AN ANALYSIS OF THE MOTION OF AN IONIZATION FRONT BY TAKING INTO ACCOUNT OUTFLOW OF THE IONIZED GAS

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A self-similar solution describing outflow of ionized gas from an ionization front is obtained for the case when the optical thickness of the gas behind the front cannot be neglected.

We solve a one-dimensional isothermal problem. A plane radiation front is incident on a half-space filled with a neutral gas at temperature T_1 and density n_1, starting with an instant of time $t = 0$; the radiation front has a quantum energy exceeding the ionization potential of the gas and an intensity I_0 quanta/cm$^2\cdot$sec. As a result of the heating produced in each act of the ionization and of the energy radiation from the gas,[1)] a temperature $T \gg T_1$ is maintained in the ionized medium, and the gas density is $n = n(x, t)$.

There is a known self-similar solution describing the outflow of hot gas from the ionization front if the proper optical thickness of the front with respect to the incident radiation is small (see, for example, [1]). In this case, the gas-ionization rate is constant, as is also the average rate of gas outflow. In the course of time, the layer of ionized gas will grow and the process of recombination in it, balanced by photoionization, will cause an insignificant fraction of the quanta I from the initial beam to reach this front. Thus, the solution of [1] cannot hold for an infinitely long time; it is valid only during the initial stage of the process. In this note we consider the opposite limiting case of a later stage, when a regular power-law decrease of the ionization rate takes place.

We recall the self-similar solution of the equations of hydrodynamics

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+c_T{}^2\frac{\partial \ln n}{\partial x}=0;$$
$$\frac{\partial \ln n}{\partial t}+v\frac{\partial \ln n}{\partial x}+\frac{\partial v}{\partial x}=0 \tag{1}$$

We have for the outflow of the hot gas from the ionization front, if the front has a small optical thickness to the radiation[1]

$$v=c_T+\frac{x}{t},\quad n(x)=n_0\exp\left\{-\frac{x}{c_Tt}\right\},\quad n_0=\frac{I_0}{c_T}. \tag{2}$$

Here x is the coordinate in a direction perpendicular to the plane separating the gas from the vacuum at the initial instant of time, $c_T = \sqrt{RT/\mu}$ is the isothermal speed of sound, and n_0 and v_0 are the density and velocity of the ionized gas at the front.

The density and velocity of the gas at the front relative to the front are connected by the well known relation:

$$n_1v_1=n_0v_0=I. \tag{3}$$

The attenuation of the flux incident on the front is determined by recombination in the column of ionized gas, where the concentration of the neutral atoms is small:

$$I_\alpha=I_0-I=\alpha\int_0^\infty n^2(x)\,dx, \tag{4}$$

where α is the recombination coefficient at temperature T. In the case of solution (2) we have $I_\alpha = \frac{1}{2}\alpha n_0^2 c_T t$, from which we see that this solution is valid only when $t \ll \tau = 2c_T/\alpha I_0$. When $t > \tau$, the density of the ionized gas at the front should obviously decrease with time in a power-law fashion, so as to compensate for the growth of the ionized layer behind the front. According to (3), it follows therefore that the rate of advance of the front in the neutral gas should also decrease in accordance with a power law, while the velocity of the hot gas is determined by the speed of sound and obviously cannot change greatly. The flux of quanta reaching the front decreases in the course of time: $I \to 0$ and $I_\alpha \to I_0$.

Let $t \gg \tau$; we seek a self-similar solution of equations (1) in the form

$$v=c_Tf(\eta),\quad n=at^{-k}\varphi(\eta), \tag{5}$$

where $\eta = x/c_Tt$ is the self-similar variable. Substituting (5) in (4)

$$I_\alpha=\alpha c_Tt^{1-2k}a^2\int_0^\infty\varphi^2(\eta)\,d\eta=A\alpha c_Ta^2t^{1-2k}, \tag{6}$$

we see that the condition $I_\alpha \to I_0$ as $t \to \infty$ can be satisfied only when the characteristic exponent is $k = \frac{1}{2}$, i.e., we seek a solution in the form

$$v=c_Tf(\eta),\quad n=at^{-1/2}\varphi(\eta). \tag{7}$$

The system (1) reduces to

$$\frac{\partial f}{\partial\eta}(f-\eta)+\frac{\partial\ln\varphi}{\partial\eta}=0,\quad (f-\eta)\frac{d\ln\varphi}{d\eta}+\frac{df}{d\eta}-\frac{1}{2}=0,$$

from which we get

$$\frac{df}{d\eta}[1-(f-\eta)^2]=\frac{1}{2}. \tag{8}$$

The solution of (8) is

$$f=\eta+1/\sqrt{2}.$$

Finally we get

$$v=\frac{c_T}{\sqrt{2}}+\frac{x}{t},\quad n=at^{-1/2}\exp\left\{-\frac{x}{\sqrt{2}c_Tt}\right\}. \tag{9}$$

1)The optical thickness of plasma for quanta with energy smaller than the ionization potential is assumed to be small, and this is true even if the thickness for the ionizing radiation is large.

The obtained solution does not satisfy the Jouguet condition, according to which the outflow of ionized gas at the front should have the speed of sound $v = c_T$. The density $n_0 = a/\sqrt{t}$ does not depend on the initial conditions in the neutral gas and is characterized only by the value of the incident flux: from (4) and (9) we have

$$a = \left(\frac{\sqrt{2}\, I_0}{\alpha c_T}\right)^{1/2}, \quad n_0 = \left(\frac{\sqrt{2}\, I_0}{\alpha c_T t}\right)^{1/2}. \tag{10}$$

The flux reaching the front, according to conditions (3) and (10), is equal to

$$I = \left(\frac{c_T I_0}{\sqrt{2}\, \alpha t}\right)^{1/2}.$$

We present also the rate of motion of the front in a neutral gas and the distance covered by it:

$$v_1 = \left(\frac{c_T I_0}{\sqrt{2}\, \alpha n_1{}^2 t}\right)^{1/2} \quad l = \int_0 v_1\, dt = \left(\frac{2\sqrt{2}\, c_T I_0 t}{\alpha n_1{}^2}\right)^{1/2}.$$

The ionization of neutral hydrogen on the periphery of galaxies by the background radiation of the metagalaxy was considered in [2], where it was shown that the neutral-hydrogen distribution observed in the line with $\lambda = 21$ cm makes it possible to limit the metagalactic isotropic background essentially to the range 912–200 Å.

Allowance for the outflow of the ionized gas makes it possible to improve greatly the limitations given in [2]. However, for complete clarity, it is necessary to take into account the presence of the gravitational and magnetic fields.

Note added in proof (28 April 1969). S. I. Anisimov has noted that the conclusions of this article are applicable also to the case of recombination in triple collisions, when

$$I_\alpha = \alpha \int n^3(x)\, dx$$

and

$$v = \frac{c_T}{\sqrt{3}} + \frac{x}{t}, \qquad n = at^{-1/3} \exp\left\{-\frac{x}{\sqrt{3}\, c_T t}\right\}.$$

[1] S. A. Kaplan, Mezhzvezdnaya gidrodinamika (Interstellar Hydrodynamics), Fizmatgiz, 1958.
[2] R. A. Sunyaev, Astrophysical Lett. 3, 33 (1969).

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