QUANTIZATION OF ELECTRON EXCITATIONS IN s-n-s FILM CONTACTS IN A MAGNETIC FIELD

V. P. GALAĬKO

Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences

Submitted April 4, 1969

Zh. Eksp. Teor. Fiz. 57, 941–948 (September, 1969)

The problem of spatial quantization of excitations in an extended layer of a normal metal bounded by two superconducting regions is considered on the basis of the microscopic equations for the two-component wave function of an "electron-hole" pair excitation. It is shown that when the magnetic field is turned on, owing to the coherent phase difference of the superconducting ordering parameter, which is proportional to the magnetic flux passing through the normal layer, the quantization becomes unstable and can be replaced by a certain "magnetic" quantization with a distance between the levels which is proportional to the magnetic field strength.

ANDREEV[1] pointed out a unique reflection of electronic excitations in a layer of normal metal from the boundary of the interface with a superconducting region. Excitation with energy lower than the energy gap Δ in the superconductor cannot penetrate inside the superconducting region. On the other hand, owing to the small value of the gap $\Delta \ll \mu$ ($\mu = p_F^2/2m$ is the chemical potential of the electrons and p_F is the Fermi momentum), whose characteristic range of variation is $\xi \sim v_F/\Delta$ ($p_F = mv_F$, $\hbar = c = 1$), the excitation momentum is of the order of the Fermi momentum p_F and remains practically unchanged on the boundary. Therefore the only possibility that is realized upon reflection is that rotation takes place in the "isotopic" "electron-hole" space, i.e., the reflected excitation is a "hole" with opposite velocity, spin, and sign of the charge.

If the normal layer is blocked between two superconducting regions, then spatial quantization should take place for excitations with energy $\epsilon < \Delta$[1]. According to the foregoing, this quantization is quite distinctive, since it occurs with almost complete conservation of the momentum, and is due in final analysis to the correlation between the electron and the "hole," a correlation characteristic of the superconducting state and produced inside the normal layer upon reflection of the excitations from the boundary. We shall show in this paper that for a macroscopically large normal layer $l \gg L$ (l—longitudinal dimension of the layer, L—thickness of the layer, $L \gg \xi$) this quantization has a peculiar instability in a magnetic field.

The Schrödinger equation for the two-component wave function of paired excitation "electron-hole" has in the presence of superconducting correlation the following form[2,3]:

$$\begin{pmatrix} \xi(\hat{\mathbf{p}} - e\mathbf{A}), & \Delta\exp(i\chi) \\ \Delta\exp(-i\chi), & -\xi(\hat{\mathbf{p}} + e\mathbf{A}) \end{pmatrix}\begin{pmatrix} \psi_1 \\ \psi_{-1} \end{pmatrix} = \varepsilon\begin{pmatrix} \psi_1 \\ \psi_{-1} \end{pmatrix},$$

$$\xi(p) = \frac{p^2}{2m} - \mu, \qquad \hat{\mathbf{p}} = -i\nabla, \tag{1}$$

where χ is the phase of the superconducting-ordering parameter and $\mathbf{A}$ is the vector potential of the magnetic field ($\mathbf{H} = \operatorname{curl} \mathbf{A}$). The equations in (1) are gauge invariant:

$$\mathbf{A} \to \mathbf{A} + \nabla U, \quad \chi \to \chi + 2eU, \quad \psi_1 \to \psi_1 \exp(2ieU), \quad \psi_{-1} \to \psi_{-1} \exp(-2ieU).$$

In order to simplify the subsequent calculations, we neglect the finite depth of penetration of the magnetic field into the superconductor and the proximity effect, and assume the following model (the x axis is directed normal to the contact, the normal layer is in the region $0 < x < L$, the z axis is directed along the magnetic field parallel to the surface of the contact):

$$\Delta(x) = \begin{cases} 0, & 0 < x < L \\ \Delta, & x < 0, \quad x > L \end{cases}, \quad A_y(x) = \begin{cases} 0\ (H = 0), & x < 0 \\ Hx, & 0 < x < L, \\ HL\ (H = 0), & x > L \end{cases}$$

$$A_x = A_z = 0. \tag{2}$$

An important factor in what follows is the correct choice of the phase $\chi(x, y)$ of the superconducting-ordering parameter in Eq. (1). Microscopically, the phase is determined from the continuity equation for the superconducting current $\operatorname{div} \mathbf{j}_s = 0$ (see, for example,[4]). In this case, in the model defined by relations (2), this condition denotes the vanishing in the superconducting region of the gauge-invariant velocity of the superconducting condensate: $\mathbf{v}_s = (\nabla\chi - 2e\mathbf{A})/2m = 0$). From this, according to (2), it follows that[1)]:

$$\chi(x, y) = \begin{cases} 0, & x < 0 \\ 2eHLy, & x > L. \end{cases} \tag{3}$$

We exclude immediately the free motion of the excitations along the magnetic field (along the z axis), assuming formally that the chemical potential μ in (1) corresponds to motion in the xy plane:

[1)]The constant phase difference between the superconducting regions $x < 0$ and $x > L$ can be eliminated by a shift along the y axis and by separating the constant phase factor in the wave functions ψ_1 and ψ_{-1}.

Relations (3) denotes the presence of a jump of the phase χ in the region $\Delta = 0$; this jump decreases linearly along the y axis and is proportional to the magnetic flux through the normal region. The need for introducing such a jump at the points $\Delta = 0$ in the description of s-n-s contacts is seen, for example, from the Ginzburg-Landau equations [4]. In the case of the intermediate state, such a jump is connected with the growth of the phase χ on going around the normal layer and with the quantization of the magnetic flux (cf. the corresponding singularities in the mixed state [5], where the phase acquires an increment of 2π on going around a vortex filament).

$$\mu-\frac{p_z^2}{2m}\to\mu=\frac{p_0^2}{2m},\quad p_0^2=p_F^2-p_z^2. \tag{4}$$

Taking (2) and (3) into account, we can rewrite Eqs. (1) in the following explicit form ($\Phi\equiv eHL$):

$$x<0\quad\begin{pmatrix}\xi(\hat p_x,\hat p_y), & \Delta\\ \Delta, & -\xi(\hat p_x,\hat p_y)\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_{-1}\end{pmatrix}=\varepsilon\begin{pmatrix}\psi_1\\ \psi_{-1}\end{pmatrix},$$

$$0<x<L\quad\begin{pmatrix}\xi(\hat p_x,\hat p_y-\Phi x/L), & 0\\ 0, & -\xi(\hat p_x,\hat p_y+\Phi x/L)\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_{-1}\end{pmatrix}=\varepsilon\begin{pmatrix}\psi_1\\ \psi_{-1}\end{pmatrix},$$

$$x>L\quad\begin{pmatrix}\xi(\hat p_x,\hat p_y-\Phi), & \Delta\exp(2i\Phi y)\\ \Delta\exp(-2i\Phi y), & -\xi(\hat p_x,\hat p_y+\Phi)\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_{-1}\end{pmatrix}=\varepsilon\begin{pmatrix}\psi_1\\ \psi_{-1}\end{pmatrix}. \tag{5}$$

Eqs. (5) have a symmetry group $y\to y+\pi/\Phi$. It is therefore convenient to classify states with a specified energy ϵ in accordance with the irreducible representations of the translation group along the y axis. We seek the corresponding wave functions in a Fourier representation with respect to the coordinate y:

$$\begin{pmatrix}\psi_1(x,y)\\ \psi_{-1}(x,y)\end{pmatrix}=\int\frac{dk}{2\pi}\begin{pmatrix}\psi_1(x,k)\\ \psi_{-1}(x,k)\end{pmatrix}\exp(iky). \tag{6}$$

After substituting this expression in (5), it is easy to find corresponding solutions for the Fourier coefficients (6) in all three regions:

$$x<0$$
$$\begin{pmatrix}\psi_1(x,k)\\ \psi_{-1}(x,k)\end{pmatrix}=\sum_{s=\pm1}A_s(k)\begin{pmatrix}1\\ e^{is\alpha}\end{pmatrix}\exp[(q(k)+isp(k))x],$$
$$x>L \tag{7}$$
$$\begin{pmatrix}\psi_1(x,k+\Phi)\\ \psi_{-1}(x,k-\Phi)\end{pmatrix}=\sum_{s=\pm1}B_s(k)\begin{pmatrix}1\\ e^{-is\alpha}\end{pmatrix}\exp[(-q(k)+isp(k))(x-L)],$$
$$0<x<L$$
$$\begin{pmatrix}\psi_1(x,k)\\ \psi_{-1}(x,k)\end{pmatrix}=\sum_{s=\pm1}\left\{C_s(k)\begin{pmatrix}1\\ 0\end{pmatrix}u_{s,\varepsilon}\left(x-\frac{kL}{\Phi}\right)+D_s(k)\begin{pmatrix}0\\ 1\end{pmatrix}u_{s,-\varepsilon}\left(x+\frac{kL}{\Phi}\right)\right\}.$$

The following notation has been introduced:

$$p(k)=\frac{1}{\sqrt2}\{[(p_0^2-k^2)^2+(2m\sqrt{\Delta^2-\varepsilon^2})^2]^{1/2}+(p_0^2-k^2)\}^{1/2},$$
$$q(k)=\frac{1}{\sqrt2}\{[(p_0^2-k^2)^2+(2m\sqrt{\Delta^2-\varepsilon^2})^2]^{1/2}-(p_0^2-k^2)\}^{1/2}, \tag{8}$$
$$e^{i\alpha}=\frac{\varepsilon+i\sqrt{\Delta^2-\varepsilon^2}}{\Delta}\qquad\left(\alpha=\operatorname{arctg}\frac{\sqrt{\Delta^2-\varepsilon^2}}{\varepsilon}\right).$$

The functions $u_{s,\epsilon}(x)$ ($s=\pm1$) represent two arbitrary fundamental solutions of the equation for the oscillator:

$$[\hat p_x^2+(\Phi x/L)^2-2m(\mu+\varepsilon)]u_{s,\varepsilon}(x)=0. \tag{9}$$

It remains to write down the condition of continuity of the functions (7) and of their derivatives at the points $x=0$ and $x=L$. In matrix form, these conditions are given by:

$$\hat R(k)\begin{pmatrix}A_1(k)\\ A_{-1}(k)\end{pmatrix}=\hat W_\varepsilon\left(-\frac{kL}{\Phi}\right)\begin{pmatrix}C_1(k)\\ C_{-1}(k)\end{pmatrix},$$
$$\hat R(k)\begin{pmatrix}e^{i\alpha} & 0\\ 0 & e^{-i\alpha}\end{pmatrix}\begin{pmatrix}A_1(k)\\ A_{-1}(k)\end{pmatrix}=\hat W_{-\varepsilon}\left(\frac{kL}{\Phi}\right)\begin{pmatrix}D_1(k)\\ D_{-1}(k)\end{pmatrix},$$
$$\hat R'(k-\Phi)\begin{pmatrix}B_1(k-\Phi)\\ B_{-1}(k-\Phi)\end{pmatrix}=\hat W_\varepsilon\left(L-\frac{kL}{\Phi}\right)\begin{pmatrix}C_1(k)\\ C_{-1}(k)\end{pmatrix},$$
$$\hat R'(k+\Phi)\begin{pmatrix}e^{-i\alpha} & 0\\ 0 & e^{i\alpha}\end{pmatrix}\begin{pmatrix}B_1(k+\Phi)\\ B_{-1}(k+\Phi)\end{pmatrix}=\hat W_{-\varepsilon}\left(L+\frac{kL}{\Phi}\right)\begin{pmatrix}D_1(k)\\ D_{-1}(k)\end{pmatrix}, \tag{10}$$

where for brevity we have introduced the notation

$$\hat R(k)=\begin{pmatrix}1, & 1\\ r(k), & r^*(k)\end{pmatrix},\quad \hat R'(k)=\begin{pmatrix}1, & 1\\ -r^*(k), & -r(k)\end{pmatrix},$$
$$\hat W_\varepsilon(x)=\begin{pmatrix}u_{1,\varepsilon}(x), & u_{-1,\varepsilon}(x)\\ u'_{1,\varepsilon}(x), & u'_{-1,\varepsilon}(x)\end{pmatrix}, \tag{11}$$
$$r(k)=q(k)+ip(k).$$

Eliminating from (10) the vectors C and D, and taking into account Eq. (9) and the definition of the matrix $W_\epsilon(x)$ (11), we obtain

$$\hat R'(k)\begin{pmatrix}B_1(k)\\ B_{-1}(k)\end{pmatrix}=\hat U_\varepsilon\left(-\frac{kL}{\Phi},-L-\frac{kL}{\Phi}\right)\hat R(k+\Phi)\begin{pmatrix}A_1(k+\Phi)\\ A_{-1}(k+\Phi)\end{pmatrix}.$$
$$\hat R'(k)\begin{pmatrix}e^{-i\alpha} & 0\\ 0 & e^{i\alpha}\end{pmatrix}\begin{pmatrix}B_1(k)\\ B_{-1}(k)\end{pmatrix}= \tag{12}$$
$$=\hat U_{-\varepsilon}\left(\frac{kL}{\Phi},\frac{kL}{\Phi}-L\right)\hat R(k-\Phi)\begin{pmatrix}e^{i\alpha} & 0\\ 0 & e^{-i\alpha}\end{pmatrix}\begin{pmatrix}A_1(k-\Phi)\\ A_{-1}(k-\Phi)\end{pmatrix},$$

where the matrix $\hat U_\epsilon(x,x_1)=W_\epsilon(x)\hat W_\epsilon^{-1}(x_1)$ satisfy the equation

$$\frac{d\hat U_\varepsilon(x,x_1)}{dx}=\begin{pmatrix}0, & 1\\ -2m(\mu+\varepsilon)+(\Phi x/L)^2, & 0\end{pmatrix}\hat U_\varepsilon(x,x_1), \tag{13}$$
$$\hat U_\varepsilon(x_1,x_1)=1.$$

The formally-obtained equations (12) must be simplified with allowance for the strong inequalities $\epsilon<\Delta\ll\mu$ and $\Phi\ll p_0$[2]. Since the principal role should be played by states near the Fermi surface, the values $|k|<p_0$ are important in (12). Therefore we obtain from (8) and (11), in the principal approximation,

$$|k|<p_0;\quad p(k)\approx\sqrt{p_0^2-k^2}\gg q(k)\approx m\sqrt{\Delta^2-\varepsilon^2}/p(k),$$
$$r(k)\approx ip(k),\quad \hat R(k)\approx\hat R'(k)\approx\begin{pmatrix}1, & 1\\ ip(k), & -ip(k)\end{pmatrix}. \tag{14}$$

At the same values of k, it is necessary to take for the matrix $\hat U_\epsilon(x,x_1)$ in (12) the following quasiclassical solution of Eq. (13):

$$\hat U_\varepsilon(x,x_1)=\begin{pmatrix}\cos S_\varepsilon(x,x_1), & P_\varepsilon^{-1}(x)\sin S_\varepsilon(x,x_1)\\ -P_\varepsilon(x)\sin S_\varepsilon(x,x_1), & \cos S_\varepsilon(x,x_1)\end{pmatrix},$$
$$S_\varepsilon(x,x_1)=\int_{x_1}^{x}P_\varepsilon(x)\,dx,\quad P_\varepsilon(x)=\sqrt{2m(\mu+\varepsilon)-(\Phi x/L)^2}. \tag{15}$$

Substituting expressions (14) and (15) in (12) and retaining throughout the first nonvanishing terms, we obtain after simple calculations the following final equation for the amplitudes $A_s(k)$:

$$A_s(k+2\Phi)\approx\exp\left\{2is\left[\alpha-\frac{mL}{p(k)}\left(\varepsilon-\frac{k\Phi}{2m}\right)\right]\right\}A_s(k). \tag{16}$$

In the absence of a magnetic field ($\Phi=0$) this leads to the quantization condition obtained by Andreev[1]:

$$\frac{mL\varepsilon}{p(k)}=\operatorname{arctg}\frac{\sqrt{\Delta^2-\varepsilon^2}}{\varepsilon}+\pi n,\quad \varepsilon\approx\frac{\pi n}{mL}\sqrt{p_0^2-k^2}\quad(|n|\gg1) \tag{17}$$

We note that the energy levels (17) are degenerate in the quantum number s. Therefore in formulas (7) we can put, for example, $A_{-1}=B_{-1}=0$, which means almost complete conservation of the large momentum $p(k)$ in the direction of the x axis. It follows from (17) that the number of levels at a specified momentum

[2]The last inequality holds because the magnetic field H is smaller then the critical field H_c of the superconductor. Therefore, for reasonable thicknesses of the normal layer $L<10^{-2}$ cm, we get $eHL<eH_cL\ll p_0$.

$p_y = k$ is of the order of magnitude of $L\Delta/v_0 \sim L/\xi$. Accordingly, the number of states at a specified energy is also of the order of L/ξ.

For any $\Phi \neq 0$, the finite-difference equation (16), as can be readily seen, always has bounded solutions up to the "turning points":

$$k_{min} = k + 2\Phi n_{min}, \quad k_{max} = k + 2\Phi n_{max},$$
$$n_{min} = -\left[\frac{p_0 + k}{2\Phi}\right], \quad n_{max} = \left[\frac{p_0 - k}{2\Phi}\right], \tag{18}$$

where $[x]$ denotes the integer part of x. These solutions have the following form:

$$A_s(k + 2\Phi n) = a_s \exp\left[-is\varphi(k + 2\Phi n)\right],$$
$$\varphi(k + 2\Phi n) = \varphi(k_{min}) +$$
$$+ \sum_{n'=n_{min}}^{n-1} 2\left[\frac{mL}{p(k + 2\Phi n')}\left(\varepsilon - \frac{\Phi}{2m}(k + 2\Phi n')\right) - \alpha\right]. \tag{19}$$

For a final construction of the solutions of Eqs. (16) it is necessary to find the boundary conditions for the functions (19) at the "turning points" (18). To this end, we consider the initial equations (12) at $|k| > p_0$. In this case, according to (8) and (11), we have in the principal approximation

$$|k| > p_0; \quad q(k) \approx \sqrt{k^2 - p_0^2} \gg p(k) \approx m\sqrt{\Delta^2 - \varepsilon^2}/q(k),$$
$$\hat{R}(k) \approx \begin{pmatrix} 1, & 1 \\ q(k), & q(k) \end{pmatrix}, \quad \hat{R}'(k) \approx \begin{pmatrix} 1, & 1 \\ -q(k), & -q(k) \end{pmatrix},$$
$$r(k) \approx q(k), \tag{20}$$
$$\hat{R}^{-1}(k) \approx -\frac{1}{2ip(k)}\begin{pmatrix} q(k), & -1 \\ -q(k), & 1 \end{pmatrix},$$
$$\hat{R}'^{-1}(k) \approx -\frac{1}{2ip(k)}\begin{pmatrix} -q(k), & -1 \\ q(k), & 1 \end{pmatrix}.$$

In the final approximation, the matrix $\hat{U}_\epsilon(x, x_1)$ (13) is equal to

$$U_\varepsilon(x, x_1) = \begin{pmatrix} \operatorname{ch} S_\varepsilon'(x, x_1), & P_\varepsilon^{-1}(x)\operatorname{sh} S_\varepsilon'(x, x_1) \\ P_\varepsilon'(x)\operatorname{sh} S_\varepsilon'(x, x_1), & \operatorname{ch} S_\varepsilon'(x, x_1) \end{pmatrix},$$
$$S_\varepsilon'(x, x_1) = \int_{x_1}^{x} P_\varepsilon'(x)\,dx, \quad P_\varepsilon'(x) = \sqrt{\left(\frac{\Phi x}{L}\right)^2 - 2m(\mu + \varepsilon)}. \tag{21}$$

Substituting relations (20) and (21) in (12), we obtain in the first nonvanishing approximation

$$\begin{pmatrix} A_1(k + 2\Phi) \\ A_{-1}(k + 2\Phi) \end{pmatrix} \approx Q(k)\begin{pmatrix} e^{2i\alpha} - 1, & 1 - e^{-2i\alpha} \\ -(e^{2i\alpha} - 1), & -(1 - e^{-2i\alpha}) \end{pmatrix}\begin{pmatrix} A_1(k) \\ A_{-1}(k) \end{pmatrix}, \tag{22}$$
$$\begin{pmatrix} A_1(k - 2\Phi) \\ A_{-1}(k - 2\Phi) \end{pmatrix} \approx Q(k)\begin{pmatrix} e^{-2i\alpha} - 1, & e^{-2i\alpha} - 1 \\ -(e^{2i\alpha} - 1), & -(e^{2i\alpha} - 1) \end{pmatrix}\begin{pmatrix} A_1(k) \\ A_{-1}(k) \end{pmatrix}; \tag{23}$$
$$Q(k) = \left(\frac{q^2(k)}{m\sqrt{\Delta^2 - \varepsilon^2}}\right)^2 \exp(2q(k)L).$$

The eigenvalues of the transformation matrices in formulas (22) and (23), as can be readily seen, are equal to a certain large number $\sim[\exp(2p_0L)]\mu/\Delta$ and to zero[3]. It follows therefore that to obtain a bounded solution of the initial equations (12) it is necessary to stipulate that the solution (19) of (17) be proportional to the eigenvector of the transformation matrices (22) and (23), which belong to the zero eigenvalues, in the vicinity of the "turning points" (18). Thus, the boundary conditions "on the left" and "on the right" have the following form:

$$\begin{pmatrix} A_1 \\ A_{-1} \end{pmatrix}_- \sim \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \begin{pmatrix} A_1 \\ A_{-1} \end{pmatrix}_+ \sim \begin{pmatrix} e^{-i\alpha} \\ -e^{i\alpha} \end{pmatrix}. \tag{24}$$

It is easy to verify that the solution (18) can be subjected to the obtained boundary conditions (24) only at those values of the energy ϵ, which satisfy the equation

$$\sum_{n=n_{min}}^{n_{max}} \frac{2mL}{p(k + 2\Phi n)}\left[\varepsilon - \frac{\Phi}{2m}(k + 2\Phi n)\right] - \alpha(n_{max} - n_{min} + 2) = \pi n.$$

In the last equation, in view of the inequality $\Phi \ll p_0$, the sum can be approximately replaced by the integral

$$\frac{mL}{\Phi}\int_{k_{min}}^{k_{max}} dk \frac{\varepsilon - \Phi k/2m}{\sqrt{p_0^2 - k^2}} - \alpha(n_{max} - n_{min} + 2) = \pi n.$$

From this, after simple calculations, taking into account formulas (18) and confining ourselves throughout to the first non-vanishing approximation, we obtain finally

$$\varepsilon_n(k) = \varepsilon_0(k) + \frac{\Phi n}{mL}\left[1 + \frac{2}{\pi}\sqrt{\frac{\Phi}{p_0}}\left(\sqrt{\left\{\frac{p_0 + k}{2\Phi}\right\}} + \sqrt{\left\{\frac{p_0 - k}{2\Phi}\right\}}\right)\right],$$
$$\varepsilon_0(k) = \frac{\Phi p_0}{m}\sqrt{\frac{\Phi}{p_0}}\left(\sqrt{\left\{\frac{p_0 + k}{2\Phi}\right\}} - \sqrt{\left\{\frac{p_0 - k}{2\Phi}\right\}}\right) + \frac{\alpha p_0}{\pi mL}\left(\frac{\Phi}{mL} = \frac{eH}{m}\right), \tag{25}$$

where $\{x\} = x - [x]$ denotes the fractional part of x.

Just as in the absence of a magnetic field (see formula (17)), the spectrum (25) has a band character. In formula (25), however, unlike in (17), the distance between the levels at a fixed "quasimomentum"[4] k ($-\Phi \le k \le \Phi$) and for arbitrary $\Phi \neq 0$ is determined by the magnetic field $\delta\epsilon_n \sim \Phi/mL = eH/m$ and is much smaller than the distance between the levels in (17), namely (Φ/mL): $(p_0/mL) \sim \Phi/p_0 \ll 1$. It follows from (25) that there exists a strong overlap of the bands and that the number of states with a specified energy $\sim(L/\xi)\sqrt{p_0/\Phi}$, i.e., the degeneracy increases compared with the case $\Phi = 0$ by a factor $\sqrt{p_0/\Phi} \gg 1$.

The fundamental circumstance is that the spectrum (25) is determined by only one discrete quantum number n, which runs, as can be readily seen, through $Lp_0/\xi\Phi$ values. It can be stated qualitatively that at a fixed "quasimomentum" k this number labels simultaneously both the old energy levels (17) and the band structure corresponding to the periodicity in k. Such a "mixing" of the levels gives rise to an absolute instability of the spatial quantization (17). Indeed, no reasonable limiting transition to the relation (17) is possible in formula (25) when $\Phi \to 0$.[5] Thus, for all $\Phi \neq 0$ there exists in place of the spatial quantization (17) a unique "magnetic" quantization of the excitations, determined by formula (4). It must be emphasized that this quantization differs qualitatively from the usual magnetic Landau quantization in strong fields ($\Phi = eHL \gg p_0$), when the electron orbit lies entirely

---

[3] Actually the second eigenvalue is very small and is close to $\exp(-2p_0L)\Delta/\mu$.

[4] In a magnetic field, k is the x coordinate of the center of the excitation orbit.

[5] Using the strong degeneracy of the levels (25), we can construct special superpositions of wave functions (7) with specified energy, which differ from zero in the limit $\Phi = 0$ only under the condition (17), and are proportional to the wave functions corresponding to the spectrum (17).

within the normal layer. The quantization (25) is due to the same "electron-hole" correlation as in the case when $\Phi = 0$, and is the result of the additional correlation of the phases of the superconducting-ordering parameter of two superconducting regions making contact through a normal layer[6] (see footnote[1]).

Using (25), it is easy to calculate the density of the states near the Fermi surface. In the limit as $\Phi/p_0 \to 0$ we obtain

$$L\nu(\varepsilon)\,d\varepsilon = \int \frac{dp_z}{2\pi}\int_{-\Phi}^{\Phi}\frac{dk}{2\pi}\sum_n 1, \qquad \nu(\varepsilon)\approx\nu(\varepsilon_F)=\frac{mp_F}{\pi^2},$$

$$\varepsilon < \varepsilon(n, p_z, k) < \varepsilon + d\varepsilon,$$

i.e., the usual state density on the Fermi boundary in normal metal. At the same time, the density of states for the spectrum (17) is given by

$$\nu(\varepsilon)=\nu(\varepsilon_F)\frac{L\varepsilon}{\pi v_F}\sum_{n=[L\varepsilon/\pi v_F]}^{\infty}\frac{1}{n^2}.$$

The absence of a limiting transition is mathematically connected with the fact that as $\Phi \to 0$ the period of π/Φ along the y axis tends formally in Eqs. (5) to infinity at a constant value of Δ. Actually, in a sufficiently weak magnetic field, the period π/Φ becomes comparable with the dimension l of the normal layer along the y axis ($\Phi l \sim 1$). In fact, however, at finite l the quantization described by formula (25) no longer holds in much weaker fields $\Phi l \sim p_0 L \gg 1$. Actually, for finite l in Eqs. (5), the vector potential gauge $A_y = Hx$, $A_x = A_z = 0$ is no longer preferable and it is possible to transform the wave functions in the following manner:

$$\begin{pmatrix}\psi_1\\ \psi_{-1}\end{pmatrix}=\begin{pmatrix}\varphi_1\exp[i\Phi\chi(x)y]\\ \varphi_{-1}\exp[-i\Phi\chi(x)y]\end{pmatrix},\quad \chi(x)=\begin{cases}0, & x<0\\ x/L, & 0<x<L.\\ 1, & x>L\end{cases}$$

[6] In the case of the d-n-s contact (d – dielectric or vacuum), the phase of the ordering parameter can be eliminated, and a slight shift of the levels (17) occurs at small values of Φ.

Then, as can be readily seen, the magnetic field is completely eliminated from the equations for the superconducting region, and in the normal region we have

$$\begin{pmatrix}\xi(\hat p_x+eHy,\ \hat p_y), & 0\\ 0, & -\xi(\hat p_x-eHy,\ \hat p_y)\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_{-1}\end{pmatrix}=\varepsilon\begin{pmatrix}\psi_1\\ \psi_{-1}\end{pmatrix}.$$

It follows therefore that when $eHl \ll p_0$ the magnetic field is not important and usual spatial quantization of the excitations takes place (formula (17) at $k = \pi n/l$) in the region $0 < x < L$, $0 < y < l$. From this point of view, the magnetic quantization (25) takes place only in sufficiently "strong" fields, namely: it is necessary that the radius of the orbit be small compared with the longitudinal dimension of the normal layer l ($L \ll p_0/eH \ll l$). It is possible to trace in detail the transition from a weak level shift (17) to the spectrum (25) with increasing magnetic field, but the corresponding calculations are rather cumbersome and are not presented here.

The author is grateful to V. I. Peresada for a discussion of certain mathematical aspects of the work and for valuable remarks.

[1] A. F. Andreev, Zh. Eksp. Teor. Fiz. 49, 655 (1965) [Sov. Phys.-JETP 22, 455 (1966)].
[2] N. N. Bogolyubov, Usp. Fiz. Nauk 67, 549 (1959) [Sov. Phys.-Usp. 2, 236 (1959)].
[3] P. G. de Gennes, Superconductivity of Metals and Alloys, Benjamin, 1966.
[4] V. L. Ginzburg, and L. D. Landau, Zh. Eksp. Teor. Fiz. 20, 1064 (1950).
[5] A. A. Abrikosov, Zh. Eksp. Teor. Fiz. 32, 1442 (1957) [Sov. Phys.-JETP 5, 1174 (1957)].

Translated by J. G. Adashko
108