EFFECT OF OPTICAL PHONONS ON THE SHAPE OF THE CYCLOTRON RESONANCE LINE IN SEMICONDUCTORS

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A theory is developed for the shape of the cyclotron resonance line due to the interaction between electrons and longitudinal optically polarized phonons. For temperatures below the Debye temperature and magnetic fields such that the cyclotron frequency $\Omega \approx \omega_0$ (phonon frequency), the theory predicts a strong broadening of the absorption line (if $\Omega > \omega_0$). The line becomes much narrower at frequencies located to the left of the region of total internal reflection. The cases of Fermi and Boltzmann statistics are considered. It is shown that for $\Omega > \omega_0$ the mass operator used for describing the line shape can be calculated by perturbation theory involving the electron-phonon coupling constant; for $\Omega < \omega_0$, other line-broadening mechanisms must be taken into account. The theory is compared with available experimental data.

INTERESTING experiments have been carried out at the present time in which the form of the cyclotron resonance lines in InSb has been studied in quantized magnetic fields such that the cyclotron frequency (Ω) is close to the frequency of the longitudinal optical phonons (ω_0).[1,2] It has been established that the width of the absorption line depends on the value of the magnetic field. As $\Omega \to \omega_0$ from the side of high magnetic fields, the width of the line increases sharply (several fold) in comparison with the width far away from ω_0. This effect, predicted by Harper,[3] is connected with the increase in the probability of decay of the electron level with quantum number $N = 1$ to an optical phonon and the level with $N = 0$. Such a decay can come about only if $\Omega \geq \omega_0$, and it is especially intense for the case $\Omega = \omega_0$, since the electron transition takes place between regions with higher density of states in both Landau levels. If $\Omega < \omega_0$, then decay with emission of an optical phonon is forbidden by the law of conservation of energy. Therefore, for temperatures below the Debye temperature, when the optical phonons are not excited, this mechanism of broadening does not occur and the line is narrowed (see also[4]).

As is known,[5,6] the interaction of electrons with optical phonons leads to the splitting of the interband magneto-optical oscillations, while the more interesting region of magnetic fields is $\Omega = \omega_0$, since then two split-off peaks are very clearly visible. In cyclotron resonance, it has not been possible to observe two peaks simultaneously, since, in the case $\Omega = \omega_0$, a band of total internal reflection extending from the light frequencies $\omega \cong \omega_t$ (frequency of the transverse optical phonon) to $\omega \cong \omega_0$ is superposed on the spectrum. In the present communication, results are given of a theory of the shape of the cyclotron resonance line with systematic account of the interaction of the electron with the longitudinal optical vibrations of the lattice.

1. CHOICE OF A MODEL AND FUNDAMENTAL EQUATIONS

We consider a semiconductor of cubic symmetry, with a small fraction of ionic bonding (A_3B_5 type), placed in a magnetic field directed along one of the axes of the crystal and satisfying the conditions $\hbar\Omega \gg T$ and $\Omega \cong \omega_0$. If these conditions are satisfied, then the electrons populate only the lowest Landau level ($N = 0$), the optical phonons are practically not excited, and the entire effect is connected with the spontaneous emission of phonons. All the calculations are carried out in an approximation that is linear in the electron concentration. For the determination of the complex electrical conductivity tensor $\sigma_{\mu\nu}$, it is convenient to use the matrix densities $f^{\nu}_{\gamma\gamma'}$, connected with $\sigma_{\mu\nu}$ by the relation

$$\sigma_{\mu\nu} = \sum_{\gamma\gamma'} j^{\mu}_{\gamma\gamma'} f^{\nu}_{\gamma\gamma'}, \quad \mu, \nu = x, y, z, \tag{1}$$

where $f^{\mu}_{\gamma\gamma'}$ is the matrix element of the current density operator, and γ is the quantum number of the electron in the magnetic field (N_γ, k^{γ}_{z}, k^{γ}_{x}). The function $f^{\nu}_{\gamma\gamma'}$ in both cases is a solution of the kinetic equation. However, in the model chosen, the lowest electron level is stationary (there are no optical phonons) and the shape of the line is determined by the nonstationarity of the upper level only. In this case, the kinetic equation reduces to an algebraic one. Then,

$$f^{\nu}_{\gamma\gamma'} = \frac{ij^{\nu}_{\gamma'\gamma} n_\gamma (\hbar\omega_{\gamma'\gamma})^{-1}}{\omega - \omega_{\gamma'\gamma} - iW + is}, \quad s \to +0, \tag{2}$$

n_γ is the mean occupation number of the electrons, ω is the light frequency, $\omega_{\gamma'\gamma} = \omega_{\gamma'} - \omega_\gamma$, and $\hbar\omega_\gamma$ is the energy of the electron in a magnetic field. The function W determines the absorption-line shift and shape. The interaction Hamiltonian of the electrons with phonons is written in the form

$$\mathcal{H}=\sum_{q}\sum_{\alpha\alpha'}[C_q J_{\alpha\alpha'}(q)\,b_q+\text{h.c.}]\,a_\alpha^+ a_{\alpha'}, \tag{3}$$

$J_{\alpha\alpha'}(q)$ is the matrix element of the operator $\exp(iq\cdot r)$, computed with the wave functions of the electron in the magnetic field, a_α^+ and a_α are the creation and annihilation operators of the electron in the state α, and b_q^+ and b_q are the corresponding operators for phonons with wave vector q. The coupling constant of electrons with phonons is conveniently represented in the form

$$|C_q|^2=(\hbar\omega_0)^2(8\pi\alpha_0 l_0^3)\,(l_0 q)^{-2},$$
$$l_0^2=\hbar/2m\omega_0,\quad \alpha_0=e^2(4\hbar\omega_0 l_0)^{-1}(\varepsilon_\infty^{-1}-\varepsilon_0^{-1}), \tag{4}$$

m is the effective mass of the electron in the conduction band, e its charge, ϵ_0 and ϵ_∞ the static and high-frequency dielectric constants, α_0 the dimensionless coupling parameter; the normalized volume is everywhere set equal to unity. The dispersion law of the electron is assumed to be quadratic: $\omega_\gamma=\Omega(N+1/2)+\hbar(k_z\gamma)^2/2m$, which always holds for small k_z, which are the only ones considered in the problem. In order to avoid absorption at the plasma frequency, which can, in certain cases, be close to Ω, the measurements were usually carried out in the Faraday configuration, when the electromagnetic wave is directed along the magnetic field, and has right-hand polarization.[2] In the case in which the interaction of the electrons with the phonons is small, which occurs in compounds of the A_3B_5 type, the absorption coefficient is

$$K(\omega)=[2\pi/cn_0]\,\mathrm{Re}\,\sigma, \tag{5}$$

where n_0 is the index of refraction, c the speed of light in vacuum, and $\sigma=\sigma_{xx}+i\sigma_{yx}$.

2. ANALYSIS OF THE PERTURBATION-THEORY SERIES FOR THE FUNCTION W

For the calculation of the function W, it is convenient to use the method developed in[7]. The function W is represented by an infinite series of diagrams. For their analysis, one should take it into account that of all the modified diagrams, those should be kept in which all the electron lines are directed along the ordering contour. By the same token, the concentration corrections to W are not taken into account, it being assumed that they are small. Moreover, the interaction of the electrons with phonons should be taken into account only in the electron line with N = 1, i.e., on the upper horizontal portion of the ordering contour. The diagrams with phonon lines on the lower horizontal portion of the contour are small in the absence of phonons, since they do not contain resonance denominators. The simplest diagram corresponds to

$$W_1=(i\hbar)^{-2}i\sum_{q\sigma}\frac{|C_q|^2 J_{\alpha\delta}^*(q)J_{\alpha\delta}(q)\,(1-n_\delta)}{\omega-\omega_{\delta\gamma}-\omega_0+is}. \tag{6}$$

For what follows, it suffices to take into account only two Landau levels with quantum numbers N = 1 and N = 0, since a resonant transition of an electron with emission of a phonon is only possible between them. The remaining, non-resonant terms appearing in (6) can be removed by renormalization of the cyclotron frequency and the frequency of the optical phonon, as is done in[8]. (The diagrams which must be summed here are shown in Fig. 1b.)[1)]

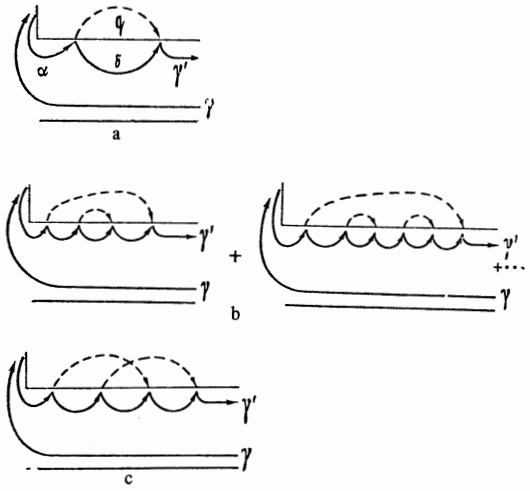

FIG. 1. Diagrams determining the function W: a—diagram which is important in the range of frequencies $\gamma\gtrsim\alpha_0^2$; b—diagram separated by renormalization of Ω and ω_0; c—diagram which becomes important in the region $\gamma<\alpha_0^2$.

The resonance term in (6), with accuracy up to terms $\sim\alpha_0^{1/3}$ is equal to

$$W_1=-\alpha_0\omega_0(\omega_0/\Omega)^{1/2}(\gamma+k^2+is)^{-1/2}, \tag{7}$$

where

$$\gamma=(\omega-\omega_0)/\Omega,\quad k^2=k_z^2R^2/2,\quad R=(c\hbar/eH)^{1/2}.$$

As is seen from (7), W_1 becomes large if $k\to 0$ and $\gamma\to 0$. The next diagram, which cannot be removed by renormalization (Fig. 1c) gives

$$W_2\sim i\alpha_0^2(\gamma+k^2+is)^{-1}. \tag{8}$$

For $\gamma\cong\alpha_0^2$ (and small k^2), $W_2\cong W_1$. Thus, in the region of frequencies $\gamma>\alpha_0^2$, the function W can be replaced by W_1 and perturbation theory is inapplicable for the region $\gamma<\alpha_0^2$. It will be shown below that the second frequency range is unimportant.

3. CASE OF FERMI STATISTICS

Let us compute σ in the very simple and lucid case in which the distribution of electrons in the lower Landau level can be replaced by a step. To this end, one must retain n_δ in the resonance term of the series (6), so that it "eliminates" the more dangerous portion of the integration, where the integrand is singular. As a result, we get

$$W_1=-\Omega\left(\frac{\omega_0}{\Omega}\right)^{3/2}\frac{\alpha_0}{\sqrt{\gamma+k^2+is}}\left[1-\theta(x_0-\sqrt{\gamma+k^2})-\frac{i}{\pi}\ln\frac{x_0+\sqrt{\gamma+k^2}}{|x_0-\sqrt{\gamma+k^2}|}\right],\quad k^2+\gamma>0,$$
$$W_1=i\Omega\left(\frac{\omega_0}{\Omega}\right)^{3/2}\frac{\alpha_0}{\sqrt{-\gamma-k^2-is}}\left[1-\frac{2}{\pi}\operatorname{arctg}\frac{x_0}{\sqrt{-\gamma-k^2-is}}\right],\quad k^2+\gamma<0, \tag{9}$$

[1)] Making use of the occasion, we note that in [8] the last term of Eq. (10) should have the form $\sum_{n=2}^{\infty}\Omega\eta/(n-1)\sqrt{n}$, so that this sum is finite. Therefore, the constant that renormalizes the energy is finite and does not depend on the upper bound of the allowed energy band. The details of calculation of the diagram under study are given in [9].

$x_0 = 2^{-3/2}(2\pi)^2R^3n$, n being the electron concentration, $\theta(y) = 1$ for $y > 0$ and $\theta(y) = 0$ for $y < 0$, and by the root in the second formula we mean the branch with a positive real part. Substituting (2) in Eq. (1) and taking it into account that, first, the matrix elements of the current density are proportional to the delta functions $\delta(k_z^\gamma - k_z^{\gamma'})\delta(k_x^\gamma - k_x^{\gamma'})$, and, second, the function n_γ in (2) is a step function, we put σ in the form

$$\sigma = \frac{i\cdot 2\sqrt{2}\,e^2}{(2\pi)^2 mR^3\Omega\eta^{1/2}}\int_0^{y_0}\frac{dx}{\Gamma + V(y_0, M, x)}, \qquad (10)$$

where

$$\Gamma = \frac{\omega-\Omega}{\Omega\eta^{2/3}},\quad y_0 = \frac{x_0}{\eta^{1/2}},\quad M = \frac{\gamma}{\eta^{2/3}},\quad x = \frac{k}{\eta^{1/2}},\quad \eta = \alpha_0\left(\frac{\omega_0}{\Omega}\right)^{3/2},$$

while the function $V(y_0, M, x)$ is obtained from (9), if we transform there to the variables y_0, M, and x:

$$V(y_0, M, x) = -i\Omega^{-1}W_1(y_0, M, x).$$

The experiments mentioned above were carried out for small concentrations of conduction electrons in strong magnetic fields, such that v_0^2 changed within the range $10^{-2} - 10^{-5}$. Therefore, if we limit ourselves to the frequency range $|M| > y_0^2$, then we can neglect the dependence $V(y_0, M, x)$ on y_0 and x. Then

$$V(M) = \begin{cases} i(M+is)^{-1/2}, & M>0,\\ (|M|-is)^{-1/2}, & M<0,\end{cases}\quad s\to+0. \qquad (11)$$

This approximation is equivalent to the neglect of the quantity n_δ in Eq. (6). It is known to be valid when the dimensionless Fermi level, calculated from the bottom of the Landau level with $N = 0$, $y_0^2 \lesssim \alpha_0^{4/3}$, since the replacement of W by W_1 is not valid for small M. Furthermore, it must be kep in mind that small M are generally not achieved, because of the band of total internal reflection of the crystal. It follows from (11) that, if $M < 0$, then $V(M)$ is real and in this range of frequencies the absorption line has a delta-like shape. Substituting (11) in (10), and separating Re σ, we finally obtain

$$\mathrm{Re}\,\sigma = \frac{e^2n}{m\omega_0\alpha_0^{3/2}}\begin{cases}\dfrac{\sqrt{\varkappa}}{\varkappa(\varkappa+\tau)^2+1}, & \varkappa>0,\\ \pi\delta[\varkappa+\tau+1/\sqrt{|\varkappa|}], & \varkappa<0.\end{cases} \qquad (12)$$

Here, $\kappa = (\omega - \omega_0)/\omega_0\alpha_0^{2/3}$ and $\tau = (\omega_0 - \Omega)/\omega_0\alpha_0^{2/3}$ characterizes the departure of the magnetic field from resonance.

In the region $\omega < \omega_0$, the absorption of the infinitely narrow line is determined by solution of the equation

$$|\varkappa|^{3/2} - \tau|\varkappa|^{1/2} - 1 = 0, \qquad (13)$$

the only real positive root of which is equal to

$$|\varkappa_0| = \left[R_1^{1/3} + R_2^{1/3}\right]^2,\quad R_{1,2} = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{\tau^3}{27}}. \qquad (14)$$

If $\tau = 0$, then $\kappa_0 = -1$, which corresponds to $\omega = \omega_0(1 - \alpha_0^{2/3})$. For $\tau < 0$, $\kappa_0 \to 0$ from the side of the negative values, and for $\tau > 0$, $|\kappa_0| > 1$ and the narrow peak appears from the opaque band, when it can be observed.

The location of the singular peak (in the κ scale) for $\tau = 2$, 4 and 6 is -2.62, 4.47 and -6.39, respectively, which, in application to InSb ($\alpha_0 = 0.01$, $\hbar\omega_0 = 2.3 \times 10^{-2}$ eV) corresponds to $\hbar\Omega = 2.1 \times 10^{-2}$, 1.9×10^{-2} and 1.7×10^{-2} eV, respectively; $\hbar\omega = 2.0 \times 10^{-2}$, 1.8×10^{-2} and 1.6×10^{-2} eV. Thus, in the region $\Omega < \omega_0$, interaction with phonons leads to a shift in the cyclotron line.

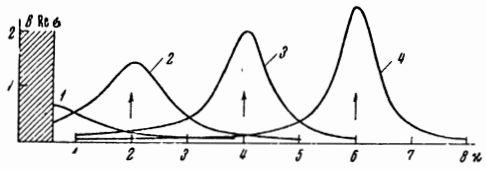

FIG. 2. Frequency dependence of the cyclotron absorption in the case of Fermi statistics for different values of the magnetic field. Curve 1 corresponds to $\tau = 6$, 2—$\tau = 4$, 3—$\tau = -2$, 4—$\tau = 0$. For InSb, the cyclotron energy $\hbar\Omega$ is equal to 2.9×10^{-2}, 2.7×10^{-2}, 2.5×10^{-2} and 2.3×10^{-2} eV, respectively. The shaded portion is the beginning of the band of total internal reflection. The arrows indicate the positions of the cyclotron lines (except for the case $\tau = 0$) in the absence of interaction. For transition to the scale $\omega-\Omega$, one should transpose the curves so that the arrows coincide.

In the region $\kappa > 0$, there is a broad peak, the shape of which depends on the value and the sign of τ. For $\tau = 0$, its maximum is situated at the point $\kappa = 0.59$ and shifts in the direction of lower x with increase in τ, being flattened out and masked in the final analysis by the opaque region. If $\tau < 0$, it even increases in magnitude, which corresponds to the case $\Omega > \omega_0$, then the peak shifts in the direction of larger κ, its maximum value increases and the halfwidth decreases (see Fig. 2).

Let us find the dependence of the width of the peak on the value of the magnetic field in the region $|\tau| > 1$, i.e., on the side of the band of lattice absorption. The value of κ corresponding to the maximum Re σ is determined from the equation

$$\varkappa(\varkappa - |\tau|)(3\varkappa - |\tau|) - 1 = 0, \qquad (15)$$

the root of which that is of interest to us being

$$\varkappa_m = |\tau|\left[1 + \frac{1}{4|\tau|^3} + \ldots\right]. \qquad (16)$$

Substituting (16) in (12), we obtain the value of Re σ at the maximum:

$$\mathrm{Re}\,\sigma_m = \frac{2\pi e^2n\sqrt{|\tau|}}{m\omega_0\alpha_0^{3/2}}\left[1 + \frac{1}{16|\tau|^3} + \ldots\right]. \qquad (17)$$

The width of the absorption peak is determined from the equation

$$\frac{\sqrt{\varkappa_m + y}}{(\varkappa_m + y)(\varkappa_m - |\tau| + y)^2 + 1} = \frac{\sqrt{\varkappa_m}}{2[\varkappa_m(\varkappa_m - |\tau|)^2 + 1]},\quad \varkappa = \varkappa_m + y, \qquad (18)$$

which, with accuracy up to terms $\sim |\tau|^{-3}$ can be reduced to a quadratic equation. As a result, the obtained width of the peak Δ is equal to

$$\Delta = \frac{2}{\sqrt{|\tau|}}\left[1 - \frac{1}{16\tau^2} + \ldots\right]. \qquad (19)$$

It is seen from Eqs. (16), (17) and (19) that upon decrease of $|\tau|$ the width of the peak increases; with increase in $|\tau|$, it narrows, becomes higher and shifts in the direction of higher light frequency.

The electron concentration n entering in Re σ is a function of the magnetic field and the temperature, and also depends on the energy of ionization of the donors. However, this circumstance does not affect the fre-

quency dependence of the absorption coefficient and consequently, that of the half-width.

4. CASE OF BOLTZMANN STATISTICS

It is of interest to consider small electron concentrations, when $T \gg \zeta$ (ζ is the chemical potential) and Boltzmann statistics apply. In this case, the quantity $n_\sigma = \exp\{(\zeta - \hbar\omega_\delta)/T\}$ entering in Eq. (6) is small and can be neglected in comparison with unity. k Then W_1 will be determined by the expression (11) and the combination $\sigma_{xx} + i\sigma_{yx}$ of interest to us reduces to the form

$$\sigma = \frac{ie^2 n}{m\omega_0}\,\frac{2\sqrt{\beta\hbar\Omega}}{\sqrt{\pi}}\,\alpha_0^{-2/3}F(\varkappa,\tau), \quad \beta = T^{-1}, \tag{20}$$

$$F(\varkappa,\tau) = \int_0^\infty \frac{\exp(-\xi y^2)\,dy}{\varkappa + \tau + i(\varkappa + y^2)^{-1/2} + is}; \quad \xi = \beta\hbar\omega_0\alpha_0^{2/3}.$$

The quantity Re σ corresponds to $-\mathrm{Im}\,F(\kappa, \tau) = P(\kappa, \tau)$, which is equal to

$$P(\varkappa,\tau) = \int_0^\infty \frac{\sqrt{\varkappa + y^2}\exp(-\xi y^2)\,dy}{(\varkappa+\tau)^2(\varkappa+y^2)+1}, \quad \varkappa > 0; \tag{21}$$

$$P(\varkappa,\tau) = \pi\int_0^{\sqrt{|\varkappa|}} \delta\left[\tau - |\varkappa| + \frac{1}{\sqrt{|\varkappa| - y^2}}\right] e^{-\xi y^2}dy$$

$$+ \int_{\sqrt{|\varkappa|}}^\infty \frac{\sqrt{y^2 - |\varkappa|}\,e^{-\xi y^2}\,dy}{(\tau - |\varkappa|)^2(y^2 - |\varkappa|) + 1}, \quad \varkappa < 0. \tag{22}$$

We first consider the frequencies located to the left of the opaque region ($\kappa < 0$). The first integral in (22) (we denote it by P_1) differs from zero if the zero of the delta function lies in the interval of the integration, i.e., if $0 \le y_0^2 \le |\kappa|$, where y_0 is the value of y which causes the expression to vanish that is under the sign of the delta function. Since we mean by the square root in the delta function the positive branch, then the solution exists only if $\tau - |\kappa| < 0$. As a result, we get

$$P_1 = \frac{(|\varkappa| - y_0^2)^{3/2}}{y_0}\,e^{-\xi y_0^2}; \quad y_0^2 = \frac{|\varkappa|(\tau - |\varkappa|)^2 - 1}{(\tau - |\varkappa|)^2}.$$

The maximum of the peak is located at the point $y_0 = 0$ (where P_1 is singular), i.e., for $|\kappa_0|$ satisfying Eq. (13). If $|\kappa| < |\kappa_0|$, then $P_1 = 0$, since here the zero for the delta function does not lie in the integration interval. For $|\kappa| > |\kappa_0|$, the peak falls off exponentially. In contrast with the Fermi case, the line obtained here, although singular, is not infinitesimally narrow. This is connected with the fact that for arbitrary $\Omega < \omega_0$ there are always electrons located at sufficiently large k_z in the level $N = 0$. The light excites them into the level $N = 1$ with the same k_z, and their energy turns out to be sufficient for transition to the level $N = 0$ with emission of an optical phonon. Thus, for $\Omega < \omega_0$, the upper electron level is nonstationary for $k_z \ge k_{lim}$, which increases with decrease in Ω. The states with $k_z < k_{lim}$ are stationary for $\Omega < \omega_0$ and transitions to these states lead to singularities in Re σ. The location of the maximum peak depends on τ in the same way as in the case of Fermi statistics.

We consider further the shape of the peak when $\kappa > 0$, which is determined by the integral (21). It is convenient to study two limiting cases, in which it is possible to get analytic expressions for $P(\kappa, \tau)$. If $\kappa \ll \xi^{-1}$, then $\sqrt{\kappa + y^2}$ in the integrand is replaced by y and

$$P(\varkappa,\tau) = -2^{-1}(\varkappa+\tau)^{-2}e^p\,\mathrm{Ei}(-p), \quad p = \xi[\varkappa + (\varkappa+\tau)^{-2}] \tag{23}$$

(Ei(x) is the integral exponential function). If $\kappa \gg \xi^{-1}$, then

$$P(\varkappa,\tau) = \frac{\pi\sqrt{\varkappa\xi}\,e^p\,\mathrm{erfc}\,(\sqrt{p})}{2(\varkappa+\tau)^2\sqrt{p}}. \tag{24}$$

If now $\xi \gg 1$, then the frequency dependence is described by Eqs. (24) and (23) refers to the narrow and unimportant range of frequencies close to $\kappa = 0$. The parameter p in this case is large and, by using the asymptotic value erfc ($\sqrt{p}$), we get[10]

$$P(\varkappa,\tau) = \left(\frac{\pi}{4\xi}\right)^{1/2}\frac{\sqrt{\varkappa}}{\varkappa(\varkappa+\tau)^2+1}, \tag{25}$$

i.e., the frequency dependence in almost the entire region is identical with the Fermi case. The half-width of the peak is determined by Eq. (19), and Re σ_m differs from what is given in Eq. (17) by the factor $2^{-1}(\pi/\xi)^{1/2}$. If $|\tau| > 1$, and consequently $\kappa_m > 1$, then Eq. (25) is valid in the region $\xi \approx 1$.

In the case $\xi \ll 1$, the frequency dependence is determined by Eq. (23), and (24) describes the "tail" of the peak for large κ. At the point $\tau = 0$, we have $P(\kappa, 0) = \xi^{-1}(\kappa^3 + 1)$, which means complete masking of the peak by the opaque band. If $\tau < 0$ and increases in absolute value, then for $|\tau| \sim 1$, we have $P(0, \tau) \approx -\tau^{-2}\ln(\xi/\tau^2) < \xi$. On the other hand, at the point $\kappa = -\tau$, we have $P(-\tau, \tau) = \xi^{-1} > P(0, \tau)$, i.e., the peak becomes different, beginning with some $|\tau| \sim 1$. The second integral entering into $P(\kappa, \tau)$ (Eq. (22)) describes in all cases the part of the studied peak in the unimportant region $\kappa < 0$. Thus, it should be expected that, in the case $\xi \ll 1$, a broad peak will be seen at larger $|\tau|$ than in the case $\xi \gg 1$. If, again, $|\tau| > 1$, then the maximum of the peak is located in the region $\kappa = |\tau|$, i.e., at large p. Using the asymptotic value of Ei(x), we get for $P(\kappa, \tau)$[10]

$$P(\varkappa,\tau) = (2\xi)^{-1}[\varkappa(\varkappa - |\tau|)^2 + 1]^{-1},$$

whence the width of the peak is

$$\Delta \approx 2/|\tau|. \tag{26}$$

A temperature dependence of Δ appears when account is taken of the next terms of the asymptotic expansion of Ei(x) in (26). The shape of the band of cyclotron resonance is shown in Fig. 3 for those same values of the magnetic field which were studied in the case of Fermi statistics. The calculations have been carried out for $T = 15°K$ and $88°K$ according to Eq. (21). It is seen that the line narrows with increasing temperature (for otherwise identical conditions). In the framework of the model considered, this is explained by the fact that, with increase in T, the fraction of electrons located at higher k_z increases. For higher k_z, the resonance condition $\Omega = \omega_0$ is less satisfactorily satisfied than for smaller k_z, which leads, in the final analysis, to a narrowing of the line. Actually, however, there are mechanisms which broaden the line with increase in T (interaction with acoustical phonons and account of absorption of optical phonons), and their inclusion can compensate for this narrowing.

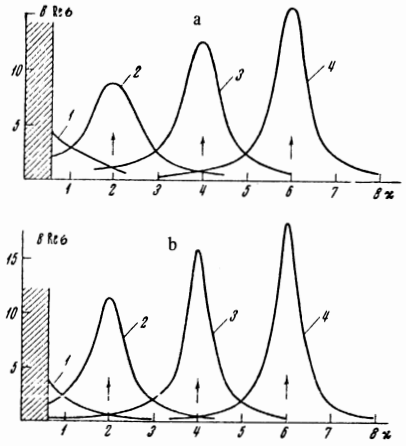

FIG. 3. Frequency dependence of the cyclotron resonance in the case of Boltzmann statistics. a–$\xi = 0.88$ (T = 15°K); b–$\xi = 0.15$ (T = 88°K). The remaining designations were identified in the legend of Fig. 2.

5. DISCUSSION OF RESULTS

The theory discussed above, in all cases considered, leads to a broadening of the cyclotron resonance line upon approach of Ω to ω_0 from the side of large Ω. This is explained by the fact that for $\Omega \approx \omega_0$ (but $\Omega > \omega_0$), the electron thrown into the level N = 1 by the light goes over to the level N = 0, emitting an optical phonon, the transition taking place in the region of small k_z, where the density of states is large. As the temperature increases, the fraction of electrons with $k_z \neq 0$ increases, which leads to a decrease in the transitions between levels with small k_z (with emission of a phonon). Therefore, the line is narrowed with decrease in $\xi \sim T^{-1}$. This is seen from comparison of the width of the peak for the Fermi and Boltzmann cases (for $\xi \gg 1$) on the one hand, and for the Boltzmann cases for $\xi \ll 1$ on the other. Here, of course, one must take into account the fact that $|\tau| > 1$, where these expressions are valid. The shape of the peak under consideration is completely described by the theory, since its maximum (if it is not masked by a region of nontransparency of the crystal) is in all cases located at frequencies $\kappa \gtrsim \alpha_0^{2/3}$, where the contribution of the vertex parts can be neglected, as was shown above. The neglected terms have a smallness $\sim \alpha_0^{2/3}$. In InSb, $\hbar\omega_0 = 2.3 \times 10^{-2}$ eV, $\hbar\omega_t = 2.2 \times 10^{-2}$ eV, $\alpha_0 = 10^{-2}$. The nontransparency region $\hbar(\omega_0 - \omega_t) \approx 10^{-3}$ eV is identical with the energy of splitting of the levels, which is equal to $\hbar\omega_0\alpha_0^{2/3} \approx 10^{-3}$ eV. It is therefore evident that it is impossible to observe two peaks simultaneously for any ω.

The formulas above describe the basic feature of the observed phenomenon accurately—the increase in the width of the peak upon approach of the magnetic field to resonance. The measurements described in[2] were made for T = 15°K and 88°K and the concentration of carriers was changed from 5.7×10^{13} cm^{-3} to 1.0×10^{15} cm^{-3}. This concentration corresponds in Fermi level from 10^{-7} to 10^{-4} eV. Thus, in the range of temperatures and concentrations studied, the distribution of the electrons was close to a Boltzmann one (the Fermi distribution would apply for concentrations 5×10^{15} cm^{-3} and T ≤ 10°K). For T = 15°K, the parameter $\xi \approx 1$, and $\kappa_{max} > 1$, so that a comparison with experiment should be made in accord with Eq. (19). Upon change of the magnetic field from 5×10^4 to 3.5×10^4 Oe, the computed width changes by a factor of 2.1, and the measured one by 2.5. The computed limiting value of the half-width (for a magnetic field equal to 6×10^4 Oe) is equal to 1.7×10^3 Oe, while the measured one (after subtraction of the background) is $\approx 0.8 \times 10^3$ Oe. This discrepancy is explained by the fact that the field of 6×10^4 Oe is very far from resonance, where the diagram taken into account in the function W (Fig. 1a) becomes $\sim \alpha_0$. But then one should also take into account the other diagrams of order α_0 for W.

In the region $\Omega < \omega_0$, the peak of the absorption is singular. This is connected with the fact that the considered mechanism of scattering (the spontaneous emission of phonons) does not lead to nonstationarity of both the upper and the lower electron levels in this region. To achieve this end, one should also take into account the two mechanisms of line broadening: the appearance of nonstationarity because of the absorption of optical phonons and their spatial dispersion. However, the consideration of these rather complicated questions goes beyond the framework of the present paper.

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245