PLASMA OSCILLATIONS IN AN ELECTRON-HOLE PINCH

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Submitted December 28, 1969

Zh. Eksp. Teor. Fiz. 59, 155–161 (July, 1970)

We consider finite-amplitude oscillations of the $m = 0$ mode in an electron-hole pinch. It is shown that nonlinearity does not stabilize the sausage modes in an intrinsic semiconductor. A stabilizing effect due to plasma drift in a longitudinal magnetic field does appear in extrinsic semiconductors.

1. INTRODUCTION

As is well-known, even the first observations of the pinch effect in an electron-hole plasma exhibited weak oscillations of the electric field.[1] These oscillations may be attributed to transient processes during the formation of the pinch, these processes being associated with periodic heating and cooling of the lattice.[2] Furthermore, it has been established experimentally by Ancker-Johnson[3] that real instabilities can appear in an electron-hole pinch, in the form of traveling density perturbations that are amplified as they propagate. In the main, these waves are sausage modes characterized by $m = 0$ with modulation amplitudes of approximately 10–20%. In the absence of a longitudinal magnetic field, modes characterized by $m = 1$ exhibit an extremely low amplitude.

Inasmuch as the oscillations are amplified along the length of the sample it is reasonable to associate them with a sausage instability, such as that which is well-known in gaseous plasmas, rather than with contact effects. It has also been shown by one of the present authors[4] that a similar instability appears in an electron-hole plasma even with unmagnetized carriers.

In the present work we consider finite-amplitude oscillations taking into account the stabilizing effect of impurities.

2. BASIC EQUATIONS

To be definite we consider a p-type semiconductor with an equilibrium concentration of holes p_0; it is assumed that the plasma is produced by injection of carriers from the cathode. By virtue of neutrality the concentrations of holes n_p and electrons n_e must satisfy the condition

$$n_p = n_e + p_0. \tag{1}$$

Although the concentration p_0 is small in the region of the pinch, as we shall see below, it must still be taken into account. The concentrations of holes and electrons satisfy the continuity equations

$$\frac{\partial n_e}{\partial t} + \operatorname{div}\left(-\mu_e n_e \mathbf{E} - D_e \nabla n_e + n_e \frac{\mu_e^2}{c}[\mathbf{EB}]\right) = 0, \qquad (2)^*$$

$$\frac{\partial n_p}{\partial t} + \operatorname{div}(\mu_p n_p \mathbf{E} - D_p \nabla n_p) = 0, \tag{3}$$

where μ_e and μ_p are the mobilities, D_e and D_p are the diffusion coefficients for electrons and holes respectively, $\mathbf{E}$ is the electric field, and $\mathbf{B}$ is the magnetic field. In the electron equations we take account of the Hall term under the assumption

$$\mu_e B / c \equiv \Omega_e \tau_e \equiv eB\tau_e / m_e c \ll 1,$$

and in the equations for the holes, which have a much lower mobility, in general we can neglect the magnetic field.

In (3), n_p is expressed in terms of n_e through (1). Then, multiplying (2) by μ_p and Eq. (3) by μ_e and adding, we obtain an ambipolar diffusion equation for the electron density n_e (for simplicity we omit the subscripts):

$$\frac{\partial n}{\partial t} = D_a \Delta n - \frac{\mu_a \mu_e}{c} \operatorname{div}(n[\mathbf{EB}]) - \mu_a p_0 \operatorname{div} \mathbf{E}, \tag{4}$$

where

$$\mu_a = \mu_e \mu_p / (\mu_e + \mu_p), \quad D_a = (\mu_e D_p + \mu_p D_e) / (\mu_e + \mu_p), \quad n \equiv n_p.$$

Under conditions of the pinch effect in a semiconductor with hole conductivity the electron current is much greater than the hole current so that

$$\operatorname{rot} \mathbf{B} = \frac{4\pi e}{c} \mu_e n \mathbf{E}, \tag{5}$$

i.e., we can neglect the hole current inside the pinch.[1)]

We now substitute $\mathbf{E}$ from (5) in (4). Confining ourselves to terms with azimuthal symmetry, that is to say, terms which are independent of the azimuthal coordinate θ in a cylindrical coordinate system r, θ and z, and assuming that there is no magnetic field, we write (4) in the form

$$\frac{\partial n}{\partial t} = D_a \Delta n + \frac{\mu_a I_0^2}{2\pi c^2 r^2}\left\{r \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \chi^2}{\partial r}\right) + \frac{\partial^2 \chi^2}{\partial z^2}\right\} + \frac{Q}{n^2 r}\left\{\frac{\partial \chi}{\partial r}\frac{\partial n}{\partial z} - \frac{\partial \chi}{\partial z}\frac{\partial n}{\partial r}\right\}, \tag{6}$$

where I_0 is the pinch current

$$\chi = \frac{c}{2I_0} rB, \quad Q = \frac{I_0}{2\pi e}\frac{\mu_a p_0}{\mu_e} = \text{const}.$$

In what follows we only consider perturbations with wavelength much greater than the radius of the pinch, because it is only these perturbations that are observed experimentally (furthermore, the linear approximation indicates that shortwave perturbations are stabilized by diffusion[4]). For these perturbations we need only consider the z-component of (5):

$$\frac{1}{r}\frac{\partial \chi}{\partial r} = \frac{2\pi e}{I_0} \mu_e n E_z. \tag{7}$$

Furthermore, from the equation of continuity for the

*$[\mathbf{EB}] \equiv \mathbf{E} \times \mathbf{B}$.

1)In Eq. (5) we have also neglected the diffusion component of the current; this is permissible when $\Omega_e \tau_e \ll 1$.

electric current $\operatorname{div} n\mathbf{E} = -\operatorname{div}(n\nabla\varphi) = 0$, where φ is the potential, it follows that for long-wave perturbations in the zeroth approximation $\partial\varphi/\partial r = 0$, that is to say, E_z can be assumed to be independent of r. Integrating this equation with respect to r (after first multiplying it by r) we find that the solution condition for the following approximation is the relation $E_z(z)N(z) = \text{const} = E_0N_0$. Here, E_0 is the mean longitudinal field, N(z) is the running density

$$N(z) = \int_0^\infty 2\pi rn\,dr, \tag{8}$$

and N_0 is the average value over the length of the pinch. Writing $I_0 = \mu_e E_0 e N_0$ we can now cast Eq. (7) in the form

$$\partial\chi / \partial r = 2\pi nr / N. \tag{9}$$

Equations (6) and (9) are sufficient for the description of long-wave plasma oscillations of an electron-hole pinch. We note that the function χ is of order unity (it is exactly equal to unity when $r = \infty$). Taking this fact into account it is easy to show that the last term in (6) (as compared with the next-to-last term) is of order $(p_0/n)(\Omega_e\tau_e)^{-1}$. Inasmuch as the quantity $\Omega_e\tau_e$ is usually smaller than unity even for modest currents in an electron-hole pinch, then even for relatively small values of p_0/n the last term in (6) can be important.

3. SAUSAGE INSTABILITIES IN AN INTRINSIC SEMICONDUCTOR

First we consider the case of an intrinsic semiconductor $p_0 = 0$, i.e., $Q = 0$ in (6). For oscillations characterized by long wavelengths in z this equation can be solved by perturbation theory. For this purpose we write it in the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(D_a r\frac{\partial n}{\partial r} + \frac{\mu_a I_0^2}{2\pi c^2}\frac{1}{r}\frac{\partial\chi^2}{\partial r}\right) = \frac{\partial n}{\partial t} - D_a\frac{\partial^2 n}{\partial z^2} - \frac{\mu_a I_0^2}{2\pi c^2}\frac{\partial^2\chi^2}{\partial z^2} \tag{10}$$

and assume that the right side is a small perturbation.

In the zeroth approximation the right side is set equal to zero. Multiplying (10) by r^3dr and integrating between 0 and ∞ we obtain the familiar pinch condition

$$I_0^2 = \frac{2D_a}{\mu_a}c^2N_0 = 2(T_e + T_p)c^2N_0, \tag{11}$$

where T_e and T_p are the carrier temperatures. It is evident from (11) that equilibrium is possible only if the running density N_0 is independent of z.

Furthermore, from (10) for the zeroth approximation and (9) we find the equilibrium Bennett distribution for the functions n_0 and χ_0 of the zeroth approximation:

$$n_0 = \frac{N_0}{\pi a^2}f\left(\frac{r}{a}\right), \quad \chi_0 = g\left(\frac{r}{a}\right), \tag{12}$$

where the functions f and g are given by

$$f(\rho) = \frac{1}{(1+\rho^2)^2}, \quad g(\rho) = \frac{\rho^2}{1+\rho^2}, \quad \rho \equiv \frac{r}{a}. \tag{13}$$

It will be evident that the zeroth approximation has an arbitrary parameter a, the radius of the pinch; this will be assumed to be a slowly varying function of z and t. The dependence of this quantity on z and t must be determined from a higher approximation.

In the next approximation, on the right side of (10) we substitute the functions obtained in the zeroth approximation, n_0 and χ_0. In this case, since the time derivative is reasonably expected to be a first-order quantity in the ratio of the radius of the pinch to the wavelength, the second and third terms on the right side of (10) can be neglected, that is to say,

$$D_a\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n_1}{\partial r} + \frac{2N_0}{\pi}\frac{1}{r}\frac{\partial}{\partial r}(\chi_0\chi_1)\right) = \frac{\partial n_0}{\partial t}. \tag{14}$$

Here, we have used the pinch equilibrium condition (11). To first order, (9) assumes the form

$$n_1 = \frac{N_0}{2\pi}\frac{1}{r}\frac{d\chi_1}{dr} + \frac{N_1}{2\pi}\frac{1}{r}\frac{d\chi_0}{dr}, \tag{15}$$

where N_1 is the perturbation of the running density. Substituting (12) and (15) in (14) we obtain for χ_1 an equation that can be solved by quadratures:

$$\chi_1 = \frac{\rho^2}{(1+\rho^2)^2}\int_{\rho^2}^{R^2/a^2}\int_0^x \left[\frac{a}{2D_a}\frac{\partial a}{\partial t}\frac{x'}{(1+x')^2} - 2\frac{N_1}{N_0}\frac{x'}{(1+x')^3}\right]dx'\left(1+\frac{1}{x}\right)^2 dx, \tag{16}$$

where we have made use of the boundary conditions $\chi_1 = 0$ for $r = 0$ and $r = R$, where R is the radius of the sample. The second condition expresses the conservation of current through the sample and can also be obtained by integration of Eq. (15), multiplied by rdr, from 0 to R. When $R \gg a$, we have to logarithmic accuracy from (16)

$$\chi_1 = \frac{R^2}{a^2}\frac{\rho^2}{(1+\rho^2)^2}\left\{L\frac{a}{D_a}\frac{\partial a}{\partial t} - \frac{N_1}{N_0}\right\}, \tag{17}$$

where $L = \ln(R/a) \approx \text{const}$. We see that when $R/a \gg 1$ the expression for χ_1 is very large. For perturbation theory to hold, we require $\chi_1 \ll \chi_0$, that is to say, the expression in brackets in (17) must be small; hence, we obtain the following relation between the time derivative of a^2 and $N_1 \equiv N - N_0$:

$$\frac{\partial a^2}{\partial t} = \frac{2D_a}{L}\frac{N-N_0}{N_0}. \tag{18}$$

Equation (18) is a solution condition for the next higher approximation in perturbation theory.

We now consider the next approximation. Actually, in the determination of the next approximation for n_2 and χ_2 we cannot use (9) since it is necessary to take account of terms containing φ_1. However, the second-order corrections n_2 and χ_2 are not of direct interest in themselves. For the present purposes it is adequate to use the solution condition for (10), which reduces to the vanishing of the integral of the right side of (10), multiplied by r, with respect to r. Substituting the zeroth-approximation functions in the second and third terms, we obtain the following equation:

$$\frac{\partial N}{\partial t} = -D_aN_0\frac{\partial^2}{\partial z^2}\ln a^2. \tag{19}$$

In obtaining this equation we have taken account of the fact that in the zeroth approximation $N = N_0 = \text{const}$; also, the second derivative with respect to z cannot be taken out from under the integral sign in the last integral in (10), since χ^2 does not vanish at infinity.

Substituting N from (18) in (19) we obtain a single equation for the radius of the pinch a:

$$\frac{\partial^2 a^2}{\partial t^2}=-\frac{2D_a{}^2}{L}\frac{\partial^2}{\partial z^2}\ln a^2. \quad (20)$$

In the linear approximation, for perturbations of the form $\exp(\gamma t + ik_z z)$, we then obtain the growth rate

$$\gamma=\left(\frac{2}{L}\right)^{1/2}\frac{D_a k_z}{a}. \quad (21)$$

As we have assumed, the growth rate is proportional to k_z.

Equations (18) and (19) provide a qualitative picture of the development of the sausage instability into the nonlinear stage, in which the modulation amplitude of the pinch radius becomes of the same order as the pinch itself. In the nonlinear stage the logarithm is a slowly varying function so that the right side of (19) can be regarded as constant, i.e., $\delta N = N - N_0$ will be a linear function of t. Correspondingly, in accordance with Eq. (18), a^2 will execute uniform accelerated motion and the pinch will contract to zero radius in the region of the sausage within a finite time. By that time the depth of modulation of the density $\delta N = N - N_0$, being small (of order $k_z a$), will still remain small.

Under actual conditions, of course, the pinch does not contract to zero radius, because a number of effects that have not been considered in our analysis come into play; these include recombination of carriers, heating, ionization, etc. Hence, under actual conditions we expect certain oscillations, possibly irregular ones, in which the amplitude of modulation of the density will be the smaller, the smaller the ratio of pinch radius to wavelength.

4. SLOW OSCILLATIONS IN EXTRINSIC SEMICONDUCTORS

We now consider the role of the last term, proportional to the hole concentration p_0, in (6).

We first consider the limiting case in which the last term in (6) is much greater than the other terms, making use of the method of successive approximations. In the zeroth approximation we retain only the last term; setting this term equal to zero we have $\chi = \chi(n)$. But then it follows from (9) that n is a function only of $s = r^2/N(z)$:

$$n=n(s), \quad d\chi/ds=\pi n(s). \quad (22)$$

In order to obtain the next approximation we substitute $n = n(s)$ and $\chi = \chi(s)$ in the other terms of (6) and replace the variable r by the variable s. The quantities n_1 and ψ_1 in the next approximation are related by (9), so that n_1 can be expressed in terms of χ_1:

$$n_1=\pi^{-1}d\chi_1/ds.$$

After this substitution, the last term of (6) can be reduced to the following form in a first approximation:

$$\frac{2Q}{N}\frac{\partial^2}{\partial z\,\partial s}\left(\frac{\chi_1}{n(s)}\right),$$

where $n(s)$ is the density in the zeroth approximation.

We assume that $N(z)$, n_1, and χ_1 are periodic functions of z. Then if we multiply (6) of the first approximation by $N(z)$ and average over the period, the last term vanishes since it is equal to the derivative of a periodic function with respect to z. Furthermore, the last term vanishes upon integration over s. Thus, we obtain two equations for the functions in the zeroth approximation

$$\frac{\partial n}{\partial t}=\frac{4D_a}{N_0}\frac{\partial}{\partial s}\left(s\frac{\partial n}{\partial s}\right)+\frac{2\mu_a I_0{}^2}{\pi c^2 N_0}\left\langle\frac{1}{N}\right\rangle\frac{\partial^2\chi^2}{\partial s^2}, \quad (23)$$

$$\frac{\partial N}{\partial t}=D_a\frac{\partial^2 N}{\partial z^2}-\frac{\mu_a I_0{}^2}{2c^2}\frac{\partial^2}{\partial z^2}\ln N, \quad (24)$$

where the angle brackets denote averages over z. In the derivation of (23) we have neglected small terms proportional to the square of the ratio of the radius of the pinch to the longitudinal wavelength. Equation (23) is of exactly the same form as the equation for a pinch that is uniform along z, the only difference being that the quantity $1/N_0$ is replaced by $\langle 1/N\rangle$ in the last term. Since $\langle 1/N\rangle\langle N\rangle = \langle 1/N\rangle N_0 > 1$, in the presence of modulation of N, in accordance with Eq. (23), the equilibrium current I_0 must be somewhat smaller than is indicated by the pinch condition (11).

For a given current I_0, Eq. (24) for N is not connected at all with (23). In the linear approximation for the equilibrium pinch, in which the current I_0 is related to the density N_0 by the pinch equilibrium condition (11), Eq. (24) leads to a stationary perturbation with frequency precisely equal to zero. In the case of finite-amplitude oscillations the second term in (24) is reduced compared with the first since the quantity ln N varies more slowly than N; furthermore the equilibrium current [in accordance with (23)] is much smaller than in the cylindrical case (11). Consequently, the nonlinear oscillations are damped.

Thus, we have shown that there is a complete stabilization of the pinch under conditions of a rather high equilibrium hole density p_0. Under these conditions it is possible to have only very slowly damped (in the linear approximation, stationary) perturbations of the running density $N(z)$. For these perturbations, the distribution density varies along the pinch in a similar fashion: $n = n(r^2/N(z))$. Under these conditions the running density varies in phase with the radius of the pinch, as in the case $Q = 0$.

5. DISCUSSION OF THE GENERAL CASE

Having the results for the limiting cases $Q = 0$ and $Q = \infty$ we can now discuss qualitatively the case of arbitrary Q. First we note that under conditions of the pinch effect the actual parameter that characterizes the role of the last term in (6) is the quantity

$$\lambda=\frac{p_0}{n}\frac{c}{\mu_c B}=\frac{p_0}{n}\frac{1}{(\Omega_c\tau_c)}. \quad (25)$$

In fact, compared with the second term in (6), the last term contains in addition to containing λ also one small parameter $k_z a$. However, as is evident even from (18) and (19), we are not interested in the second term itself, but the part that is not in equilibrium with respect to z and is proportional to $k_z a$.

Thus, the intermediate case between two limiting cases is the case $\lambda \sim 1$. This case is extremely complicated for an analytic treatment, so that we shall discuss it only qualitatively.

We note first that for a specified running density N the parameter λ is a strong function of the pinch radius $\lambda \sim a^2$. This means that in the case $\lambda \sim 1$ the expansion of the pinch carries it into a region of stability, while compression brings it into a region of instability; hence, the development of the instability is a case of "hard" excitation, that is to say, a strong sausage on a pinch cannot be stabilized by p_0 if p_0 remains constant. However, it is possible that the amplitude of the perturbation is so small that strong sausage instabilities do not occur. In this case the stabilizing (last) term in (6) will act in the following way. At the axis itself, taking account of (9), we find this term to be of the form $(2\pi Q/Nn)\,\partial n/\partial z$, that is to say it describes the drift transfer of the density perturbations in the unneutralized plasma, which is well-known in semiconductor physics.[5,6] In the linear approximation we can obtain a transfer rate $v_0 = -2\pi Q/Nn$ in the direction from the cathode to the anode (in a p-type semiconductors), whereas in the nonlinear approximation we can have the effect of breaking of the density-perturbation wave. Assuming that perturbations with $k_z a > 1$ are damped by diffusion[4], we might expect stabilization of finite-amplitude perturbations. However, as λ increases the amplitude of the stationary wave should decrease; for very large λ there must be a complete stabilization of the waves traveling with the drift velocity v_0, because this velocity depends on r, so that perturbations with different r travel with different velocities, in which case transverse diffusion causes quenching. Under these conditions it is only possible to have almost stationary perturbations, which have been considered in the preceding section. In some range $\lambda \gtrsim 1$ one then expects steady-state oscillations with finite but modest amplitude, and is observed experimentally.

6. CONCLUSION

Thus, we have shown that intrinsic semiconductors can support sausage type instabilities which, within the framework of the diffusion analysis (neglecting recombination, generation, and heating of the carriers), must lead to collapse of the sausage. In an extrinsic semiconductor there is a stabilizing effect associated with the plasma drift in the electric field. In a p-type semiconductor the characteristic parameter that determines the magnitude of this effect is the quantity $\lambda = p_0 c/n\mu_e B$, where p_0 is the equilibrium density of holes, n is the density of carriers in the pinch, and μ_e is the electron mobility. When $\lambda \gg 1$ the pinch is completely stable and can only exhibit small stationary perturbations of the pinch radius with pinch density constant along the axis. When $\lambda \gtrsim 1$ it is possible to have small stationary perturbations in the form of waves that travel from cathode to anode. When $n_0 c/p\mu_p B \gg 1$ a similar stabilization effect is to be expected in n-type semiconductors. However, since rather large currents are generally used in this case, it cannot be observed experimentally.

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Translated by H. Lashinsky
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