# *TRANSFORMATION OF FIRST (SECOND) SOUND INTO FOURTH SOUND IN SUPERFLUID HELIUM*

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Normal incidence of sound waves from superfluid helium onto the surface of a porous body saturated with helium is considered. The porous body is assumed to consist of plane-parallel capillaries which are normal to the surface. The coefficient of transformation of first (second) sound into fourth sound, and also the reflection coefficient and the coefficient of transformation of the sounds at the surface are considered both for pure $He^4$ and for $He^3$—$He^4$ solutions.

As is well known,[1] two types of oscillations can propagate in superfluid helium: first and second sound. Moreover, near the walls, there exist rapidly decaying transverse waves that are characteristic for any viscous liquid (viscous waves).

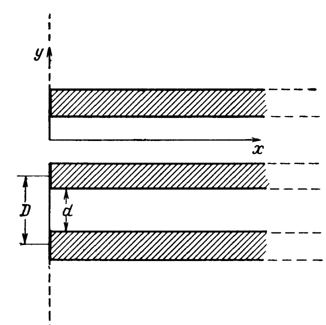
Pellam[2] and Atkins[3] called attention to the fact that the wave motion in helium changes significantly upon retardation of the normal component. In sufficiently thin capillaries, in which the normal component is retarded by friction at the walls, oscillations called fourth sound can be propagated along the superfluid component.[3] The velocity of fourth sound is equal to

$$u_4 = \left(\frac{\rho_s}{\rho} u_1^2 + \frac{\rho_n}{\rho} u_2^2\right)^{1/2},$$

where $u_1$ and $u_2$ are the velocities of first and second sounds, $\rho_n$ the density of the normal component, $\rho_s$ the density of the superfluid component, $\rho = \rho_n + \rho_s$. Measurements of the propagation velocity of fourth sound[4] have shown excellent agreement between theory and experiment.

The absorption of fourth sound is generally due to slippage of the normal component[5] (as a consequence of the finite viscosity). Sound propagation has also been studied in capillaries of finite width;[6] it has been shown that the velocity of fourth sound can be regarded as the limiting value of the velocity of first sound as $d/\lambda_v \to 0$ (here d is the width of the capillary and $\lambda_v$ is the length of the viscous wave).

Although experiments on the study of fourth sound propagations are always carried out on capillaries in which the oscillations are excited from free helium, all the calculations have been made only for infinite capillaries. In the present research we deal with the reflection of waves of first and second sound of frequency $\omega$ incident normally on a periodic system of plane-parallel capillaries filling a half space (see the drawing, which explains the choice of coordinate axes). Waves of fourth sound, the amplitudes of which are also computed here, are excited in the capillaries. In addition, the transformation of fourth sound into first and second upon emergence of the waves from the capillaries into free helium is investigated.




The problem is solved in the hydrodynamic approximation, which can be used if the free path length $l$ of the elementary excitations (phonons and rotons) is much smaller than the capillary width d ($l \ll d$). The width of the capillaries, according to the conditions of propagation of fourth sound, should be much smaller than the viscous wavelength $\lambda_v = \sqrt{2\eta/\omega\rho_n}$ ($\eta$ is the viscosity of the normal component, $\omega$ the sound frequency):

$$l \ll d \ll \sqrt{2\eta/\omega\rho_n}. \tag{1}$$

Condition (1) limits our consideration to not too high frequencies and not too low temperatures (for more detail, see[7]).

The wave processes in helium are described by a linearized set of hydrodynamic equations which, in the transition to "normal coordinates" can be written in the form of three independent equations

$$\begin{gathered} \Delta Q_1 + k_1^2 Q_1 = 0, \quad k_1^2 = \omega^2/u_1^2; \\ \Delta Q_2 + k_2^2 Q_2 = 0, \quad k_2^2 = \omega^2/u_2^2; \\ \Delta \mathbf{u} + k_3^2 \mathbf{u} = 0, \quad \operatorname{div} \mathbf{u} = 0, \quad k_3^2 = i\omega\rho_n/\eta, \end{gathered} \tag{2}$$

and all the oscillating quantities (the velocities of the superfluid and normal components $\mathbf{v}_s$ and $\mathbf{v}_n$, the deviation of the pressure P′ and temperature T′ from the equilibrium values) are described in the form of linear combinations of $Q_1$, $Q_2$, $\mathbf{u}$ and their derivatives:

$$\begin{gathered} \mathbf{v}_n = \nabla Q_1 + \nabla Q_2 + \mathbf{u}, \quad \mathbf{v}_s = \nabla Q_1 + P_2 \nabla Q_2, \quad P_2 = -\rho_n/\rho_s; \\ P' = M_1 Q_1, \quad M_1 = i\omega\rho; \\ T' = D_2 Q_2, \quad D_2 = i\omega\rho_n/\rho_s\sigma, \end{gathered} \tag{3}$$

whence it is seen that $Q_1$ and $Q_2$ are the velocity potentials in waves of first and second sound, and **u** is the velocity of the normal component in the viscous wave. In the definition of $k_1$ and $k_2$, we have neglected the dissipative components and in the definition of $k_3$, only the first viscosity of the dissipative processes is kept, since the first viscosity is the principal reason for the retardation of the normal component in the capillary and, consequently, for the transition from first and second sound to fourth sound.

Boundary conditions should be added to the set of equations (2)—(3), which are valid both in free helium and in capillaries. They are connected essentially with the properties of the solid from which the capillaries are made. We shall assume that the solid is an absolutely rigid, ideal heat insulator. Because of the great difference in the densities of the helium and the solid, the assumption of the absolute rigidity of the solid does not lead to additional restrictions, while the assumption of the absence of heat conduction in the solid is not too essential, since the heat effects play an insignificant role in the propagation of fourth sound (however, see[5]). The normal component of matter flux, the normal component of heat flux, and the tangential component of velocity of the normal (nonsuperfluid) component should vanish at an absolutely rigid, ideal heat insulator. From this follows the vanishing of all velocity components at the wall except the tangential component of the superfluid velocity.

The enumerated boundary conditions are of course sufficient for the exact solution of the problem. However, in such a setup, the problem is not solvable exactly. We shall make use of the fact that in the case of interest to us, the period D of the system of capillaries is much smaller than the characteristic wavelengths

$$k_1 D \ll 1, \quad k_2 D \ll 1, \quad |k_3| D \ll 1, \quad k_4 D \ll 1. \tag{4}$$

At large enough distances from the boundary ($|x| \gg D$), the structure of the solution is known to us. To the left, we have the incident and reflected waves of first and second sound:

$$\begin{gathered} Q_1 = C_1 e^{ik_1 x} + \tilde{C}_1 e^{-ik_1 x}, \\ Q_2 = C_2 e^{ik_2 x} + \tilde{C}_2 e^{-ik_2 x}, \quad \mathbf{u} = 0, \\ x < 0, \quad |x| \gg D, \end{gathered} \tag{5}$$

(here $C_{1(2)}$ is the amplitude of the incidence wave of first (second) sound, and $\tilde{C}_{1(2)}$ the amplitude of the reflected wave). To the right is the wave of fourth sound[1)]

$$Q_1 = C_4 e^{ik_4 x}, \quad Q_2 = \frac{\rho_s u_2^2 (u_1^2 - u_4^2)}{\rho_n u_1^2 (u_2^2 - u_4^2)} C_4 e^{ik_4 x}, \quad x \gg D \tag{6}$$

($C_4$ is the amplitude of fourth sound).

Near the boundary ($|x| \lesssim D$) the distribution of all the quantities is very complicated; it changes materially at distances of the order of d, D − d ("ripple"). This means that the spatial derivatives of $Q_1$, $Q_2$, and **u** are much larger than the corresponding values of $k_1^2 Q_1$, $k_2^2 Q_2$, and $k_3^2 \mathbf{u}$ (see Eq. (2) and the inequality (4)). In other words, the distribution of the velocities and other quantities near the boundary, after separation of the incident, transmitted, and reflected waves satisfies static equations. This statement requires explanation.

The exact solution in the capillary can be constructed out of damped traveling waves, in which the dependence on the coordinates has the following form:

$$\left\{ \begin{matrix} \sin k_{\perp i}{}^j y \\ \cos k_{\perp i}{}^j y \end{matrix} \right\} \exp(i k_{\parallel}{}^j x),$$

$$k_{\perp i}^{j2} = k_i^2 - k_{\parallel}^{j2}, \quad i = 1, 2, 3, \quad j = 1, 2, 3, 4, \ldots, \tag{7}$$

and the permissible values of $k_{\parallel}^j$ are found from the complex dispersion equation, which is introduced by starting from the boundary conditions on the walls of the capillary (see[5], Eq. (48)). As $k_i d \to 1$, (i = 1, 2, 3) one of the roots of this equation is the wave vector of fourth sound $k_4 = \omega/u_4$; for all the remaining wave vectors, $|k_{\parallel}^j| \gg k_i$ and all the waves corresponding to the wave vectors $k_{\parallel}^j \neq k_4$ decay rapidly in the interior of the capillary. This also means that the solution is a quasiparticle one after the separation of the wave of fourth sound and can be constructed from the solutions of Laplace's equation. Thus the desired solution is first, second, fourth sound and the "ripple"—the superposition of harmonics of the form

$$\left\{ \begin{matrix} \sin q_n y \\ \cos q_n y \end{matrix} \right\} e^{-q_n |x|}, \quad q_n = \left\{ \begin{matrix} 2\pi n/d \\ 2\pi n/D \end{matrix} \right. \tag{8}$$

(the upper line in the capillary, the lower in free helium), the amplitude of which is determined from the boundary conditions for x = 0. The conditions on the boundaries of the capillary are satisfied by choosing the $q_n$.

The significant consequence of the boundary conditions is the vanishing of the velocity of the normal component at x = 0 (in capillaries $v_n \equiv 0$, since $|k_i d| \ll 1$, and on the wall because of friction); this allows us to find the connection between the amplitudes of the incident ($C_1$, $C_2$) and the reflected waves of first and second sound;[2)]

$$(C_1 - \tilde{C}_1)/u_1 + (C_2 - \tilde{C}_2)/u_2 = 0. \tag{9}$$

Since the "quasistatic ripple" does not take part in the transfer of matter, the continuity of the x component of the material flux density through the capillary is assured only by the wave of fourth sound:

$$\frac{1}{u_1}(C_1 - \tilde{C}_1) - \frac{\rho_n}{u_2 \rho_s}(C_2 - \tilde{C}_2) = \frac{d}{D} C_4 \frac{u_4 (u_2^2 - u_1^2)}{u_1^2 (u_2^2 - u_4^2)}, \tag{10}$$

The missing boundary condition (one must determine three quantities: $C_1$, $C_2$, $C_4$) follows from the law of energy conservation. Equating the energy flux density in the incident waves to the flux density in the reflected and transmitted waves, we obtain the condition

$$\frac{1}{u_1}(C_1^2 - \tilde{C}_1^2) + \frac{1}{u_2}\frac{\rho_n}{\rho_s}(C_2^2 - \tilde{C}_2^2) = \frac{d}{D}\frac{\rho_s}{\rho_n}\frac{u_4^3 (u_2^2 - u_1^2)^2}{u_1^4 (u_2^2 - u_4^2)^2} C_4^2, \tag{11}$$

which, after simple transformations with the use of Eqs. (9) and (10), can be written in the much simpler form

$$C_1 + \tilde{C}_1 - \frac{\rho_n}{\rho_s}(C_2 + \tilde{C}_2) = C_4 \frac{u_4^2 (u_2^2 - u_1^2)}{u_1^2 (u_2^2 - u_4^2)}. \tag{12}$$

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1) The quantities $Q_1$ and $Q_2$, the superposition of which is fourth sound, generally depend on the coordinates in different fashion (owing to the different dependence on the coordinate y), but in the approximation of interest to us ($k_i d \ll 1$) this difference disappears.

2) From the "viewpoint" of normal motion, a body with capillaries behaves like a continuous medium.

The possibility of using the law of energy conservation is connected with the fact that the dissipative losses near the boundary are small; as $k_i d \to 0$, they tend to zero. Actually, in accord with[8], the dissipative function is, in order of magnitude,

$$\eta \frac{v_n^2}{d^2} = \frac{\omega\rho}{(k_3 d)^2} v_n^2.$$

But $v_n$ in the boundary layer is a quantity proportional to $(k_i d)^2$, which is not difficult to establish by considering the next approximation in $k_i d$. Consequently, the energy density dissipated per unit time in the boundary layer is proportional to $(k_i d)^2$.

The validity of Eqs. (9)–(12) can be proved by analysis of the exact boundary conditions.

Solving Eqs. (10)–(12), we obtain the values for the amplitudes:

$$\tilde{C}_1 = \frac{C_1}{\Xi}\left[\frac{u_4}{u_1}\rho + \frac{d}{D}\left(\rho_n \frac{u_2}{u_1} - \rho_s\right)\right] + \frac{2C_2}{\Xi}\frac{d}{D}\rho_n,$$

$$\tilde{C}_2 = \frac{2C_1}{\Xi}\frac{d}{D}\frac{u_2}{u_1}\rho_s + \frac{C_2}{\Xi}\left[\frac{u_4}{u_1}\rho + \frac{d}{D}\left(\rho_s - \frac{u_2}{u_1}\rho_n\right)\right],$$

$$C_4 = 2\frac{\rho u_1 (u_2^2 - u_4^2)}{\rho_s u_4 (u_2^2 - u_1^2)} \frac{C_1 \rho_s - C_2 \rho_n}{\Xi}, \qquad (13)$$

$$\Xi = \frac{u_4}{u_1}\rho + \frac{d}{D}\left(\rho_n \frac{u_2}{u_1} + \rho_s\right),$$

with the help of which we find the coefficients for the excitation of fourth sound from first sound $A_1 = d\gamma_4/D\gamma_1$ and from second sound $A_2 = d\gamma_4/D\gamma_2$, the reflection coefficients of first and second sound, $R_{11} = \tilde{\gamma}_1/\gamma_1$ and $R_{22} = \tilde{\gamma}_2/\gamma_2$, and the coefficients of transformation of first sound into second and second into first, $R_{12} = \tilde{\gamma}_2/\gamma_1$ and $R_{21} = \tilde{\gamma}_1/\gamma_2$ ($\gamma_i$ is the energy flux density in i-th sound, i = 1, 2, 4).

Taking it into account that $u_2 \ll u_1$ for not too low temperatures, we get

$$A_1 = 4\frac{d}{D}\sqrt{\frac{\rho_s}{\rho}}\left(1 + \frac{d}{D}\sqrt{\frac{\rho_s}{\rho}}\right)^{-2}, \quad A_2 = \frac{\rho_n}{\rho_s}\frac{u_2}{u_1}A_1, \quad R_{11} = 1 - A_1,$$

$$R_{22} = 1 - 4\frac{d}{D}\frac{\rho_n}{\rho_s}\frac{u_2}{u_1}\sqrt{\frac{\rho_s}{\rho}}\left(1 + \frac{d}{D}\sqrt{\frac{\rho_s}{\rho}}\right)^{-1},$$

$$R_{12} = R_{21} = 4\left(\frac{d}{D}\right)^2 \frac{u_2}{u_1}\frac{\rho_n}{\rho}\left(1 + \frac{d}{D}\sqrt{\frac{\rho_s}{\rho}}\right)^{-2}. \qquad (14)$$

It is seen that fourth sound is better excited by first sound than by second. In the immediate neighborhood of the λ point, this is evidently not so: $A_1|_{T \to T_\lambda} \to 0$, while $A_2|_{T \to T_\lambda} \to A_2 \neq 0$. However, $A_{2\lambda} \ll 1$ ($A_{2\lambda} \sim \sqrt{T_\lambda / u_1^2 M}$, where M is the mass of the $He^4$ atom). The sound transformation coefficients are small (they contain the factor $u_2/u_1$).

In a similar fashion, we can study the reflection of second sound from the end of the capillary and the excitation by it of first and second sound in free helium. In this case, one must assume as given the amplitude of fourth sound $C_4$ and determine $C_1$, $C_2$, and $\tilde{C}_4$, where $\tilde{C}_4$ is the amplitude of reflected fourth sound, $C_1$ and $C_2$ are the amplitudes of first and second sound:

$$\tilde{C}_4 = \frac{C_4}{\Lambda}\left[\frac{d}{D}\frac{\rho_s}{\rho}\frac{u_1}{u_4}\left(1 + \frac{\rho_n}{\rho_s}\frac{u_2}{u_1}\right) - 1\right],$$

$$C_1 = \frac{2C_4}{\Lambda}\frac{d}{D}\frac{\rho_s}{\rho}\frac{u_4}{u_1}\frac{(u_2^2 - u_1^2)}{(u_2^2 - u_4^2)}, \quad C_2 = \frac{-2C_4}{\Lambda}\frac{d}{D}\frac{\rho_s}{\rho}\frac{u_4}{u_2}\frac{(u_2^2 - u_1^2)}{(u_2^2 - u_4^2)},$$

$$\Lambda = \frac{d}{D}\frac{\rho_s}{\rho}\frac{u_1}{u_4}\left(1 + \frac{\rho_n}{\rho_s}\frac{u_2}{u_1}\right) + 1. \qquad (15)$$

Using these amplitudes, it is easy to find: $R = R_{11}$, $F_1 = A_1$, and $F_2 = A_2$, where R is the reflection coefficient of fourth sound from the end of the capillary, and $F_1$ and $F_2$ are the excitation coefficients of first and second sound from fourth sound.[3)]

The study of the transformation of waves in $He^3$–$He^4$ solutions is of interest. The velocity of fourth sound in $He^3$–$He^4$ solutions is the same as in pure $He^4$, and is expressed in terms of the velocities of first and second sound.[9] The square of the velocity of fourth sound is equal to

$$u_4^2 = \frac{\rho_s}{\rho}u_1^2(1+\beta)^2\left(1 + \frac{\rho_s}{\rho_n}\beta^2\right)^{-1} + \frac{\rho_n}{\rho}u_2^2\left(1 + \frac{\rho_s}{\rho_n}\beta^2\right), \qquad (16)$$

where $u_1$ and $u_2$ are respectively the velocities of first and second sound in $He^3$–$He^4$ solutions, $\beta = (c/\rho)\partial\rho/\partial c$ (c is the mass concentration of $He^3$).

Measurements of the velocity of fourth sound in a wide range of temperatures and concentrations[10] are in excellent agreement with those calculated by Eq. (16).

The wave processes in $He^3$–$He^4$ solutions, just as in pure $He^4$, are described by the linear set of equations (2). The velocities $v_n$ and $v_s$, the departures of the pressure, temperature and concentration in the sound field from their equilibrium values can be expressed in terms of $Q_1$, $Q_2$ and u (see, for example,[11]). However, in view of the fact that the connection between these quantities in the $He^3$–$He^4$ solution has a much more complicated form than in pure $He^4$, it is convenient to write down the boundary conditions in the variables P and T.

The condition (9) of the vanishing of the velocity of the normal component of the liquid on the boundary (in the variables P and T) for the $He^3$–$He^4$ solution is written in the following form:

$$\frac{1 - \rho_s\beta/\rho_n}{\rho u_1}(P_1 - \tilde{P}_1) + \frac{u_2}{\tilde{\sigma}}\frac{\partial\sigma}{\partial T}(1+\beta)(T_2 - \tilde{T}_2) = 0, \qquad (17)$$

where $P_1$ and $\tilde{P}_1$, $T_1$ and $\tilde{T}_1$ are the amplitudes of the oscillations of pressure and temperature in the incident and reflected waves of first and second sound, σ is the entropy per unit volume of the $He^3$–$He^4$ solution, and $\tilde{\sigma} = \sigma - c\partial\sigma/\partial c$.

The condition (10) of continuity on the boundary of the velocity of the superfluid component of the solution takes the form

$$\frac{1+\beta}{\rho u_1}(P_1 - \tilde{P}_1) - \frac{\rho_n}{\rho_s}\frac{\partial\sigma}{\partial T}\frac{u_2}{\tilde{\sigma}}\left(1 - \frac{\rho_s}{\rho_n}\beta\right)(T_2 - \tilde{T}_2) = \frac{d}{D}\frac{1+\beta}{\rho u_4}P_4, \qquad (18)$$

where $P_4$ is the amplitude of oscillation of the pressure in the wave of fourth sound.

The continuity on the boundary of the energy flux in the sound wave is written down in the form

$$\frac{1}{2\rho u_1}\left(1 + \frac{\rho_s}{\rho_n}\beta^2\right)(P_1^2 - \tilde{P}_1^2) - \frac{\rho\rho_n}{2\rho_s}\frac{1}{\tilde{\sigma}^2}\left(\frac{\partial\sigma}{\partial T}\right)^2 u_2^3\left(1 + \frac{\rho_s}{\rho_n}\beta^2\right)(T_2^2 - \tilde{T}_2^2)$$
$$= \frac{d}{D}\frac{\rho_s}{2\rho^2 u_4}(1+\beta)^2 P_4^2. \qquad (19)$$

3)We note that the formulas obtained here explain the ineffectiveness of the excitation of second sound by forcing helium through a porous diaphragm.

By using Eqs. (17)—(19), we can compute the excitation coefficients of fourth sound by first and second ($A_1$ and $A_2$), the reflection coefficients of first and second sound ($R_{11}$, $R_{22}$), the transformation coefficient of first sound into second ($R_{12}$) and of second into first ($R_{21}$). By taking it into account that $u_2 \ll u_1$ always in $He^3$—$He^4$ solutions, we get

$$
\begin{gathered}
A_1 = 4\frac{d}{D}\sqrt{\frac{\rho_s}{\rho}}\frac{1+\beta}{N}L^2,\\
A_2 = 4\frac{d}{D}\frac{\rho_n}{\rho_s}\frac{u_2}{u_1}\sqrt{\frac{\rho_s}{\rho}}\left(1-\frac{\rho_s}{\rho_n}\beta\right)^2\frac{1}{(1+\beta)N}L^2,\\
R_{11} = 1 - A_1,\\
R_{22} = 1 - 4\frac{d}{D}\frac{\rho_n}{\rho_s}\frac{u_2}{u_1}\sqrt{\frac{\rho_s}{\rho}}\left(1-\frac{\rho_s}{\rho_n}\beta\right)^2\frac{1}{(1+\beta)N}L, \qquad (20)\\
R_{12} = R_{21} = 4\left(\frac{d}{D}\right)^2\frac{\rho_n}{\rho}\frac{u_2}{u_1}\left(1-\frac{\rho_s}{\rho_n}\beta\right)^2\frac{L^2}{N^2};\\
N = \sqrt{1+\frac{\rho_s}{\rho_n}\beta^2}, \quad L = \left(1+\frac{d}{D}\sqrt{\frac{\rho_s}{\rho}}\frac{1+\beta}{N}\right)^{-1}
\end{gathered}
$$

As follows from the formulas that have been given, fourth sound is better excited in $He^3$—$He^4$ solution by first sound than by second, just as is the case for pure $He^4$. The coefficients of reflection and transformation of waves in $He^3$—$He^4$ solutions depend on the parameter $\beta = (c/\rho)\partial\rho/\partial c$, which is not small for highly concentrated solutions ($\beta \approx -0.3-0.4$).

The transformation coefficients of fourth sound into first and second sound, and the reflection coefficient of fourth sound can be computed in similar fashion. It turns out that for solutions, just as for pure $He^4$, $R = R_{11}$, $F_1 = A_1$, $F_2 = A_2$, where R is the reflection coefficient of fourth sound, and $F_1$ and $F_2$ are the coefficients of excitation of first and second sound by fourth.

Although Eqs. (14) and (20) were introduced for very special assumptions on the form and location of the capillaries, it is evident from the conclusion that they are valid under much more general assumptions. In particular, if the boundary plane $x = 0$ is homogeneous in the mean, then the replacement of d/D by the ratio of the total area of the capillaries to the entire surface allows us to make use of the formulas introduced in the case of capillaries of arbitrary shape. Finally, the radius R of curvature of the capillaries here (for straight capillaries, $R = \infty$) should be greater than the wavelength of fourth sound. Equations (14) and (20) can be used for the study of fourth sound by measurement of the coefficients of reflection and transformation.

The passage of sound through a set of capillaries of finite length can also be considered by the method suggested here; however, the formulas are somewhat more difficult to obtain.

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