LOW-FREQUENCY PROPERTIES OF A WEAKLY-TURBULENT PLASMA IN THE REGION OF FREQUENT COULOMB COLLISIONS

V. G. MAKHAN'KOV and B. G. SHCHINOV

Joint Institute of Nuclear Studies

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Using the hydrodynamical equations we obtain the non-linear tensor of the dielectric constant of a weakly turbulent plasma for the case of frequent Coulomb collisions. We show that in that frequency range new unstable branches of oscillations (of the second sound type) may occur. The condition for instability of these oscillations turns out to be appreciably less strict compared to the excitation of ionic sound in a collisionless plasma.

1. INTRODUCTION

IT is well known that the electromagnetic properties of a plasma depend essentially on to which range of characteristic plasma frequencies the oscillation frequency considered belongs.[1)] In an unperturbed plasma the characteristic frequencies are the electron and ion plasma frequencies, the cyclotron-hybrid, drift, and other frequencies.[1-3] In a sufficiently dense plasma, moreover, there are still the frequencies of binary collisions ν_e (electron-electron and electron-ion) and ν_i (ion-ion). It is then natural that when we change to the range of frequencies which are lower than the frequencies of the binary particle collisions the dispersion properties of the plasma are appreciably changed (it is possible that new forms of instability and so on appear).[4,5]

Apart from the above mentioned characteristic frequencies there appear in a turbulent plasma new ones connected with the effective turbulent collisions of particles with plasmons and of plasmons with one another. It then turns out that even when the turbulence is weak when the relative energy density included in the pulsations is small, $W/n_0T_e \ll 1$ (n_0T_e is the thermal energy density of the plasma) the frequencies of the turbulent collisions ν_{eff} can be sufficiently large, especially if high-frequency (for instance, Langmuir) turbulence is developed.[6-8] They may become comparable to or even appreciably exceed the frequency of the binary particle collisions. It is natural to assume that the dispersive properties of a weakly turbulent plasma in the frequency range below ν_{eff} may differ appreciably from those of a "quiescent" plasma in which no collective degrees of freedom are excited. If, moreover, $\nu_{eff} \gg \nu_e$ we can consider the plasma to be collisionless in the range $\nu_e \ll \omega \ll \nu_{eff}$.

Such a study was carried out for an isotropic and uniform plasma in[9,10] (see also[6]). In those papers it was observed that ion-acoustic-like oscillations (in a non-isothermal plasma $T_e \gg T_i$) become unstable when there is a background of Langmuir waves present, and the criterion for the occurrence of this instability was found:

$$W^l / n_0T_e > 12v_{Te}^2 / (v_{ph}^l)^2,$$

where v_{Te} and v_{ph}^l are the appropriate velocities.

Finally, a general method was proposed in[11,12] to study the dispersive properties of a plasma in the range of frequencies less than the maximum frequency of turbulent collisions.[2)] Apart from the fact that in a dense plasma (produced, for instance, by a laser[14]) the frequency of pair collisions ν_e may be rather large (10^{15} sec^{-1}), under astrophysical conditions there is also interest in the excitation of long-wavelength oscillations.

Here we shall consider the case when

$$\omega \ll \nu_{eff}, \ \nu_c. \tag{1.1}$$

In Sec. 2 we use a method similar to the one developed in[11,12] to obtain an expression for the longitudinal part of the non-linear dielectrical constant of a weakly turbulent plasma $\epsilon^l(\omega, \mathbf{k}, W^l)$ in the frequency range (1.1). When obtaining $\epsilon^l(\omega, \mathbf{k}, W^l)$ we used the results of[15] in which we found a relatively simple set of hydrodynamic equations which give an adequate description of the non-linear phenomena in a plasma in the range of frequencies of interest to us.

We study in Sec. 3 the dispersion relation $\epsilon^l(\omega, \mathbf{k}, W^l) = 0$ and find a criterion for the instability and the build-up increments of acoustic-type oscillations; we also find new non-linear branches of oscillations which can be considered as "second" sound in a plasmon gas.

Finally, in Sec. 4 we find the limits of applicability of the results.

2. NON-LINEAR DIELECTRIC CONSTANT OF A PLASMA

Since we are interested in the electromagnetic properties of a weakly-turbulent (quasi-stationary) plasma[3)], we shall look for the linear response of such a plasma to

[1)] Here and henceforth we understand by the term electromagnetic properties of a plasma the linear response of the plasma to an arbitrary weak electromagnetic field.

[2)] For more details see [13].

[3)] See [16] for the quasi-stationary spectra of a weakly-turbulent plasma.

a weak regular electric field E^R. In[15] we obtained a set of hydrodynamic equations which are very convenient for further calculations. We also showed there that with sufficient accuracy h.f. turbulence (which we shall here assume to be Langmuir turbulence) can be described by the equations of collisionless hydrodynamics

$$\frac{\partial n_e}{\partial t} + \operatorname{div} n_e \mathbf{V}_e = 0, \tag{2.1}$$

$$\frac{\partial \mathbf{V}_e}{\partial t} + (\mathbf{V}_e \nabla \mathbf{V}_e) = \frac{e}{m_e}\mathbf{E}, \tag{2.2}$$

while the low frequency field is described by the following equations

$$\frac{\partial n_e}{\partial t} + \frac{\partial n_e \mathbf{V}_e}{\partial \mathbf{r}} = 0, \qquad \frac{\partial n_i}{\partial t} + \frac{\partial n_i V_i}{\partial \mathbf{r}} = 0, \tag{2.3}$$

$$m_e n_e \left[\frac{\partial}{\partial t} + \left(\mathbf{V}_e \frac{\partial}{\partial \mathbf{r}}\right)\right] V_{e,\alpha} = -\frac{\partial n_e T_e}{\partial x_\alpha} - \frac{\partial \pi_{e\alpha\beta}}{\partial x_\beta} - e n_e E_\alpha + R_\alpha, \tag{2.4}$$

$$m_i n_i \left[\frac{\partial}{\partial t} + \left(\mathbf{V}_i \frac{\partial}{\partial \mathbf{r}}\right)\right] V_{i,\alpha} = -\frac{\partial n_i T_i}{\partial x_\alpha} - \frac{\partial \pi_{i\alpha\beta}}{\partial x_\beta} + e n_i E_\alpha - R_\alpha, \tag{2.5}$$

$$\frac{3}{2} n_e \left(\frac{\partial}{\partial t} + \mathbf{V}_e \frac{\partial}{\partial \mathbf{r}}\right) T_e + n_e T_e \frac{\partial}{\partial \mathbf{r}} \mathbf{V}_e = -\frac{\partial}{\partial \mathbf{r}} \mathbf{q}_e - \pi_{e,\alpha\beta} \frac{\partial V_{e\alpha}}{\partial x_\beta} + Q_e, \tag{2.6}$$

$$\frac{3}{2} n_i \left(\frac{\partial}{\partial t} + \mathbf{V}_i \frac{\partial}{\partial \mathbf{r}}\right) T_i + n_i T_i \frac{\partial}{\partial \mathbf{r}} \mathbf{V}_i = -\frac{\partial}{\partial \mathbf{r}} \mathbf{q}_i - \pi_{i,\alpha\beta} \frac{\partial V_{i\alpha}}{\partial x_\beta} + Q_i, \tag{2.7}$$

where $\mathbf{R} = \mathbf{R}_U + \mathbf{R}_T$, $\mathbf{R}_U$ is the friction force, $\mathbf{R}_T$ the thermoforce and, similarly, $q_e = q_U^e + q_T^e$, $\mathbf{U} = \mathbf{V}_e - \mathbf{V}_i$, m_e, m_i are the masses, n_e, n_i the concentrations, and T_e, T_i the temperatures of the electrons and ions, respectively. The value $\mathbf{R}_U = -0.51 \times m_e n_e \nu_e \mathbf{U}$, if the value of $\mathbf{U}$ corresponds to low-frequency oscillations and $\mathbf{R}_U = -m_e n_e \nu_e \mathbf{U}$, if $\mathbf{U}$ corresponds to high-frequency oscillations;

$$\mathbf{R}_T = -0.71 n_e \frac{\partial T_R}{\partial \mathbf{r}}, \qquad \mathbf{q}_U{}^e = 0.71 n_e T_e \mathbf{U},$$

$$\mathbf{q}_T{}^e = -3.16 \frac{n_e T_e}{m_e \nu_e} \frac{\partial}{\partial \mathbf{r}} T_e, \qquad \mathbf{q}_i = -3.9 \frac{n_i T_i}{m_i \nu_i} \frac{\partial T_i}{\partial \mathbf{r}},$$

$$Q_e = -(\mathbf{RU}) - Q_i, \qquad Q_i = 3\frac{m_e}{m_i} n_e \nu_e (T_e - T_i),$$

$$\pi_{\alpha\beta}{}^e = -0.73 \frac{n_e T_e}{\nu_e} W_{\alpha\beta}^{(e)}, \qquad \pi_{\alpha\beta}{}^i = -0.96 \frac{n_i T_i}{\nu_i} W_{\alpha\beta}^{(i)},$$

$$W_{\alpha\beta} = \frac{\partial V_\alpha}{\partial x_\beta} + \frac{\partial V_\beta}{\partial x_\alpha} - \delta_{\alpha\beta} \frac{2}{3} \frac{\partial \mathbf{V}}{\partial \mathbf{r}}.$$

Appropriately to our problem we expand all required functions, both the random (turbulent) and the regular ones in a power series in the amplitude E^R of the weak regular field and retain terms which are not of higher order than linear. Introducing the following notation:

$$\mathbf{E} = \mathbf{E}^R + \tilde{\mathbf{E}}, \quad n = n^R + \tilde{n}, \quad \mathbf{V} = \mathbf{V}^R + \tilde{\mathbf{V}}, \tag{2.8}$$

(here $\mathbf{E}^R$, n^R, and $\mathbf{V}^R$ are the values of the functions $\mathbf{E}$, n, and $\mathbf{V}$, averaged over an ensemble) we shall have

$$\tilde{\mathbf{E}} = \tilde{\mathbf{E}}^{(0)} + \tilde{\mathbf{E}}^{(1)}, \quad \tilde{n} = \tilde{n}^{(0)} + \tilde{n}^{(1)}, \quad \tilde{V} = \tilde{V}^{(0)} + \tilde{V}^{(1)} \tag{2.9}$$

and so on, where the quantities $\tilde{E}^{(0)}$, $\tilde{n}^{(0)}$, and $\tilde{V}^{(0)}$ are independent of E^R, while $\tilde{E}^{(1)}$, $\tilde{n}^{(1)}$, and $\tilde{V}^{(1)}$ are proportional to E^R.

We substitute the expansions (2.8) and (2.9) into Eqs. (2.1)—(2.7) and retain only the terms linear in E^R. The zeroth order equations in E^R are then satisfied since, by assumption, the plasma turbulence is quasi-stationary. Equation (2.6) leads to a weak collisional heating of the plasma[17] which can be taken into account in the adiabatic approximation if the inequality

$$\frac{\nu_e}{\omega} \frac{W}{n_0 T_e} \ll 1 \tag{2.10}$$

or if not (2.10) the inequality

$$1/\omega t \ll 1, \tag{2.11}$$

is satisfied. The temperature $T_e^{(0)}$ occurs as a constant parameter in the equations, which are linear in E^R, if inequality (2.10) is satisfied. When (2.11) is satisfied the temperature itself is a slowly varying function of time and in general is proportional to W, so that the quantity $T_e^{(0)}$ depends on the moment that quasi-stationarity is established.

We turn to the set of equations which are linear in E^R. As we already mentioned in the Introduction we shall consider only turbulent collisions of lowest order in the turbulent energy W so that in the equations (2.1) and (2.2) we can retain terms linear in $\tilde{E}$ while the set of equations for low-frequency oscillations (2.3)—(2.7) contain terms of second order in $\tilde{E}$ or $\tilde{V}$. We get then for the Fourier components of the required functions

$$\left(-i\omega + i\frac{k^2 v_{Te}^2}{\omega}\right) V_{ke}{}^R = -0.51 \nu_e U_k{}^R - 1.71 ik v_{Te}^2 \frac{T_{ke}{}^R}{T_{0e}} - \frac{e}{m_e} E_k{}^R,$$

$$\left(-\frac{3}{2} i\omega + 3.16 \frac{k^2 v_{Te}^2}{\nu_e} + 3\frac{m_e}{m_i}\nu_e\right) \frac{T_{ke}{}^R}{T_{0e}} = -ikV_{ke}{}^R - 0.71 ikU_k{}^R$$
$$+ 3\frac{m_e}{m_i}\nu_e \frac{T_{ki}{}^R}{T_{0e}} - \frac{m_e \nu_e}{T_{0e}} \int \langle \tilde{V}_{k_1}^{(0)} \tilde{V}_{k_2}^{(1)} \rangle \delta(k - k_1 - k_2)\, dk_1\, dk_2, \tag{2.12}$$

$$\left(-i\omega + 1.28 \frac{k^2 v_{Ti}^2}{\nu_i} + i\frac{k^2 v_{Ti}^2}{\omega}\right) V_{ki}{}^R$$
$$= -ik v_{Ti}^2 \frac{T_{ki}{}^R}{T_{0i}} + 0.71 ik v_{Ti}^2 \frac{T_{ke}{}^R}{T_{0i}} + 0.51 \frac{m_e}{m_i}\nu_e U_k{}^R + \frac{e}{m_i} E_k{}^R,$$

$$\left(-\frac{3}{2} i\omega + 3.9 \frac{k^2 v_{Ti}^2}{\nu_i} + 3\frac{m_e}{m_i}\nu_e\right) \frac{T_{ki}{}^R}{T_{0i}} = -ikV_{ki}{}^R + 3\frac{m_e}{m_i}\nu_e \frac{T_{ke}{}^R}{T_{0i}}.$$

The expressions

$$\tilde{V}_k^{(0)} = \frac{e}{m_e} \frac{(\tilde{\mathbf{E}}^{(0)}\mathbf{k})}{\omega_{\mathbf{k}} k}, \tag{2.13}$$

$$\tilde{V}_{\mathbf{k}}{}^1 = \frac{1}{k\varepsilon^{л}(\omega, \mathbf{k})} \int \frac{k_1'}{\omega_1'} \frac{(\mathbf{k}\mathbf{k}_2')}{k_2'} V_{k_1'}{}^R \tilde{V}_{k_2'}^{(0)} \delta(k - k_1' - k_2')\, dk_1'\, dk_2' \tag{2.14}$$

are obtained as the result of solving Eqs. (2.1), (2.2) linearized in E^R together with div $\mathbf{E} = 4\pi e(n_e - n_i)$. Substituting (2.13) and (2.14) into (2.12) and solving the set obtained together with

$$\partial \mathbf{E}^R / \partial t = -4\pi \mathbf{j}^R,$$

we obtain the longitudinal part of the non-linear dielectric constant tensor of the plasma:

$$\varepsilon^n(\omega, \mathbf{k}) = 1 + i\frac{\omega_{0e}^2}{\omega_e \varkappa \omega (1+\beta)} \left(1 - \frac{m_e}{m_i}\frac{A_i \omega_e}{A_e \omega_i}\beta\right) + i\frac{m_e}{m_i}\frac{\omega_{0e}^2}{\omega_i \varkappa \omega}, \tag{2.15}$$

where

$$\omega_i = -i\omega + i\frac{k^2 v_{Ti}^2}{\omega} + 1.28 \frac{k^2 v_{Ti}^2}{\nu_i} + \frac{k^2 v_{Ti}^2}{\Omega_i}$$
$$- \frac{T_{0e}}{T_{0i}} \frac{k^2 v_{Ti}^2}{\Omega_e} \left(0.71 - \frac{\delta\nu}{\Omega_i}\right)\left(1 + \frac{\delta\nu}{\Omega_i}\right),$$

$$\omega_e = -i\omega + i\frac{k^2 v_{Te}^2}{\omega} + 1.71 \frac{k^2 v_{Te}^2}{\Omega_e}\left(1 + \frac{\delta\nu}{\Omega_i}\right)$$

$$\Omega_i = -\frac{3}{2} i\omega + \delta\nu + 3{,}9 \frac{k^2 v_{Ti}^2}{\nu_i},$$

$$\Omega_e = -\frac{3}{2} i\omega + \delta\nu + 3.16 \frac{k^2 v_{Te}^2}{\nu_e} - \frac{(\delta\nu)^2}{\Omega_i}, \qquad \delta\nu = 3\frac{m_e}{m_i}\nu_e,$$

$$\varkappa = 1 + \left[0.51 \nu_e + 1.71\left(0.71 - \frac{\delta\nu}{\Omega_i}\right)\frac{k^2 v_{Te}^2}{\Omega_e}\right]\left(\frac{1}{\omega_e} + \frac{m_e}{m_i}\frac{1}{\omega_i}\right),$$

$$A_e = 1.71 + \frac{m_e}{m_i}\frac{1}{\omega_i}\left(1+\frac{\delta\nu}{\Omega_i}\right)\left[0.51\nu_e + 1.71\left(0.71 - \frac{\delta\nu}{\Omega_i}\right)\frac{k^2 v_{Te}^2}{\Omega_e}\right]$$

$$A_i = 0.71 - \frac{\delta\nu}{\Omega_i} - \frac{1}{\omega_e}\left(1+\frac{\delta\nu}{\Omega_i}\right)\left[0.51\nu_e + 1.71\left(0.71 - \frac{\delta\nu}{\Omega_i}\right)\frac{k^2 v_{Te}^2}{\Omega_e}\right],$$

$$\beta = -\frac{ik^2\nu_e A_e e^2}{8\pi^2\varkappa\Omega_e\omega_e m_e^2\omega\varkappa}\int\frac{(\mathbf{k}_1(\mathbf{k}-\mathbf{k}_1))^2}{k_1^2|\mathbf{k}-\mathbf{k}_1|^2}\frac{(\mathbf{k}\partial N_{\mathbf{k}_1}{}^l/\partial\mathbf{k}_1)}{\omega-\mathbf{k}\mathbf{v}_g}d\mathbf{k}_1,$$

$\mathbf{v}_g = \partial\omega_\mathbf{k}/\partial\mathbf{k}$ is the group velocity of the high-frequency oscillations, $N_{\mathbf{k}_1}^l$ the number of plasmons which can be defined by the relation

$$W^l = \int \omega_{\mathbf{k}_1} N_{\mathbf{k}_1}{}^l d\mathbf{k}_1.$$

3. LOW-FREQUENCY SPECTRA OF A WEAKLY TURBULENT PLASMA

We consider here the dispersion equation

$$\varepsilon^n(\omega, \mathbf{k}, W^l) = 0. \tag{3.1}$$

We have noted earlier that its solution is valid in the range $|\omega - kv_{T\alpha}| \ll \nu_e$, α = e, i. Moreover, in order to obtain overseeable results it is expedient to split this range in a few smaller regions defined by the inequalities

$$1)\ \delta\nu \gg \omega \gg kv_{Te}, \tag{3.2}$$

$$2)\ \omega \ll kv_{Te},\ \ \delta\nu,\ \ k^2v_{Te}^2/\nu_e, \tag{3.3}$$

$$3)\ k^2v_{Te}^2/\nu_e,\ \ \delta\nu \ll \omega \ll kv_{Te}, \tag{3.4}$$

$$4)\ \delta\nu \ll \omega \ll kv_{Te},\ \ k^2v_{Te}^2/\nu_e, \tag{3.5}$$

$$5)\ k^2v_{Te}^2/\nu_e \ll \omega \ll kv_{Te},\ \ \delta\nu. \tag{3.6}$$

We emphasize here that in the limit of weak non-linearity only two branches of oscillations would occur separated by the region of strong absorption,[18] a high-frequency one $\omega_s < k^2v_{Te}^2/\nu_e$ and a low-frequency one $\omega_s > k^2v_{Te}^2/\nu_e$. Both in this and in other cases the quantity ω_s can be estimated to be approximately $\omega_s \sim kv_{Ti}(1 + T_e/T_i)^{1/2}$. As we shall see below in a weakly turbulent plasma there may appear completely new branches of oscillations (amongst them some of the "second" sound type) which under well-defined conditions become unstable.

In the first range (3.2) the solution of the dispersion equation (3.1) under the conditions

$$W^l/n_0T_e \gg kv_{Te}m_i/\nu_e m_e \tag{3.7}$$

is

$$\omega^5 = ik^4v_{Te}{}^4\delta\nu\frac{W^l}{n_0T_e}. \tag{3.8}$$

In the limit which is the opposite of (3.7) there is no solution in this range.

In the second range (3.3) solutions of (3.1) exist only for a very low level of Langmuir turbulence (and $v_{ph} > v_{Te}(m_i/m_e)^{1/2}$)

$$\left(\frac{m_e}{m_i}\right)^2\left(\frac{T_i}{T_e}\right)^2 \ll \frac{W^l}{n_0T_e} \ll \left(\frac{m_e}{m_i}\right)^2, \tag{3.9}$$

$$\omega^3 = -ik^2v_{Te}{}^2\nu_e W^l/n_0T_e. \tag{3.10}$$

We now turn to the most interesting solution of the dispersion equation (3.1) (from the point of view of the magnitude of the increment). To do this we consider the range (3.5). We note in passing that just in this range there exists a linear "high-frequency" acoustical branch $\omega_s = kv_{Ti}(T_e/T_i + 5/3)^{1/2}$. The equation has the form

$$\omega^4 - k^2v_{Ti}{}^2\left(\frac{5}{3}+\frac{T_e}{T_i}\right)\omega^2 - k^2v_{Ti}{}^2\frac{T_e}{T_i}0.24\frac{W^l}{n_0T_e}\nu_e^2 = 0, \tag{3.11}$$

whence

$$\omega^2 = \frac{1}{2}k^2v_{Ti}{}^2\left(\frac{5}{3}+\frac{T_e}{T_i}\right) \pm\left[\frac{1}{4}k^4v_{Ti}{}^4\left(\frac{5}{3}+\frac{T_e}{T_i}\right)^2 + k^2v_{Ti}{}^2\nu_e^2\frac{T_e}{T_i}0{,}24\frac{W^l}{n_0T_e}\right]^{1/2}. \tag{3.12}$$

One sees easily that one can obtain from (3.12) for sufficiently small values of W^l linear solutions.

If

$$\frac{W^l}{n_0T_e} > \frac{k^2v_{Te}{}^2}{\nu_e^2}\frac{m_e}{m_i}\left(\frac{5}{3}\frac{T_i}{T_e}+1\right)^2 = \frac{W_{cr}{}^l(\nu)}{n_0T_e}, \tag{3.13}$$

the oscillations considered, (3.12), become unstable and the increment of this instability is

$$\gamma \sim kv_{Te}\left(\frac{\nu_e}{kv_{Te}}\right)^{1/2}\left(\frac{m_e}{m_i}\frac{W^l}{n_0T_e}\right)^{1/4}. \tag{3.14}$$

Let us compare the results obtained with the non-linear oscillations of a collisionless weakly turbulent plasma.

It is well known that in the case $\omega \gg \nu_e$ the equation which is similar to (3.11) has the form, when $kv_{Ti} < \omega < kv_{Te}$,[6] 4)

$$\omega^4 - \omega^2(\omega_*{}^2 + \omega_s^2) - (\omega_\sim{}^2 - \omega_*{}^2)\omega_s^2 = 0.$$

$$\omega_* = k^2v_g^2,\quad \omega_\sim{}^2 = {}^3/_4 W^l k^2/n_0m_e. \tag{3.15}$$

and unstable solutions occur when

$$\frac{W^l}{n_0T_e} > 12\frac{v_{Te}^2}{v_\Phi^2} \sim \frac{m_e}{m_i} = \frac{W_{cr}{}^l}{n_0T_e}. \tag{3.16}$$

Comparing (3.16) and (3.13) we see that the excitation of non-linear acoustic oscillations is possible for considerably smaller turbulent energies

$$W_{cr}{}^l(\nu)/W_{cr}{}^l \sim k^2v_{Te}{}^2/\nu_e^2 \ll 1. \tag{3.17}$$

Moreover, since $W_{cr}^l(\nu)$ is proportional to k^2, the long-wave oscillations become unstable at relatively lower W^l. The solution of (3.14) is limited to the following ranges of the quantities k and W^l:

$$\frac{\nu_e}{v_{Te}}\left(\frac{m_e}{m_i}\frac{W^l}{n_0T_e}\right)^{1/6} \ll k \ll \frac{\nu_e}{v_{Te}}\left(\frac{m_i}{m_e}\frac{W^l}{n_0T_e}\right)^{1/2}, \tag{3.18}$$

$$\frac{W_{cr}{}^l(\nu)}{n_0T_e} < \frac{W^l}{n_0T_e} < \left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{T_e}{T_i}\right)^{9/2}. \tag{3.19}$$

Finally the solutions of Eq. (3.1) in the ranges (3.6), (3.4) immediately go over into one another when we change the frequency from $\omega < \delta\nu$ to $\omega > \delta\nu$ and are the same as (3.8):

$$\omega^5 = ik^4v_{Te}{}^4\delta\nu\frac{W^l}{n_0T_e}. \tag{3.20}$$

The following inequality must then be satisfied:

$$W^l/n_0T_e > (m_e/m_i)^2. \tag{3.21}$$

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4) In [6] it is shown that Eq. (.15) is valid only in a very narrow range of phase velocities of Langmuir waves, $v_{ph} < 3v_{Te}(m_i/m_e)^{1/2}$.

4. BRIEF CONCLUSIONS

This study allows us to reach the following conclusions.

1. The dispersion properties of a weakly turbulent plasma in the region $\omega \ll \nu_e$ (as also in the collisionless case) may differ radically from those for the case when there is no turbulence.

2. Non-linear instabilities occur in the range considered at a relatively low (compared to the case when $\omega > \nu_e$) level of turbulence (see Eq. (3.17)).

3. For a given level of turbulence W_0^l there is a critical value of the wave vector k_0 which is determined by the left-hand side of inequality (3.18). The build-up of oscillations with $k \ll k_0$ is described by Eq. (3.20) and for $k > k_0$ by Eq. (3.14) which describes the maximum increment. If $W/n_0T_e < (m_e/m_i)^2$ the solutions of the dispersion relation (3.1) have the form (3.10).

4. When we obtained the expression for $\epsilon^n(\omega, k, W^l)$ we considered only terms of first order in the turbulent energy W^l, dropping higher-order terms. We shall obtain the conditions under which such considerations are valid. According to[12] we can neglect turbulent collisions proportional to second and higher powers of W^l, if the following inequality is satisfied

$$\frac{k}{k_1}|\omega - kv_g| \gg \omega_{cor}; \quad (4.1)$$

here ω_{cor} is the real part of the non-linear correction to the frequency which is connected with the non-linear interactions between Langmuir waves. The maximum of the quantity ω_{cor} occurs when $v_{ph} > v_{Te}(m_e/m_i)^{1/2}$,

$$\omega_{cor} \sim \omega_{0e} W^l / n_0 T_e. \quad (4.2)$$

Substituting (4.2) into (4.1) we see that when $k < k_0$ (solution of the form (3.20)) there are no further restrictions, but in the region $k_0 < k < k_{max}$ (for the solution (3.14)) we have

$$\frac{v_{ph}}{v_{Te}} > \left(\frac{\omega_{0e}}{\nu_e}\right)^2 \left(\frac{m_e}{m_i}\right)^2.$$

5. The condition that terms $\sim W^2$ and higher-order terms can be neglected in the case considered when $\omega \ll \nu_e$ does not lead to an appreciable reduction of the range of applicability of the results obtained, in contrast to the collisionless limit where it is necessary to take into account turbulent collisions of higher order in W^l except for a narrow range of parameters.[6] 5)

In conclusion the author expresses his gratitude to V. N. Tsytovich for discussions of the statement of the problem and of the results obtained.

5) This statement refers only to the build-up of the ionic sound of Langmuir waves and is not valid in the general case of the interaction of other modes of oscillations.

[1] V. P. Silin and A. A. Rukhadze, Elektromagnitnye svoĭstva plazmy i plazmopodobnykh sred (Electromagnetic Properties of plasmas and plasma-like media) Atomizdat, 1961.

[2] A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin, A. G. Sitenko, and K. N. Stepanov, Kolektivnye kolebaniya v plazme (Collective oscillations in a plasma) Atomizdat, 1964 [English translation published by Pergamon Press, 1966].

[3] A. B. Mikhaĭlovskiĭ, Voprosy teorii plazmy (Plasma Theory Problems) Vol. 3, 1963 [English translation published by Consultants Bureau, 1966].

[4] A. A. Galeev, S. S. Moiseev, and R. Z. Sagdeev, Atomn. Energiya 15, 183 (1966).

[5] A. A. Rukhadze and V. P. Silin, Usp. Fiz. Nauk 96, 87 (1968) [Sov. Phys.-Uspekhi 11, 659 (1969)].

[6] V. N. Tsytovich, Nelineĭnye effekty v plazme (Nonlinear effects in a plasma) Nauka, 1967.

[7] A. A. Vedenov, E. P. Velikhov, and R. Z. Sagdeev, Nucl. Fusion 1, 81 (1961).

[8] W. E. Drummond and D. Pines, Nucl. Fusion 3, 1049 (1962).

[9] A. A. Vedenov and L. I. Rudakov, Dokl. Akad. Nauk SSSR 159, 767 (1964) [Sov. Phys.-Doklady 9, 1073 (1965)].

[10] A. A. Vedenov, A. V. Gordeev, and L. I. Rudakov, Plasma Phys. 9, 719 (1967).

[11] V. N. Tsytovich, Dokl. Akad. Nauk SSR 181, 60 (1968) [Sov. Phys.-Doklady 13, 672 (1969)].

[12] E. Krivorutskiĭ, V. G. Makhan'kov, and V. N. Tsytovich, Dubna preprint R9-3982, 1968; Nucl. Fusion 9, 97 (1969).

[13] V. N. Tsytovich, Lebedev Institute preprint No. 172, 1968; Zh. Eksp. Teor. Fiz. 57, 141 (1969) [Sov. Phys.-JETP 30, 83 (1970)].

[14] U. Ascoli-Bartoli, B. Brunelli, A. Caruso, A. De Angelis, G. Gatti, R. Gratton, F. Parlange, and H. Salzmann, Report CN-24/F-6, Third Conference, IAEA, Novosibirsk, 1968.

[15] V. G. Makhan'kov and V. N. Tsytovich, Dubna preprint R9-4042, 1968.

[16] S. B. Pikel'ner and V. N. Tsytovich, Zh. Eksp. Teor. Fiz. 55, 977 (1968) [Sov. Phys.-JETP 28, 507 (1969)].

[17] V. L. Ginzburg and A. V. Gurevich, Usp. Fiz. Nauk 70, 201 (1960) [Sov. Phys.-Uspekhi 3, 115 (1960)].

[18] V. G. Makhan'kov and V. N. Tsytovich, Zh. Eksp. Teor. Fiz. 56, 1872 (1969) [Sov. Phys.-JETP 29, 1004 (1969)].

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