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A NEW TYPE OF SKIN EFFECT IN A MAGNETIC FIELD

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An argument is presented that in investigating the high frequency electrical conductivity of a massive metal specimen placed in a strong magnetic field (the Larmor radius of the electron r is much less than the mean free path l) it is necessary to take into account the collisions of the conduction electrons with the surface. For example, in a metal with an equal number of holes and electrons over a wide range of frequencies corresponding to $r \ll \delta \ll l$ (δ is the thickness of the skin layer) collisions with the surface determine the frequency dependence of the surface impedance. In this frequency range in the case of diffuse scattering of electrons by the boundary the impedance does not depend on the frequency and agrees in order of magnitude with the magnetoresistance of a thin plate in a constant field. A complete calculation of the impedance has been carried out for an arbitrary Fermi surface and an arbitrary angle of inclination of the magnetic field to the surface of the sample. The case when the normal to the surface coincides with a high symmetry axis and is parallel to the magnetic field is an essentially special case.

1. In recent years a large number of both theoretical and experimental papers has been devoted to the study of electrical high frequency (HF) properties of a metal situated in a strong magnetic field ($r \ll l$, where r is the Larmor radius, l is the mean free path). The possibility of propagation of electromagnetic waves in an unbounded metal has been investigated in detail (cf., the review article[1]), and in this connection both the spatial and the frequency dispersion of the conductivity tensor has been investigated. In solving the problem of the penetration of the HF field into a bounded sample one must, generally speaking, take into account the fact that the conduction electrons collide in their motion with the surfaces bounding the sample. But it was usually assumed that in order to calculate the current one can without introducing a large error utilize the conductivity tensor for unbounded space.

As will be shown in the present paper such an assumption is not always valid. In particular, over a wide range of frequencies corresponding to

$$r \ll \delta \ll l \tag{1}$$

(δ is the effective depth of penetration of the electric field), for metals with an equal number of holes and electrons ($n_1 = n_2$) in the case of diffuse scattering at the surface the current in the sample is entirely determined by collisions of electrons with the surface. This leads to an unusual frequency dependence of the surface impedance. Independently of the shape of the Fermi surface (FS) of the metal and of the angle of inclination of the magnetic field to the surface of the sample, if the electrons move along closed sections of the FS (cf.[2]), the active part of the impedance in the region (1) does not depend on the frequency and in order of magnitude is equal to the resistance of a thin plate to a constant current[3].

The physical picture in this case is analogous to the "static skin effect"[3]: in colliding with the surface the conduction electron, generally speaking, jumps from one spiral trajectory to another one, and this gives rise to a current flowing along the surface. If the field within the sample varies sufficiently slowly, $\delta \gg r$, the magnitude and the depth of decay of this "surface" current both depend only weakly on the distribution of the electric field in the sample and are determined by the field at the surface. When the condition $n_1 = n_2$ is satisfied the "usual" current which is not associated with collisions with the surface is not great throughout the bulk of the sample (since the Hall current is absent), and the "surface" mechanism can provide the predominantly greatest contribution to the total current. We shall show that for this it is sufficient that the inequalities (1) should be satisfied. Under suitable conditions (1) corresponds to a wide frequency range (for normal metals for $H \sim 10^4$ Oe and $l \sim 1$ cm approximately from 10^4 to 10^7 Hz, and even broader for semimetals). The region indicated above where new types of dependence should be observed lies between the regions of the normal skin effect and the anomalous Reuter-Sondheimer skin effect[4].

If the surface of the sample scatters the electrons specularly, the "surface" current does not affect the magnitude of the impedance.

The problem of the penetration of the HF field into the sample (which reduces to a linear inhomogeneous integro-differential equation with a non-difference kernel) can be solved in limiting cases ($r \ll \delta$ or $r \gg \delta$) for any conditions of reflection at the boundary. In this paper we give a solution for a semi-infinite sample with $r \ll \delta$. In brief it reduces to the following. The integral (non-difference) operator for the conductivity is broken up into two parts: a difference part corresponding to an infinite sample, and a "surface" part. For both parts we obtain their asymptotic expressions for $r \ll \delta$ (they have a quite different form). After continuing the electric field and the current over the whole space the fundamental equation for the problem is solved by means of a Fourier transformation.

In Sec. 2 of this paper a discussion is given of the mathematical formulation of the problem and of the method of its solution. The "surface" part of the conductivity operator is investigated in Appendix 1.

In Sec. 3 we give a classification of the possible dependences for the surface impedance of a half-space[1)] in the range of frequencies which are small compared to the Larmor frequency Ω. In doing this we utilize the results of the solution of the difference problem carried out in Appendix 2, where we have also proved the Onsager relation for the difference conductivity operator.

2. The electric field E(z) in a flat semi-infinite sample is obtained by a simultaneous solution of the equations for the field

$$a_{ij}E_j''(z) = -\frac{4\pi i\omega}{c^2} j_i(z), \quad z > 0, \qquad a_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad i, j = x, y, z \tag{2}$$

(cf. Fig. 1) and of the linearized kinetic equation for the conduction electrons

$$v_z\frac{\partial f}{\partial z} + \frac{\partial f}{\partial t} + \tilde{\nu} f = v_i E_i, \qquad \tilde{\nu} = \nu - i\omega \tag{3}$$

(we employ the usual variables t, p_z, ϵ, cf.,[2]). The current $j_i(z)$ is expressed in terms of the distribution function f:

$$j_i(z) = -\frac{2e^2}{(2\pi\hbar)^3}\left|\frac{eH}{c}\right| \int dp_z \int_0^T dt\, v_i(t) f(z, t, p_z) \equiv \langle v_i f\rangle. \tag{4}$$

The function f must depend periodically on t and must satisfy the given boundary conditions at z = 0.

The function f can, evidently, be sought in the form of a sum of a fixed solution of (3) and a solution of the corresponding homogeneous equation and one can satisfy any arbitrary boundary conditions by choosing the required solution of the homogeneous equation. For the fixed solution of the inhomogeneous equation we choose the well known solution in an unbounded medium (cf., for example,[1]);

$$f^\infty = \int_{-\infty}^{t} d\tau \exp[-\tilde{\nu}(t-\tau)] E_i[z - z(t) + z(\tau)] v_i(\tau).$$

Here z(t) and $v_i(t) = d_i(t)/dt$ are the coordinate and the velocity of the particle; $v_i(t)$ is a periodic function. The function f^∞ is proportional to the energy which the particle has acquired moving along a helical trajectory in the electric field. For particles moving along z in the positive direction the solution (4) is expressed in terms of E(z) both for z > 0, and for z < 0, therefore (4) presupposes that E(z) is given over the region z < 0. It is convenient to continue the field as an even function, E(−z) = E(z). (We emphasize that $f^\infty(z)$ is so far defined only for z > 0.)

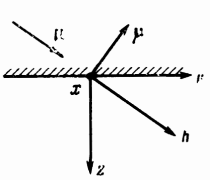

FIG. 1. The orientation of the coordinate axes associated with the sample (x, y, z) and with the magnetic field (x, μ, h).

We write the general periodic solution of the homogeneous equation corresponding to (3) in the form

$$C(\lambda)\exp[-\tilde{\nu}(t-\lambda)], \tag{5}$$

Here $C(\lambda)$ is an arbitrary periodic function (which is to be determined from the boundary conditions), $\lambda(z, t)$ satisfies the equation

$$v_z\partial\lambda/\partial z + \partial\lambda/\partial t = 0 \tag{6}$$

and the condition

$$\lambda(0, t) = t. \tag{6'}$$

(If for certain z, t there exist several values of λ, one should select the smallest one.) Physically $\lambda(z, t)$ denotes the instant of reflection from the surface for a particle which at the time t is at the point with the coordinate z. The current obtained by substitution of (5) into (4) we shall refer to as a "surface" current:

$$j_i^{\rm surf} = \langle v_i C(\lambda)\exp[-\tilde{\nu}(t-\lambda)]\rangle. \tag{7}$$

From the manner in which it was introduced it follows that $j^{\rm surf}$ is the difference between the current flowing in the sample and the current which would flow in an unbounded metal in the same electric field.

We shall solve Eq. (2) with the aid of a two-sided Fourier transformation, by continuing E(z) and j(z) (and consequently, also f(z)) as even functions into the region z < 0. On going over to Fourier components we obtain the relation

$$\left\{a_{ij}k^2 + \frac{4\pi i\omega}{c^2}\sigma_{ij}(k)\right\}\mathscr{E}_j(k) = 2\frac{4\pi i\omega}{c^2}\{a_{ij}J_j - j_i^{\rm surf}(k)\}, \tag{8}$$

which differs from the one ordinarily considered (cf.,[1]) by the term $j^{\rm surf}$. (In deriving (8) we have utilized the relation for the conductivity tensor $\sigma_{ij}(-k) = \sigma_{ji}(k)$; $J = (c^2/4\pi i\omega)E'(0)$ is the total current flowing through the sample.)

We investigate $j^{\rm surf}$ in detail in Appendix 1. Here we give only the results (which follow directly from the form adopted by us for writing 7)). Since $C(\lambda)$ depends periodically on z (cf., Appendix 1) for any kind of scattering at the surface $j^{\rm surf}(z)$ consists of a "fast" (j^r) and a "slow" (j^l) part which vary over distances of the order of r and l. Correspondingly we have

$$j^{\rm surf}(k) = j^l(k) + j^r(k);$$

j^r in the basic approximation does not depend on k for $1/l \ll k \ll 1/r$ and has the form

$$j_\alpha{}^r(k) \approx S_{\alpha\beta}E_\beta(0), \quad \alpha, \beta = x, y, \tag{9}$$

where $E_\beta(0)$ is the field at the surface. In the case of diffuse scattering the value of $S_{\alpha\beta}$ is in order of magnitude equal to

$$S \sim \sigma_0\gamma r, \tag{10}$$

and this coincides with the resistance of a thin plate to a constant current in the case of diffuse boundaries which was calculated in[3] (σ_0 is the conductivity for H = 0; for the sake of graphic illustration we shall continually be using in writing down formulas the characteristic quantities σ_0, r and $\gamma = r/l$). The simplicity of expression (9) is explained by the fact that j^r is calculated, essentially, in the approximation of a constant field (for details cf. Appendix 1). The components of $j^r(k)$

[1)]The skin effect in a thin plate is investigated in a separate paper.

with $k \sim 1/r$ need not be taken into account since the large quantity k^2 on the left hand side of (8) suppresses the corresponding components of the electric field. The current $j^l(k)$ differs from zero only for small values of k, $k \sim 1/l$.

On taking (9) into account for sufficiently low frequencies when values of $k \ll 1/r$ are significant (we shall give estimates later), (8) can be written in the form

$$\left\{ a_{ij}k^2 + \frac{4\pi i\omega}{c^2}\sigma_{ij}(k)\right\}\mathscr{E}_j(k) = \frac{8\pi i\omega}{c^2}\{a_{ij}J_j - S_{ij}E_j(0) - j_i^l(k)\}. \quad (11)$$

We seek the solution in the form

$$\mathscr{E}_i(k) = \mathscr{E}_i^{(0)} + \mathscr{E}_i\ .$$

$\mathscr{E}_i^{(0)}$ satisfies the equation obtained from (11) by neglecting $j^l(k)$. Integrating it over k we obtain

$$E_\alpha = Z_{\alpha\beta}^\infty(\omega)\{J_\beta - S_{\beta\gamma}E_\gamma(0)\}, \quad (12)$$

$$Z_{\alpha\beta}^\infty(\omega) = \int_{-\infty}^{\infty} dk\frac{4\pi i\omega}{c^2}\left\{\delta_{\alpha\beta} + \frac{4\pi i\omega}{c^2}\tilde{\sigma}_{\alpha\beta}(k)\right\}^{-1} \quad (13)$$

($\mathscr{E}_z$ has been eliminated in the usual manner by introducing $\tilde{\sigma}_{\alpha\beta} = \sigma_{\alpha\beta} - \sigma_{\alpha z}\sigma_{z\beta}/\sigma_{zz}$).

$\mathscr{E}_r^{(l)}$ is obtained from the solution of the inhomogeneous integral equation

$$\left\{a_{ij}\frac{k^2c^2}{4\pi i\omega} + \sigma_{ij}(k)\right\}\mathscr{E}_j^{(l)}(k) + j_i^l\{\mathscr{E}^{(l)}(k)\} = -j_i^l\{\mathscr{E}^{(0)}(k)\}. \quad (14)$$

In Appendix 1 it is shown that the field $E^{(l)}$ is directed essentially along the h axis and for $r \ll \delta \ll l$ is of order of magnitude $E_h^{(0)}\delta/l$, i.e., it does not have to be taken into account in evaluating the impedance.

From (12) we obtain the expression for the impedance of the sample $Z_{\alpha\beta}(\omega)$:

$$[Z_{\alpha\beta}(\omega)]^{-1} = [Z_{\alpha\beta}^\infty(\omega)]^{-1} + S_{\alpha\beta}. \quad (15)$$

Thus, the impedance of the sample is expressed in terms of the impedance $Z_{\alpha\beta}^\infty$, obtained from the solution of the "unbounded" problem in which the surface current has not been taken into account and in terms of the surface conductivity $S_{\alpha\beta}$. Formula (15) corresponds to the "parallel switching on" of the "surface" and "volume" conductivities and its existence is due to the fact that in the half-space the surface current does not change the distribution of the electric field over the depth of the sample.

If the electrons are specularly scattered by the surface then apparently $S_{\alpha\beta} = 0$ (for any arbitrary inclination of the magnetic field to the surface). Indeed, in the approximation $l = \infty$ under which relation (9) has been obtained, $S_{\alpha\beta}$ can not have a dissipative symmetric part. Moreover, the quantity $S_{\alpha\beta}$ must satisfy the Onsager relations, and since $S_{\alpha\beta} \propto H^{-2}$ the antisymmetric part also vanishes. (The equality $S_{\alpha\beta} = 0$ does not contradict the existence in the case of specular scattering of a surface current (7), cf., in this regard Appendix 1.)

3. While $S_{\alpha\beta}$ has a universal meaning (cf. (10)), $Z_{\alpha\beta}^\infty$ has a different dependence on the frequency under different conditions (the shape of the FS, the geometry of the sample, the intensity of the magnetic field). We investigate the possible forms of the dependence $Z(\omega, H)$, using formula (15) and the expressions for $Z_{\alpha\beta}^\infty$ calculated for different cases in Appendix 2.

In the general case (the shape of the FS and the direction of the magnetic field are arbitrary, and the only restriction is the absence of open orbits on the FS) the nature of the dependence of $Z_{\alpha\beta}^\infty(\omega)$ is determined by the magnitude of the magnetic field. In not too strong fields[2)]

$$\alpha = \frac{c}{v_F}\frac{\Omega}{\omega_0} \ll 1. \quad (16)$$

In this case the condition $\omega \ll \alpha\nu$ corresponds to the well-known normal skin effect. At higher frequencies lies the region $\alpha\nu \ll \omega \ll \alpha\Omega$. This condition can be rewritten in the form $\delta_0 \ll r \ll \delta_\alpha$, where δ_0 $(c^2/4\pi\omega\sigma_0)^{1/2}$, $\delta_a = (c^2 l/4\pi\omega\sigma_0)^{1/3}$ are the effective depths of penetration respectively in the case of the normal and the anomalous skin effect. In this region

$$Z_{\alpha\alpha}^\infty = \frac{a_\alpha}{\sigma_0\gamma r}\left[\ln\left(\frac{r}{\delta_0}\right) + i\frac{\pi}{2}\right], \quad \alpha = x, y, \quad (17)$$

$$Z_{xy}^\infty = i\frac{b}{\sigma_0 r}\left(\frac{r}{\delta_0}\right)^2 + \frac{b}{\sigma_0 r}, \quad a_\alpha \sim 1, \quad b \sim 1.$$

The magnitude of the components $Z_{\alpha\beta}^\infty$ is small compared to $S_{\alpha\beta}$ and

$$Z_{\alpha\alpha}(\omega) \approx \frac{1}{\sigma_0\gamma r}\left(1 + i\frac{1}{\ln^2(r/\delta_0)}\right).$$

Here the "surface" conductivity (and, consequently, the taking into account of the collisions of the electrons with the surface) play a decisive role. At higher frequencies $\alpha\Omega \ll \omega \ll \Omega$ the Reuter-Sondheimer dependence is valid

$$Z_{\alpha\alpha}(\omega) = \left(\frac{\sqrt{3}\,\omega}{8\pi^2c^2\sigma_0}\right)^{1/3} e^{-i\pi/6}. \quad (18)$$

Here taking the "surface" current into account will change only the small additional term to (18) which depends on the magnetic field.

In Fig. 2 we have shown the frequency dependence of the impedance.

The initial condition (16) for normal metals is satisfied up to fields of the order of $\sim 10^7$ Oe. For semimetals of the type of bismuth $\alpha \sim 1$ for fields of the order of 10^3 Oe. Therefore for them, and a fortiori for semiconductors, one can realize the opposite case $\alpha \gg 1$. In this case the frequencies $\omega \ll \nu$ refer to the normal skin effect, while $\omega \gg \nu$ refer to the HF normal skin effect (the spatial inhomogeneity of the electric

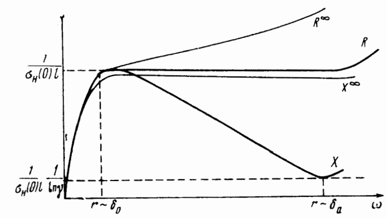

FIG. 2. The frequency dependence of the real (R) and the imaginary (X) parts of the surface impedance. The light lines correspond to specular scattering at the surface, the heavy ones to diffuse scattering; $\sigma H(O) = \sigma_0\gamma^2$.

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2) Here $\omega_0 = (ne^2/m)^{1/2}$ is the plasma frequency, $\Omega = v_F/r$, v_F is the Fermi velocity of the electron. Condition (16) is equivalent to $v_A \ll v$, were v_A is the Alfven velocity.

field is not essential, and the conductivity is obtained from the static one by replacing ν by $-i\omega$). The latter corresponds to the impedance

$$Z_{\alpha\alpha}^{\infty}(\omega)=\left(\frac{4\pi}{c^2}\frac{\Omega^2}{\sigma_0\nu}\right)^{1/2}\left(1+i\frac{\nu}{2\omega}\right).$$

Taking the surface current into account gives small corrections (of the order of α^{-1}).

The symmetric case when the magnetic field is perpendicular to the surface of the sample and is parallel to an axis of symmetry of third or higher order requires a separate investigation. (Conditions close to these can be realized in bismuth by directing the magnetic field along the trigonal axis.) For circular components of the impedance $Z_{\pm} = Z_{xx} \pm iZ_{xy}$ we have the expression

$$Z_{\pm}^{\infty}(\omega)=\frac{1}{\sigma_0\gamma^2}\left[\frac{\delta_0^{-2}\gamma^2}{(1\pm\alpha^{-2}\omega/\Omega)(\omega/\nu-i)}\right]^{1/2}$$

(cf., Appendix 2). In deriving this expression we have retained in $\sigma(k)$ terms of the order $\gamma(kr)^2$. For $\alpha \gg 1$ the symmetric case in general does not differ from the one described above. For $\alpha \ll 1$ new dependences appear. In the region $\alpha^2\Omega \ll \omega \ll \Omega$

$$Z_{\pm}^{\infty}=\frac{1}{\sigma_0\gamma r}\left[\frac{\mp\Omega}{\omega-i\nu}\right]^{1/2}. \tag{19}$$

At high frequencies, $\omega \gg \nu$, the impedance Z^{∞} for one of the polarizations is real (in this case helicons described in[5] are propagated in the metal). At low frequencies, $\omega \ll \nu$, there exists a region where Z^{∞} does not depend on the frequency (the latter is possible if $\alpha^2\Omega \ll \nu$). For one of the polarizations there exists a sharp maximum at the frequency $\omega = \alpha^2\Omega$. In order to evaluate Z^{∞} at this maximum it is necessary to retain in $\sigma(k)$ terms of the order $\gamma(kr)^4$.

The same is also necessary in the special case when in addition to the symmetric geometry described above the FS for the electrons and the holes are similar (for example, they are two spheres). For $\alpha \ll 1$ at sufficiently high frequencies $\alpha^{4/3}\Omega \ll \omega \ll \Omega$ we obtain new expressions for the impedance:

$$Z_{\pm}^{\infty}=\frac{1}{\sigma_0\gamma r}\left(\frac{\Omega}{\omega}\right)^{3/4}\exp\left[i\left(\frac{\pi}{8}\pm\frac{\pi}{8}\right)\right], \tag{20}$$

If $\omega \gg \nu$ (in this region helicons are propagated in the metal with the spectrum $\omega = v^4k^4/\Omega^3$) and

$$Z_{\pm}^{\infty}=\frac{1}{\sigma_0\gamma^{7/4}r}e^{\pm i\pi/8}, \tag{21}$$

if $\omega \ll \nu_0$. The height of the maxima referred to above is also determined by formulas (20) or (21).

A characteristic feature of the symmetric cases is the existence over the whole frequency range $\omega \ll \Omega$ of a slowly varying component of the field $kr \ll 1$. Formulas (19)–(21) take into account the contribution to the impedance of this component in particular. Moreover, for $\alpha \ll 1$ at frequencies $\omega \gg \alpha^2\Omega$ there exists a rapidly decaying Reuter-Sondheimer field the contribution of which to the impedance is given by formula (18). At sufficiently high frequencies determined by comparing (18) with one of the expressions (19)–(21) the latter is overwhelmingly large.

The symmetric case described in detail above is of particular interest since here the impedance depends very strongly on the nature of the scattering at the surface (cf., Fig. 3). For a sample with a specular surface $Z = Z^{\infty}$. In going over to diffuse scattering all the unusual dependences (19)–(21) are masked by the "surface" current. In particular, the maximum at $\omega = \alpha^2\Omega$ has a relative magnitude of $\gamma^{1/2} \ll 1$. Thus, a measurement of the impedance enables one to determine directly the degree of diffuseness.

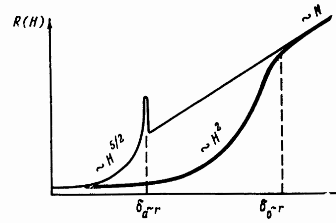

FIG. 3. The dependence on the magnetic field of the surface impedance: the heavy curve corresponds to the general case and to the case of high symmetry and diffuse scattering; the light curve corresponds to the case of high symmetry and specular scattering.

We point out that the "surface" conductivity noticeably changes the impedance also in the case $n_1 = n_2$. For one of the circular components (not the one which corresponds to the spiral wave) the real part of the impedance R^{∞} is small. We have from (15)

$$R = R^{\infty} + (X^{\infty})^2 S \sim \frac{\gamma^{1/2}}{\sigma_0\delta_0} + \frac{1}{(\sigma_0\delta_0\gamma^{1/2})^2}\sigma_0\gamma r.$$

For $\delta_0 \ll (rl)^{1/2}$ the second term is the principal one.

4. The results quoted above show that the possible dependences of the impedance of the half-space on the frequency (or on the magnetic field at a fixed frequency) are quite varied, and in many cases taking the collisions of the electrons with the surface of the sample into account qualitatively alters the expression for the impedance. An experimental investigation of the impedance can directly make evident the presence of a surface current (the existence of which follows inevitably from the kinetic equation). This possibility is of particular interest in view of the fact that the surface conductivity does not affect the dependence of the static magnetoresistance on the magnetic field.

APPENDIX 1

We investigate the expression for the "surface" current

$$j_i^{\mathrm{surf}} = \langle v_i(t)\, C(\lambda)\, e^{-\tilde{\nu}(t-\lambda)}\rangle_{A(z)}. \tag{22}$$

The angular brackets with the subscript A(z) denote that the summation is carried out over that part of the FS which corresponds to particles which arrive at the point z from the surface of the sample (cf., below). We shall require the relations

$$t-\lambda=\frac{z}{\bar{v}_z}-\frac{1}{\bar{v}_z}[\xi(t)-\xi(\lambda)],$$

$$\lambda(z+\bar{v}_zT,\, t)=\lambda(z,\, t)+T,$$

which follow from the equation $z(t) - z(\lambda) = z$ which is equivalent to (6) and (6′). We represent the function z(t) in the form

$$z(t)=\bar{v}_z t+\xi(t),$$

where $\bar{v}_z$ is the average value of the component of the velocity

$$\bar{v}_z=\frac{1}{T}\int_0^T dt\, v_z(t),$$

while the average value of $\xi(t)$ is equal to zero.

We write (22) in the form

$$j_i^{\text{surf}} = \left\langle v_i(t)\left(C(\lambda)\exp\left\{-\frac{\tilde{v}}{\bar{v}_z}[\xi(t)-\xi(\lambda)]\right\}\right)\exp\left(-\frac{\tilde{v}z}{\bar{v}_z}\right)\right\rangle_{A(z)}. \tag{23}$$

The function contained within the figure brackets depends periodically on z with the period $\bar{v}_z T \sim r$. Its constant part gives rise to j^l, and the harmonics give rise to j^r.

We write down the expression for the Fourier component of the "surface" current:

$$j_i^{\text{surf}}(k) = \int_0^\infty dz\, e^{ikz}\langle v_i(t)\, C(\lambda)\, e^{-\tilde{v}(t-\lambda)}\rangle_{A(z)}.$$

The summation denoted by $\langle\ \rangle$, contains integration over t. Going over from integration over t to integration over λ and in place of integration over z to integration over t we obtain the compact expression:

$$j_i^{\text{surf}}(k) = \langle C(\lambda)\, v_z(\lambda) f_i^{\varkappa}(\lambda, k)\rangle_+. \tag{24}$$

Here we have

$$f_i^{\varkappa}(\lambda, k) = \int_{\varkappa}^{\lambda} dt \exp\{-\tilde{v}(t-\lambda) - ik\bar{v}_z(t-\lambda) - ik[\xi(t)-\xi(\lambda)]\}\, v_i(t), \tag{25}$$

$f_i^\kappa \mathscr{E}_i(k)$ coincides with the energy acquired by the particle moving in the field $\mathscr{E}_i(k)\exp(ikz)$; κ is either $-\infty$ if the characteristic intersects the straight line z = 0 at one point (cf. Fig. 4), or the instant of return of the particle to the surface $\lambda'(\lambda)$. Formula (23) corresponds to the fact that at first we evaluate the contribution to $j_i^{\text{surf}}(k)$ of particles moving along a fixed trajectory, and then a summation is taken over the different trajectories (for which in the case z = 0 the quantity $v_z > 0$, which is denoted in (24) by the subscript +).

We discuss first the case when the electrons are scattered by the surface diffusely, i.e.,

$$f|_{v_z>0,\, z=0} = f^\infty + C(\lambda)\, e^{-\tilde{v}(t-\lambda)} = \mathscr{C} = \text{const}.$$

The constant $\mathscr{C}$ is determined from the condition of the conservation of the current j_z at the boundary,

$$j_z(0) = \langle v_z f^\infty\rangle + \langle v_z(t)\,[\mathscr{C} - f^\infty(\lambda)]\, e^{-\tilde{v}(t-\lambda)}\rangle_{A(0)} = 0.$$

In Fig. 4 the region A(0) corresponds to the segments 1–2, 3–4, 2–3, 3′–2′. In virtue of $r/l \ll 1$ in (25) we have $\exp[-\tilde{\nu}(t-\lambda)] \approx 1$. Taking into account the fact that along the segments 1–3 and 1′–3′ we have pairs of trajectories with $v_i' = -v_i$ we obtain

$$\mathscr{C} = -\frac{1}{\langle v_z\rangle_{3-4}}\{\langle v_z f^\infty\rangle + \langle v_z f^{\lambda'}\rangle_{1-2}\}. \tag{26}$$

The expressions (24)–(26) are of a completely general nature. For $\delta \gg r$ one can consistently carry out the procedure of expanding in terms of r/δ and evaluate $f^\kappa(k)$ and $j_i^{\text{surf}}(k)$ with the required accuracy. We expand (25) in powers of $ik\xi$:

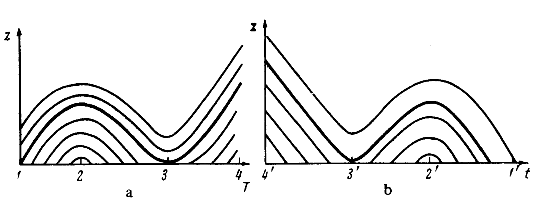

FIG. 4. Characteristics of Eq. (6).

$$f_i^\infty(t, k) = Lv_i + ik[L\xi v_i + \xi(t) Lv_i] + \frac{i^2}{2}k^2[\xi^2(t)Lv_i - 2\xi(t)L\xi v_i + L\xi^2 v_i] + \ldots$$

Here we have

$$L\psi = \int_{-\infty}^{t} d\tau\, e^{-\nu^*(t-\tau)}\psi(\tau) = \frac{\bar{\psi}}{\nu^*} + \tilde{\psi}^{(1)}(t) - \nu^*\tilde{\psi}^{(2)}(t) + \nu^{*2}\tilde{\psi}^{(3)}(t) - \ldots,$$

$$\bar{\psi} = \frac{1}{T}\int_0^T d\tau\psi(\tau), \quad \tilde{\psi} = \psi - \bar{\psi}, \quad \tilde{\psi}^{(i+1)} = \int d\tau \tilde{\psi}^{(i)},$$

$\nu^* = \tilde{\nu} + ik\bar{v}_z$, while the average value of $\tilde{\psi}^{(i)}$ is equal to $\tilde{\psi}^{(i)}_{\text{av}} = 0$. The series is obtained by repeated evaluation of the integral by parts, with the condition $\psi^{(i)}_{\text{av}} = 0$ guaranteeing the falling off of successive terms of the expansion, $\tilde{\psi}^{(i+1)} \sim (\nu^*/\Omega)\tilde{\psi}^{(i)}$.

In view of the fact that in our problem the field is even (cf., Sec. 2), the expansion can be considerably simplified by symmetrizing it with respect to k. As a result of rather lengthy calculations of a single type we obtain

$$f_i^\infty(t, k) = \frac{\tilde{v}\bar{v}_i}{\tilde{v}^2 + (k\bar{v}_z)^2} + \left\{\left[\tilde{r}_i(t) - \frac{\bar{v}_i}{\bar{v}_z}\xi(t)\right] + \frac{1}{\bar{v}_z}(\xi\tilde{v}_i)_{\text{cp}}\right\} - \frac{1}{\bar{v}_z}\frac{\tilde{v}^2}{\tilde{v}^2 + (k\bar{v}_z)^2}[(\xi\tilde{v}_i)_{\text{cp}} - \bar{v}_i\xi] + \frac{\tilde{v}k^2}{\tilde{v}^2 + (k\bar{v}_z)^2}(\tilde{v}_i\xi)_{\text{cp}}\,\xi(t) + \ldots,$$

$$\tilde{r}_i = \int d\tau\, \tilde{v}_i(\tau), \qquad (\tilde{r}_i)_{\text{cp}} = 0,$$

$$f^\infty(z)\,|_{z=0} = \int_{-\infty}^{\infty} dk\, \mathscr{E}_i(k) f^\infty(t, k). \tag{27}$$

Expressions (27) should be substituted into (23) and (26). For the sake of brevity we shall not reproduce here the detailed calculations. Making an estimate of the order of magnitude of the quantity j^l, we then make an estimate of the value of the field $E^{(l)}$ induced by it (cf., (14)):

$$E_h^{(l)}\sigma_0 \sim E_h^{(0)}\sigma_0\frac{\delta}{l} + E_{\mu,x}\sigma_0\frac{r}{l}.$$

In the region (1) of interest to us $E^{(l)} \ll E^{(0)}$ and need not be taken into account. Utilizing the results of Appendix 2 it can be shown that $E_h^{(0)} \sim (r/\delta)E^{(0)}_{\mu,x}$, and the principal role is played by the second (appearing inside the figure brackets) term in (27) (we shall refer to it as φ_i). Substituting φ_i into (23) and (26) we obtain S_{ij} (cf. (9)). Just as φ_i, S_{ij} does not depend on k (it can be easily seen that S_{ij} is the Fourier transform of the non-difference part of the kernel of the integral operator for the current $K_{ij}(k, k')$ for $l^{-1} \ll k, k' \ll r^{-1}$). Utilizing (27) one can easily verify that in our approximation ($l = \infty$) S_{ij} does not have any components with i, j = z (which is a consequence, on the one hand, of the fact that the field E_z is a potential one, and, on the other hand, of the continuity theorem which follows from the kinetic equation (3)). A concrete evaluation of $S_{\alpha\beta}$ in the general case requires calculation by numerical methods solution of a transcendental equation and an integration which for a given FS can be easily carried out with the aid of an electronic computer. In the special case when there are no particles which reach the surface of the sample twice we have

$$j_i^{\text{surf}}(k) = \int \langle f_i^\infty(k)\, v_z f_j^\infty(k')\rangle \mathscr{E}_j(k')\, dk'.$$

If the FS has an axis of symmetry oriented as in the symmetric case described in section 3 we have for each of the closed sheets of the FS[3)]

$$S_{\alpha\beta}=\langle r_\alpha(t)v_z r_\beta(t)\rangle=\frac{3}{8}\sigma_0\frac{r_0}{l}r_0\delta_{\alpha\beta},$$

Since $r_\alpha(t) = r_0 \sin \Omega t$ (here r_0 is the maximum orbit radius).

If the surface of the sample scatters the electrons specularly then, as has been pointed out already, $S_{\alpha\beta} = 0$. This does not contradict the existence of a surface current. Indeed, the periodic function $C(\lambda)$, generally speaking, can be represented in the form of a sum of $\sin(mz/\bar{v}_z T)$ and $\cos(mz/\bar{v}_z T)$ with integral values of m. But if only the cosines are present then after continuation as an even function $j^r(k)$ has only harmonics with $k = n/\bar{v}_z T$, $n \gg 1$, $S_{\alpha\beta} = 0$. The surface current is absent only in the special case $C(\lambda) = 0$ (symmetry of the fourth or the sixth order).

APPENDIX 2

We shall evaluate $Z^\infty(\omega)$ (cf., (13)), i.e., we shall solve the "infinite" problem without taking the surface current into account. In order to obtain $\sigma_{ij}(k)$ we utilize (26) setting $\kappa = -\infty$. The terms omitted in (26) give a contribution to $\sigma_{ij}(k)$ of the order of $\sigma_0\gamma(kr)^2$ and $\sigma_0\gamma^2$. The number of them is very great and there is no point in writing them out. We shall convince ourselves that the terms $\sigma_0\gamma(kr)^2 \propto H^3$ enter only into the antisymmetric part of the conductivity tensor and in order to do this we prove the Onsager relation

$$\sigma_{ij}(H,k)=\sigma_{ij}(-H,-k).$$

In order to do this, by introducing into the expression

$$\sigma_{ij}=\int dp_z\int_0^T dt\, v_i(t)\int_{-\infty}^t d\tau \exp\{-\bar{\nu}(t-\tau)-ik[z(t)-z(\tau)]\}v_j(\tau) \tag{28}$$

(cf.,[3]) the change of variables $\tau = t + \tau'$ and going over in the integral over τ' to integration over a period, we represent $\sigma_{ij}(k)$ in the form

$$\sigma_{ij}(k)=\int dp_z\frac{1}{1-\exp\{-\nu^* T\}}\int_0^T dt\int_T^0 d\tau'\, v_i(t)v_j(t+\tau')$$
$$\times\exp\{\nu\tau'-ik[z(t)-z(t+\tau')]\}.$$

In replacing H by $-H$ the derivative $\partial/\partial t$ in the kinetic equation will be replaced by $-\partial/\partial t$. A restricted solution of the new equation is given by

$$-\int_{-\infty}^t d\tau\exp\{\bar{\nu}(t-\tau)+ik[z(\tau)-z(t)]\}v_j(\tau)\mathscr{E}_j(k).$$

Transforming it similarly to (27) we obtain

$$\sigma_{ij}(-H,k)=-\int dp_z\frac{1}{1-\exp\{-\nu^* T\}}\int_0^T dt\int_T^0 d\tau v_i(t)v_j(t+\tau)$$
$$\times\exp\{-\nu\tau+ik[z(t)-z(t+\tau)]\}.$$

After the replacement $\tau = -\tau'$ and $t - \tau = t'$ (where in virtue of the periodicity of the integrand with respect to t the limits with respect to t' can, as before, be taken equal to 0 and T) we obtain the desired equality:

$$\sigma_{ij}(H,k)=\sigma_{ji}(-H,k)=\sigma_{ij}(-H,-k).$$

We write out the expression for $\sigma_{ij}(k)$ for $k \gg |\tilde{\nu}|/\bar{v}_z$ (and $|\nu - i\omega + ik\bar{v}_z| \ll \Omega$):

$$\sigma_{ij}(k)=\sum_s\Big\{\Big[\Big\langle\frac{\bar{\nu}^s}{k^2}\Big\rangle\frac{a_i a_j}{a_z^2}\Big]+\Big[\langle\tilde{v}_i\bar{r}_j\rangle+\frac{a_i}{a_z}\langle\tilde{v}_j\xi\rangle-\frac{a_j}{a_z}\langle\tilde{v}_i\xi\rangle\Big]$$
$$-\Big[A_s\frac{\bar{\nu}^s}{|k|}(a_i b_j^s-a_j b_i^s)\Big]+[A_s|k|b_i^s b_j^s]\Big\}+\sigma_0\alpha_{ij}\gamma(kr)^2+\sigma_0\beta_{ij}\gamma^2+\ldots$$

Here we have

$$a_i=\bar{v}_i/|\bar{v}|,\qquad b_i=(\xi v_i)_{cp}\Big|_{\bar{v}_z=\omega/k},$$
$$A_s=\frac{e^2}{(2\pi\hbar)^3}T^s\Big|_{\bar{v}_z=\omega/k}\Big|\frac{eH}{c}\Big|\Big(\frac{\partial\bar{v}_z^s}{\partial p_z}\Big)^{-1}\Big|_{\bar{v}_z=\omega/k}, \tag{29}$$

α_{ij} and β_{ij} are constant quantities of the order of magnitude of unity. In view of the fact that $(\bar{v}_z)_{max} \gg \omega/k$ all the quantities can be evaluated at $\bar{v}_z = 0$ (with the exception of those cases when there exist some singularities near the intersection of the FS with $\bar{v}_z = 0$). The sum over s denotes summation over the different zones.)

One can easily convince oneself that $(r_i v_j)_{av} = (-r_j v_i)_{av}$ and, therefore, $b_z^s = 0$[4)].

Groups of terms in (29) collected inside square brackets are of the order of magnitude respectively of $\sigma_0(kl)^{-2}$, $\sigma_0\gamma$, $\sigma_0\gamma(kl)^{-1}$, $\sigma_0\gamma|k|r$ (for $\omega \ll \nu$). Terms of the order of $\sigma_0\gamma|k|r$ owe their existence to particles which are accelerated in a nonpotential field in moving along almost closed trajectories. Therefore, the absence of the components j, i = z follows directly from the potential character of the field and from the Onsager relation.

In the case $n_1 = n_2$ terms of the order of $\sigma_0\gamma$ are absent from (29). In this case for the tensor $\sigma_{\alpha\beta}(k)$ we have:

$$\sigma_{\alpha\beta}(k)=-\sum_s A_s|k|b_\alpha^s b_\beta^s+\sigma_0\tilde{\alpha}_{\alpha\beta}\gamma(kr)^2+\sigma_0\tilde{\beta}_{\alpha\beta}\gamma^2+\sigma_0\pi_{\alpha\beta}(kr)^6+\ldots \tag{30}$$

(terms of the order of $\sigma_0(kl)^{-2}$ and $\sigma_0\gamma(kl)^{-1}$ did not enter into (30) in view of their "dyadic" form).

We go over to a system of coordinates where the first term in (30) (the largest one) is diagonal. Substituting (30) into (13) we calculate the inverse operator with the aid of perturbation theory utilizing in the zero order approximation only the first term in (30). This is possible in the case that

$$\gamma\ll\frac{4\pi i\omega}{c^2}\sigma_0\gamma r^2\ll 1,$$

which is equivalent to (1). Formulas (17) are the result. (We have omitted from (30) terms of little significance which are proportional to $(kr)^6$.)

If the magnetic field is sufficiently large, then the principal term in $\sigma_{ij}(k)$ will be given by $\sigma_0\gamma^2(1 - i\omega/\nu)$. On studying the dispersion equation

$$k^2+i\delta_0^{-2}\gamma r|k|+i\delta_0^{-2}\gamma^2(1-i\omega/\nu)=0,$$

one can easily convince oneself that it corresponds to

$$\Big(\frac{\omega_0}{\Omega}\Big)^2\Big(\frac{v}{c}\Big)^2\ll\Big|1+i\frac{\nu}{\omega}\Big|.$$

[3)]In this latter case the solution can be written down exactly since the problem reduces to the Wiener-Hopf problem in analogy with [4].

[4)]This was not taken into account in [1] where an expression analogous to (29) is given.

In this region the normal skin effect is realized.

If the normal and the field H are directed along an axis of order higher than the second then all the terms containing $\langle \tilde{v}_{x,y}\xi\rangle$ drop out from (27), and for $n_1 = n_2$ we have

$$\sigma_{ij} = \sigma_0 \begin{pmatrix} \gamma^2(1 - i\omega/\nu) & \gamma(kr)^2 & 0 \\ -\gamma(kr)^2 & \gamma^2(1 - i\omega/\nu) & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Investigation of different cases can be conveniently carried out after the evaluation of the impedance (cf., (19)).

In the case when the FS for the electrons and for the holes are similar, and also in calculating the height of the resonance it is necessary to retain in the dispersion equation the terms $\sigma_0\gamma(kr)^4$:

$$k^2 + i\delta_0^{-2}[\gamma^2(1 - i\omega/\nu) \pm i\gamma(kr)^4] = 0. \quad (31)$$

When we have

$$|\delta_0^{-2}r^4\gamma^3(1 - i\omega/\nu)| \gg 1$$

then the first term in (31) plays no role, and this corresponds to the expressions (20) and (21).

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