \cong R\chi V(\widetilde{H\mathscr{H}}) I_0^{-1}\omega_0^{-2}, \tag{7}$$

where $\mathscr{H} \equiv \partial H/\partial R$, χ is the magnetic susceptibility, V is the volume of the masses, I_0 is the moment of inertia of the balance and $(\widetilde{H\mathscr{H}})$ is the amplitude of the diurnal change of $(H\mathscr{H})$. It was determined by means of control measurements that $(\widetilde{H\mathscr{H}})$ did not exceed 6 6×10^{-8} Oe^2/cm and that $\Delta\varphi_{\text{mag}}$ did not exceed 2×10^{-8} rad, thus corresponding to a simulation $\tilde{\Delta}_{\text{mag}} \approx 1 \times 10^{-13}$. It must also be taken into account that $(H\mathscr{H})$ was measured outside the Permalloy shield that reduced variations of the magnetic field around the balance.

B. The radiometric pressure in the presence of a temperature gradient between the opposite walls of the vacuum chamber induces a torsional moment that deflects the balance through an angle

$$\Delta\varphi_{\text{radiom}} \approx \frac{\Delta SpR}{4I_0\omega_0^2}\left(\frac{\widetilde{\Delta T}}{T}\right), \tag{8}$$

where ΔS is the difference between the geometric cross sections of the aluminum and platinum masses, p is the pressure, and $(\widetilde{\Delta T}/T)$ is the diurnal variation of temperature at the opposite walls of the chamber. The temperature change in the thermostat in one day was at most 5×10^{-4} °C. The difference of temperature in the opposite parts of the chamber was at least one order of magnitude smaller. Therefore $\Delta\varphi_{\text{radiom}} \approx 5 \times 10^{-8}$ rad, which corresponds to the overestimate $\tilde{\Delta}_{\text{radiom}} \approx 3 \times 10^{-13}$.

C. Local perturbations of the gravitational field gradients, induced, for example, when the operator approached the room containing the apparatus, could change the amplitude of balance oscillations by

$$\Delta\varphi_{\text{grav}} \approx \gamma Mm\Delta l\tilde{\tau}R/l^3 I_0\omega_0, \tag{9}$$

where l is the distance between the "suspended" mass and the balance, $\tilde{\tau}$ is the time during which the mass was "suspended," and Δl is the largest possible inaccuracy in the construction of the balance. Assuming $M = 10^5$ g, $\Delta l = 2 \times 10^{-2}$ cm, $l = 4 \times 10^2$ cm (the shortest possible distance of approach to the balance), and $\tilde{\tau} = 1 \times 10^3$ sec, we obtain $\Delta\varphi_{\text{grav}} \approx 7.5 \times 10^{-8}$ rad, which corresponds to $\tilde{\Delta}_{\text{grav}} = 4 \times 10^{-13}$. This result should be regarded as an overestimate, because "unfavorable" values of the parameters were substituted into (9).

D. Variations of the laser light beam pressure on the mirror of the balance lead to a diurnal variation of oscillation amplitude by

$$\Delta\varphi_{\text{light}} \approx \frac{2W\Delta a}{cI_0\omega_0^2}\left(\frac{\widetilde{\Delta W}}{W}\right), \tag{10}$$

where W is the power of the laser light reaching the mirror, Δa is the displacement of the center of the laser spot from the center of the mirror, $(\widetilde{\Delta W/W})$ is the diurnal relative variation of intensity, and c is the velocity of light. Under our conditions $(\widetilde{\Delta W/W})$ did not exceed 1×10^{-4}, $W \approx 0.3$ mW, and $\Delta a \lesssim 0.1$ cm; this corresponds to $\Delta\varphi_{\text{light}} \approx 5 \times 10^{-8}$ rad and $\tilde{\Delta}_{\text{light}} \approx 2.5 \times 10^{-13}$. We note that when an unstabilized laser was used in a preliminary series of measurements violation of the principle of equivalence was simulated at the level $\tilde{\Delta}_{\text{light}} \approx 6 \times 10^{-12}$.

E. Seismic perturbations cannot be evaluated by means of an analytic relation because of their nonstochastic character. The contribution of seismic shocks to the measurement error of $\tilde{\Delta}$ is determined, in our opinion, by the change in the amplitude of "pendulum" oscillations of the balance, which in combination with the light pressure (effect D) induces the largest error in $\tilde{\Delta}$. The effect of strong seismic shocks was taken into account in treating the results. Among all the values of $\Delta\varphi_i$ we discarded only one ($\Delta\varphi \approx 1 \times 10^{-5}$ rad), which was derived from measurements on February 7, 1971, because during this period the seismic station "Moscow" registered a relatively strong earthquake (of amplitude $\sim 170\ \mu$). All the other recording time intervals in the table did not coincide with relatively large seismic shocks. We note that the discarded value should have been excluded also according to Fisher's statistical criterion.

3. DISCUSSION OF RESULTS

Our result $\tilde{\Delta} = (-0.3 \pm 0.9) \times 10^{-12}$ is approximately one and one-half orders greater than the resolution achieved in the experiments of Dicke, Krotkov, and Roll. The resolution of $\tilde{\Delta}$ in our experiment is close to the strength ratio of the weak and strong interactions.[4] If, within the framework of our experimental scheme, by increasing τ^* we attempt to reduce the confidence interval for $\tilde{\Delta}$, it would obviously be necessary to reduce the effects that simulate a violation of the principle of equivalence.

We note in conclusion that the sensitivity limit of this experiment for $\tau^* \to \infty$ is determined by the quantum conditions (see [3] for more details)

$$\tilde{\Delta} \approx \frac{4}{g\hat{\tau}}\sqrt{\frac{\hbar\omega}{m}}. \tag{11}$$

This equation yields a value of $\tilde{\Delta}$ that is at least 10 orders smaller than our present result.

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87