THEORY OF THE DOMAIN STRUCTURE OF METALS UNDER CONDITIONS OF THE DE HAAS–VAN ALPHEN EFFECT

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The domain structure of metals is investigated under the conditions of the de Haas–van Alphen effect. The boundary conditions on the phase interfaces are determined and an estimate is obtained for the layer width (see (25)). It is shown that the interfaces should move when the current flows perpendicular to the layer.

1. INTRODUCTION

IT was shown earlier that a domain structure may be produced in metals under the conditions of the de Haas–van Alphen effect[1,2]. This structure appears only when the magnetization oscillations have a sufficiently large amplitude, and the dependence of the magnetic field H on the induction B has the form shown in Fig. 1. That part of the curve between points B_1 and B_2 which is determined from the condition that the shaded areas be equal corresponds to unstable states. At a given H, the stable phase is the one with the lowest value of the thermodynamic potential

$$\tilde{\Omega} = -\frac{1}{4\pi}\int_0^H B\,dH.$$

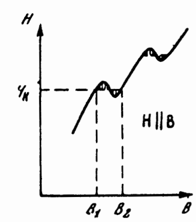
The states with $\partial H/\partial B > 0$ can exist when $B_1 < B < B_2$ as metastable states, and states with $\partial H/\partial B < 0$ are absolutely unstable. The thermodynamic potentials of phases with inductions B_1 and B_2 are equal, and these phases can coexist. Therefore, in a long cylindrical sample in a longitudinal external field $H_0 = H_c$, a first-order phase transition should occur, namely a jumplike change of the induction from B_1 to B_2[3].

In the case of a thin plate in an external field H_0 perpendicular to it, a domain structure, i.e., stratification into regions of the first and second phase, becomes thermodynamically favored in the interval $B_1 < H_0 < B_2$. If the domain structure is realized, then the dependence of the average induction and magnetization of the sample on the external magnetic field should be smooth.

The first to point out the possibility of domain formation was Condon[1]. His experiments confirmed indirectly the existence of domains in beryllium (jumps of the magnetic moment were observed in a cylinder in a longitudinal external field, and a continuous transition in a plate in a perpendicular field). One of us[2] calculated the surface-tension energy on the domain interface.

The domain structures of ferromagnets and of the intermediate state of superconductors are well known[4]. In the simplest case of a ferromagnetic or superconducting plate in a field H_0 perpendicular to it the domain structure is a system of periodically alternating plane-parallel layers. In ferromagnets, the neighboring layers have different magnetization directions, in the intermediate state of a superconductor, the normal and super-

FIG. 1



conducting layers alternate. The phase concentrations are determined by the requirement that the magnetic flux be conserved, and the domain dimensions can be obtained from the condition that the sum of the surface-tension energy on the interphase boundaries and the energy of emergence of the domains to the surface be a minimum. The latter energy is connected with the distortion of the domain structure near the surface of the plate (at a depth on the order of the domain width). As a result, the domain dimensions turn out to be proportional to $\sqrt{l}$ (l is the plate thickness). Stratification into domains results in an energy gain proportional to the sample volume.

In our case, under the condition $B_1 < H_0 < B_2$, a similar structure, i.e., periodic alternation of layers, within which the induction assumes values B_1 and B_2, is also thermodynamically favored. In the following sections we shall find the boundary conditions on the phase interfaces, investigate the properties of the domain structure, and obtain an estimate for the layer width (see (25)). We shall also show that the interfaces should move in the presence of a current flowing perpendicular to the layers.

2. BOUNDARY CONDITIONS ON THE PHASE INTERFACES

To find the shape and dimensions of the layers, it is necessary to determine the boundary conditions on the interfaces, in addition to the ordinary electrodynamic conditions that the tangential components of the field H and the normal components of the induction B be equal at the surface. We note that in the case of ferromagnets, the only condition imposed on the field H at the interface is the continuity condition. On the boundary between the normal and superconducting phases, the magnetic field in the normal phase is parallel to the boundary and equals the critical value. In the superconducting phase the field vanishes everywhere (we bear in mind the conventional method of describing a superconductor), i.e., the boundary condition is imposed only on the side of the normal phase.

In our case, the coexistence of phases at $H = H_c$ (see Fig. 1) is also possible[2]. However, if we stipulate that the equality $H = H_c$ be satisfied everywhere along the domain boundary, then such a boundary condition is excessively stringent, and the corresponding magnetostatic problem has no solution at all (see the next section). The point is that this boundary condition should be satisfied not in one of the phases, aş in superconductors, but in both, i.e., actually the number of boundary condition would be larger than in superconductors. It is therefore natural to assume that the condition $H = H_c$ on the domain boundary is satisfied only asymptotically inside the domains, and near the surface the field H on the interface differs from H_c. To prove that this is possible, we should consider the problem of the transition layer between the domains.

In the inhomogeneous case, the magnetic field $\mathbf{H}(\mathbf{x})$ is a nonlocal functional of the induction $\mathbf{B}(\mathbf{x})$:

$$\mathbf{H} = \mathbf{H}\{\mathbf{B}(\mathbf{x})\} = \mathbf{B}(\mathbf{x}) - 4\pi\mathbf{M}\{\mathbf{B}(\mathbf{x})\},$$
$$\mathbf{M}(\mathbf{x}) = -\delta\Omega_m / \delta\mathbf{B}(\mathbf{x}),$$

where Ω_m is the thermodynamic potential of the magnetic material at the specified induction $\mathbf{B}(\mathbf{x})$, without allowance for the field energy $\int (B^2/8\pi)dx$. The concrete form of this functional is given for various particular cases in[2,5].

The problem of the transition layer is one-dimensional, i.e., the component of **H** tangential to the interface, and the normal component of **B**, do not change in the transition layer (this follows from Maxwell's equations curl **H** = 0, div **B** = 0). We choose the x axis along the tangential component of H and the y axis along the normal component of B, and then $H_x = \text{const}$, $B_y = \text{const}$, $H_z = 0$, and Maxwell's equations are satisfied. Thus, it remains to find the condition for the existence of such a solution of the equations

$$\begin{aligned} H_x\{B_x(y), B_y, B_z(y)\} &= H_x, \\ H_y\{B_x(y), B_y, B_z(y)\} &= H_y(y), \\ H_z\{B_x(y), B_y, B_z(y)\} &= 0, \end{aligned} \tag{1}$$

which tends to the constant values $\mathbf{B}(\pm\infty) = \mathbf{B}^{\pm}$ and $H_y(\pm\infty) = H_y^{\pm}$ as $y \to \pm\infty$.

It is easy to verify that the equations in (1) are the Euler-Lagrange equations for the functional[1)]

$$\Omega' = \int\limits_{-\infty} dy \left\{\omega_m\{\mathbf{B}(y)\} + \frac{B^2}{8\pi} - \frac{H_x B_x}{4\pi}\right\} \tag{1a}$$

under the conditions $H_x = \text{const}$, $B_y = \text{const}$, and $H_z = 0$. Therefore a solution having the required asymptotic properties only when the integrand assumes identical values as $y \to \pm\infty$, for otherwise the functional can have no extremum. Consequently, we have the following equation for the determination of the boundary condition

$$-\int_0^{\mathbf{B}^-} \mathbf{M}d\mathbf{B} + \frac{\mathbf{B}^{-2}}{8\pi} - \frac{H_x B_x^-}{4\pi} = \int_0^{\mathbf{B}^+} \mathbf{M}d\mathbf{B} + \frac{\mathbf{B}^{+2}}{8\pi} - \frac{H_x B_x^+}{4\pi}, \tag{2}$$

which can be rewritten in the form

$$\int_0^{\mathbf{B}^-} \mathbf{H}d\mathbf{B} - H_x B_x^- = \int_0^{\mathbf{B}^+} \mathbf{H}d\mathbf{B} - H_x B_x^+. \tag{3}$$

This equation establishes the connection between H_x and B_y. Consequently, the boundary condition has the form $H_x = H_x(B_y)$, i.e.,

$$H_t = H_t(B_n) \tag{4}$$

(we recall that the direction of the tangential component of **H** was chosen to be along the x axis).

It is curious that the potential Ω', which should have a minimum at the given H_x and B_y, is the same on both sides of the interface. In the case $B_y = 0$ phases with equal values of the density of the thermodynamic potential of the magnet coexist; this is expressed in the isotropic model by the condition that the areas on Fig. 1 be equal. Relation (3) denotes a similar equality of areas on the $H_x(B_x)$ plot at $B_y = \text{const}$ and $H_z = 0$, which is natural, since B_y plays the role of a parameter in (1).

The concrete form of the solution in the transition region can be obtained for the isotropic model in the case of weak inhomogeneity, previously considered in[2]. In this case the connection between H and B as the simple form

$$H_x = H_x^{(0)}(B_x, B_y) - 2\alpha\partial^2 B_x/\partial y^2; \quad \alpha > 0. \tag{5}$$

The constant α was calculated in[2] for the case $B_y = 0$. From (5) we can obtain in implicit form the $B_x(y)$ dependence:

$$y = \alpha^{1/2} \int^{B_x} \left[\int_C^{B_x} H_x^{(0)}(\zeta, B_y)d\zeta - H_x B_x \right]^{-1/2} dB_x. \tag{6}$$

The values of H_x, $B_x^{\pm}$, and C are determined from the condition $y(B_x^{\pm}) = \pm\infty$. This means that the radicand in (6) should have multiple roots at $B_x = B_x^{\pm}$, i.e., not only the radicand but also its derivative with respect to B_x should vanish at these values of B_x. Recognizing that

$$\int_0^{\mathbf{B}} \mathbf{H}\,d\mathbf{B} = \int_0^{B_y} H_y(0, B_y)\,dB_y + \int_0^{B_x} H_x(B_x, B_y)\,dB_x,$$

it is easy to see that these conditions are equivalent to (3).

3. DOMAIN DIMENSIONS

In this section we consider the domain structure in an infinite plane-parallel plate and in an external field H_0 perpendicular to it. Since the ratio of the period of the structure to the plate thickness tends to zero as $l \to \infty$, we can put $l = \infty$ when solving the magnetostatic problem. We confine ourselves to the isotropic model ($\mathbf{H} \parallel \mathbf{B}$) and investigate a planar structure ($H_z = 0$, H_x and H_y independent of z; see Fig. 2). The interfaces bend as they emerge to the surface, just as in the case of superconductors, but these interfaces, generally speaking, are not force line, because the component of **B** normal to the interface may differ from zero.

1)In (1a) we have $\int dx\, \omega_m\{\mathbf{B}(\mathbf{x})\} = \Omega_m$; in the homogeneous case $\omega_m = -\int_0^{\mathbf{B}} \mathbf{M}d\mathbf{B}$.

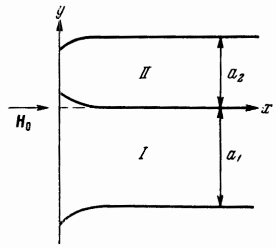


FIG. 2

Inside the metal (as $x \to \infty$) the field H approaches H_c, and the induction B approaches the values B_1 and B_2, with H and B becoming parallel to the boundaries. The connection between B and H can in this case be linearized:

$$B = \begin{cases} B_1 + \lambda_1^2 (H - H_c) & \text{in region I.} \\ B_2 + \lambda_2^2 (H - H_c) & \text{In region II.} \end{cases} \quad (7)$$

The linearized magnetostatic equations are

$$\frac{\partial H_x'}{\partial y} = \frac{\partial H_y}{\partial x}, \quad \tilde{\lambda}_i^2 \frac{\partial H_x'}{\partial x} + \frac{\partial H_y}{\partial y} = 0, \quad (8)$$

where

$$H_x' = H_x - H_c \approx H - H_c, \quad \tilde{\lambda}_i^2 = H_k \lambda_i^2 / B_i; \quad i = 1, 2 \quad (9)$$

and account is taken of the fact that $B_x \approx B$, $B_y \approx (B_i/H_c)H_y$.

It is also necessary to linearize the boundary conditions, i.e., the function $H_t(B_n)$, which should be determined from (3). It is obvious that in our model this function has at small values of B_n the form $H_t = H_c + \text{const} \cdot B_n^2$, and the dependence on B_n can be neglected upon linearization.

The linearized boundary conditions are thus

$$(H_x')_{\text{I}} = (H_x')_{\text{II}} = 0, \quad (10)$$

and these conditions are imposed on the "unshifted boundaries," i.e., on the lines $y = -a_1, 0, a_2, \ldots$. The slope of the boundary is determined from the condition that the values of B_n be equal on both sides of the boundary:

$$-B_{x\text{I}} \sin\theta + B_{y\text{I}} \cos\theta = -B_{x\text{II}} \sin\theta + B_{y\text{II}} \cos\theta.$$

At the accuracy indicated, we have

$$\theta = \frac{B_{y\text{II}} - B_{y\text{I}}}{B_2 - B_1}, \quad B_{y\text{I}} = \frac{B_1}{H_c} H_{y\text{I}}, \quad B_{y\text{II}} = \frac{B_2}{H_c} H_{y\text{II}} \quad (11)$$

(we recall that B_1 and B_2 are determined from the condition that the areas on Fig. 1 be equal).

The linearized equations (8) have a solution that decreases as $x \to \infty$ and satisfies the boundary conditions (10), can be written in the form

$$H_x'^{(i)} = \frac{H_c}{\tilde{\lambda}_i} \gamma_1^{(i)} \exp\left(-\frac{\pi x}{\tilde{\lambda}_i a_i}\right) \sin\frac{\pi y}{a_i},$$
$$H_y^{(i)} = -H_c \gamma_1^{(i)} \exp\left(-\frac{\pi x}{\tilde{\lambda}_i a_i}\right) \cos\frac{\pi y}{a_i}. \quad (12)$$

The contribution of the higher harmonics is less than the experimental error.

The coefficients $\gamma_1^{(i)}$ should be determined by solving the magnetostatic problem in all of space. Obviously, however, these coefficients are small (of the order of $(B_2 - B_1)/H_c$), for in the limit as $B_2 - B_1 \to 0$ there exists only a homogeneous solution. For this reason, the deviation of H and B from their asymptotic values should be small even near the surface of the plate.

On the basis of (11) we can estimate the angle of inclination of the boundary θ near the surface (at $x \sim \tilde{\lambda}_i a_i$). In general, this angle is not small, i.e., the shift of the boundary is appreciable; this, of course, greatly complicates the solution of the magnetostatic problem.

We shall assume below that the connection between B and H inside each of the domains is linear (see formulas (7)), and put $\tilde{\lambda}_1 = \tilde{\lambda}_2 = \tilde{\lambda}$ and $a_1 = a_2$, i.e., $H_0 = (B_1 + B_2)/2$. It will be shown that in this case the shift of the interfaces is small (it is proportional to $(B_2 - B_1)/H_c$). Therefore in the first approximation in the parameter $(B_0 - B_1)/H_c$ it is necessary to solve the linearized equations

$$\frac{\partial H_x'}{\partial y} = \frac{\partial H_y}{\partial x}, \quad \tilde{\lambda}^2 \frac{\partial H_x'}{\partial x} + \frac{\partial H_y}{\partial y} = 0 \quad (13)$$

with linearized boundary conditions (10). In addition, as will be shown below, in this approximation the solution satisfies the additional condition on the interfaces ($y = -a_1, 0, a_2, \ldots$)

$$H_y^{\text{I}} = H_y^{\text{II}}. \quad (14)$$

It follows from (11) that in this case

$$\theta = \frac{H_y^{\text{I, II}}}{H_c} \sim \frac{B_2 - B_1}{H_c} \ll 1. \quad (15)$$

Relations (10) and (14) are equivalent to the boundary condition $H^{\text{I}} = H^{\text{II}} = H_c$, which is more stringent than (4). Such a situation obtains only in the first approximation in the parameter $(B_2 - B_1)/H_c$ and only if the following conditions are satisfied: the connection between B and H inside each of the domains is linear and $\tilde{\lambda}_1 a_1 = \tilde{\lambda}_2 a_2$, i.e., $H_0 = (\tilde{\lambda}_2 B_1 + \tilde{\lambda}_1 B_2)/(\tilde{\lambda}_1 + \tilde{\lambda}_2)$ (the condition $\tilde{\lambda}_1 = \tilde{\lambda}_2$ is not fundamental, but we shall nevertheless assume that it is satisfied, so as to simplify the problem).

We seek a solution inside the metal in a form satisfying the boundary conditions (1) and (14):

$$H_x'^{(i)} = \frac{H_c}{\tilde{\lambda}} \sum_{n=0}^{\infty} \gamma_n \exp\left[-\frac{(2n+1)\pi x}{\tilde{\lambda} a}\right] \sin\frac{(2n+1)\pi y}{a},$$

$$H_y^{(i)} = -H_c \sum_{n=0}^{\infty} \gamma_n \exp\left[-\frac{(2n+1)\pi x}{\tilde{\lambda} a}\right] \cos\frac{(2n+1)\pi y}{a}. \quad (16)$$

The even harmonics appear only in the next higher approximation in the parameter $(B_2 - B_1)/H_c$ when nonlinearity is taken into account.

The solution for the field in vacuum, satisfying the condition of continuity of the tangential component at $x = 0$, is given by

$$H_x^{(e)} = H_0 - H_c \sum_{n=0}^{\infty} \gamma_n \exp\left[\frac{(2n+1)\pi x}{a}\right] \sin\frac{(2n+1)\pi y}{a},$$

$$H_y^{(e)} = -H_c \sum_{n=0}^{\infty} \gamma_n \exp\left[\frac{(2n+1)\pi x}{a}\right] \cos\frac{(2n+1)\pi y}{a},$$

$$H_0 = {}^1/_2 (B_1 + B_2). \quad (17)$$

From (7) we obtain in the linear approximation

$$B_x^{(i)} = \tilde{B}(y) + \tilde{\lambda}^2 H_x'^{(i)}, \quad (18)$$

where $\tilde{B}(y)$ is either B_1 or B_2.

The Fourier expansion of $\tilde{B}(y)$ is

$$B(y) \approx B(x\to\infty, y) = \frac{B_1+B_2}{2} + 2\frac{B_2-B_1}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1}\sin\frac{(2n+1)\pi y}{a}. \quad (19)$$

Equating the Fourier coefficients in the expansions of $B_x^{(i)}$ and $H_x^{(e)}$ at x = 0, we get

$$\gamma_n = -\frac{2(B_2-B_1)}{H_c(1+\tilde{\lambda})(2n+1)\pi}. \quad (20)$$

When $\tilde{\lambda} \ll 1$ (i.e., $(\partial H/\partial B)_{1,2} \gg 1$), the change of the induction B within each of the domains is of the order of $\tilde{\lambda}(B_2 - B_1)$, i.e., the linear extrapolation used by us in the linearization of the problem is convenient for the calculation of the energy of emergence of the domains to the surface.

In the opposite limiting case $(\partial H/\partial B)_{1,2} \ll 1$ the change of the induction is of the order of $B_2 - B_1$, i.e., the nonlinearity is of the order of unity. In this case linear extrapolation allows us to estimate the magnitude of the effects connected with the emergence of the domains to the surface. The inclination of the interface is determined by the relation

$$\frac{dy}{dx} = \frac{H_y^{(i)}(x, y=\pm na)}{H_c}.$$

Integrating, we get

$$y(x) = \pm na + (-1)^n\frac{2(B_2-B_1)\tilde{\lambda}a}{\pi^2(1+\tilde{\lambda})}\int_{\pi x/2\tilde{\lambda}a}^{\infty}\ln\operatorname{cth} z\,dz. \quad (21)$$

The Fourier series for $H_y^{(i)}$ diverges logarithmically at the points x = 0 and y = ±na, because the expansion in terms of the small parameter is not valid near the points at which the interfaces emerge to the surface. However, the linear dimensions δ of the corresponding regions are exponentially small (in spite of the fact that the shift of the interface is of first order of smallness):

$$\ln(\tilde{\lambda}a/\delta) \sim H_K(1+\tilde{\lambda})/(B_2-B_1),$$

thus justifying our approximation.

The thermodynamic potential of the magnet is

$$-\frac{1}{4\pi}\int d^3\mathbf{x}\int_0^{\mathbf{H}(\mathbf{x})}\mathbf{B}\,d\mathbf{H}.$$

Introducing $\mathbf{B}' = \mathbf{B} - \mathbf{H}_0$ and discarding the quantity

$$-\frac{1}{4\pi}\int d^3\mathbf{x}\,\mathbf{H}_0(\mathbf{H}-\mathbf{H}_0),$$

which reduces to a surface integral (see[4], Sec. 31)[2)], we represent the density of the thermodynamic potential in the form

$$\tilde{\Omega}(\mathbf{H}) = -\frac{1}{4\pi}\int_0^{\mathbf{H}_0}\mathbf{B}\,d\mathbf{H} - \frac{1}{4\pi}\int_{\mathbf{H}_0}^{\mathbf{H}(\mathbf{x})}\mathbf{B}'\,d\mathbf{H}.$$

The density of the energy of emergence of the domains to the surface is ($\mathbf{H}_c \parallel \mathbf{H}_0$)

$$\delta\tilde{\Omega}(\mathbf{H}) = \begin{cases} -\dfrac{1}{4\pi}\displaystyle\int_{\mathbf{H}_K}^{\mathbf{H}(\mathbf{x})}\mathbf{B}'\,d\mathbf{H} & \text{for } x>0 \\ -\dfrac{1}{8\pi}(\mathbf{H}(\mathbf{x})-\mathbf{H}_0)^2 & \text{for } x<0 \end{cases}. \quad (22)$$

Inside the metal we must put $B_y \approx H_y$; B_x is determined by relations (18) and (19). Therefore the energy of emergence of the domains to the surface, per unit area of the plate, can be represented in the form

$$\delta\tilde{\Omega} = -\frac{1}{4\pi a}\int_{-a}^{a}dy\left\{\int_0^{\infty}dx\left[\bar{B}(y)H_x' + \frac{1}{2}\tilde{\lambda}^2 H_x'^2 + \frac{1}{2}H_y^2\right] + \frac{1}{2}\int_{-\infty}^{0}dx\,[(H_x-H_0)^2+H_y^2]\right\}. \quad (23)$$

Performing the integration, we get

$$\delta\tilde{\Omega} = \frac{(B_2-B_1)^2 a}{2\pi^4(1+\tilde{\lambda})}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^3}. \quad (24)$$

The thermodynamic-equilibrium domain width a is determined from the condition that the sum $\delta\tilde{\Omega} + \Delta l/a$ be a minimum, where Δ is the surface tension, which is calculated in[2] and in the Appendix:

$$\Delta \sim \begin{cases} \left(\dfrac{\partial H}{\partial B}\right)_{1,2}^{1/2} r_0(B_2-B_1)^2 & \text{for } \left(\dfrac{\partial H}{\partial B}\right)_{1,2} \ll 1 \\ \left(\dfrac{\partial H}{\partial B}\right)_{1,2}^{\alpha} r_0(B_2-B_1)^2, \quad 0<\alpha\leqslant 2/3 & \text{for } \left(\dfrac{\partial H}{\partial B}\right)_{1,2} \gg 1 \end{cases}$$

(r_0 is the electron cyclotron radius). We finally get

$$a \sim \begin{cases} (r_0 l)^{1/2} & \text{for } \left(\dfrac{\partial H}{\partial B}\right)_{1,2} \ll 1 \\ \left(\dfrac{\partial H}{\partial B}\right)_{1,2}^{\alpha/2}(r_0 l)^{1/2} & \text{for } \left(\dfrac{\partial H}{\partial B}\right)_{1,2} \gg 1 \end{cases} \quad (25)$$

This estimate of the domain width is valid if the concentration of each phase is small compared with unity, in spite of the fact that the displacement of the interfaces may be relatively large in the general case (on the order of unity). This can be verified on the basis of the asymptotic formulas (12), taking into account the fact that $y_1^{(i)} \sim (B_2 - B_1)/(1+\tilde{\lambda})$, and that all the terms in (23) for $\delta\tilde{\Omega}$ do not exceed in order of magnitude the resultant value of $\delta\tilde{\Omega}$ given in (24). The latter is quite important, since these terms have opposite signs. When $\tilde{\lambda} \ll 1$ the interface shift is small, of the order of $\tilde{\lambda}a$, for in accordance with (11) and (12) the angle is $\theta \sim \exp(-\pi x/\tilde{\lambda}a)$.

4. MOTION OF DOMAIN STRUCTURE IN AN ELECTRIC FIELD

We now consider the flow of current through the sample in a direction perpendicular to the interfaces ($j = j_y$; see Fig. 2). In each of the domains there exists a Hall field parallel to the interfaces

$$E_z^{(1)} = -R_1B_1j, \quad E_z^{(2)} = -R_2B_2j;$$

the bending of the interfaces is insignificant here. Since $E_z^{(1)} \neq E_z^{(2)}$, a static domain structure is impossible, and the interfaces must move[3)]. The rate of motion $V = V_y$ can be determined by equating the tangential components of the electric field in a coordinate system moving together with the boundaries. Using the formula for the transformation of the field E*

[2)]This surface integral, generally speaking, vanishes only when $\mathbf{H}_0$ vanishes at infinity. This is the only case with physical meaning.

[3)]The motion of the structure in superconductors is well known [6]. A similar phenomenon should take place also in ferromagnets.

*[BV] ≡ B × V.

$$\mathbf{E} = \mathbf{E}' + \frac{1}{c}[\mathbf{B}, \mathbf{V}] \qquad (\mathbf{B}' \approx \mathbf{B})$$

we obtain

$$E_z^{(1)} - E_z^{(2)} = \frac{V}{c}(B_1 - B_2).$$

If the number of electrons in the metal is not equal to the number of holes, then $R_1 = R_2 = 1/n|e|c$, where n is the difference between the number of electrons and holes, and e is the electron charge. In this case

$$V = -j/n|e|. \tag{26}$$

It is easy to verify that in the laboratory frame Maxwell's equation

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\partial\mathbf{B}/\partial t$$

is satisfied on the interface.

APPENDIX

SURFACE TENSION ON THE DOMAIN INTERFACE IN THE CASE WHEN $(dH/dB)_{1,2} \gg 1$

The surface tension Δ on the domain interface is determined by the relation

$$\Delta = \int_{-\infty}^{\infty} dy \left\{\omega_m\{B(y)\} + \frac{B^2(y)}{8\pi} - \frac{H_c B(y)}{4\pi} - \text{const}\right\}, \tag{A.1}$$

where the constant is chosen such as to make the integrand vanish when $y \to \pm\infty$. The functional $\omega_m\{B(y)\}$ is of the form

$$\omega_m\{B(y)\} = \omega_m(\hat{K}B(y)), \tag{A.2}$$

where (see[2,5]; the isotropic model is assumed)

$$\hat{K}B(y) = \int_{-\infty}^{\infty} dy' K(y-y')B(y'),$$

$$K(y) = \begin{cases} \dfrac{2}{\pi r_0}\sqrt{1 - \dfrac{y^2}{r_0^2}} & \text{for} \quad |y| < r_0 \\ 0 & \text{for} \quad |y| > r_0 \end{cases} \tag{A.3}$$

and $\omega_m(B)$ is the thermodynamic potential in the homogeneous case:

$$\omega_m(B) = -\int_0^B M\,dB.$$

We assume for simplicity that in the homogeneous case

$$M(B) = \frac{a}{4\pi}\sin u, \quad u = k\left(B - \frac{B_1 + B_2}{2}\right) = k(B - H_c),$$

$$\omega_m(B) = \frac{a}{4\pi k}\cos ku, \quad k \approx \frac{2\pi}{B_2 - B_1} \quad \left(\text{for}\left(\frac{\partial H}{\partial B}\right)_{1,2} \gg 1\right). \tag{A.4}$$

This assumption does not affect in any manner the subsequent results.

Going over to the dimensionless variable $\xi = y/r_0$ and putting

$$ak = \chi \approx (\partial H/\partial B)_{1,2} \gg 1, \tag{A.5}$$

we get

$$\Delta = \frac{r_0}{4\pi k^2}\int_{-\infty}^{\infty} d\xi \left\{\chi[\cos\hat{\tilde{K}}u(\xi) - \cos u_0] + \frac{u^2(\xi)}{2} - \frac{u_0^2}{2}\right\}. \tag{A.6}$$

Here

$$\tilde{K}(\xi) = \begin{cases} 2\pi^{-1}\sqrt{1-\xi^2} & \text{for} \quad |\xi| > 1 \\ 0 & \text{for} \quad |\xi| < 1 \end{cases}. \tag{A.7}$$

The function $u(\xi)$ satisfies the equation

$$u(\xi) = \chi\hat{\tilde{K}}\sin\hat{\tilde{K}}u(\xi) \tag{A.8}$$

and the boundary conditions

$$u(\pm\infty) = \pm u_0, \qquad u_0 = \chi\sin u_0, \qquad u_0 \approx \pi(1 - 1/\chi). \tag{A.9}$$

It will be shown below that if $u(\pm\infty) = \pm\pi$ the functional

$$I_1 = \int_{-\infty}^{\infty} d\xi(1 + \cos\hat{\tilde{K}}u(\xi)) \tag{A.10}$$

has a minimum equal to zero, although it does not have a minimizing function. In this connection, it turns out that the surface tension Δ increases with increasing χ more slowly than the first power of χ. Thus, the estimate of the surface tension for the case $(\partial H/\partial B)_{1,2} \gg 1$, indicated in[2], is too high.

The absence of a function minimizing the functional I_1 means that the solution $u(\xi)$ has no limit as $\chi \to \infty$. At finite values of χ, Eq. (A.8) does, of course, have a solution minimizing the functional (A.6).

The fact that the functional I_1 cannot be equal to zero is obvious, since the right-hand side of the equation

$$\int_{-\infty}^{\infty} K(\xi - \xi')u(\xi')d\xi' = \pi\operatorname{sign}\xi',$$

is, first, discontinuous, and second, not orthogonal to the solutions of the homogeneous equations $\exp(it_j\xi)$, where t_j are the roots of the Fourier transform of the function $\tilde{K}(\xi)$:

$$\tilde{K}(t) = \int_{-\infty}^{\infty} \tilde{K}(\xi)e^{-it\xi}d\xi = \frac{2I_1(t)}{t}. \tag{A.11}$$

However, this functional can be made arbitrarily small, if $u(\xi)$ is defined as follows:

$$u(\xi) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\tau(t)g(t)}{K(t)}e^{it\xi}dt = \hat{\tilde{K}}^{-1}\hat{\tau}g(\xi), \tag{A.12}$$

where g(t) is the Fourier transform of the continuous function $g(\xi)$, which vanishes at $\xi = 0$, and approaches $\pm\pi$ asymptotically when $1 \gg |\xi| \gg \epsilon$; the function $\tau(t)$ vanishes when $t = t_j$, and is close to unity outside narrow intervals near $t = t_j$ (of width ν_j). With such a choice of $u(\xi)$, the functional I_1 turns out to be small to the extent that ϵ and ν_j are small. The second term in (A.6)

$$I_2 = \frac{1}{2}\int_{-\infty}^{\infty}(u^2(\xi) - u_0^2)d\xi \tag{A.13}$$

then turns out to be large compared with unity.

The parameters ϵ and ν_j will next be determined by a variational method. We put for concreteness (this does not affect the conclusion)

$$g(\xi) = u_0\operatorname{th}\frac{\pi\xi}{2\varepsilon}, \qquad g(t) = \frac{2u_0}{it}\frac{\varepsilon}{\operatorname{sh}\varepsilon t},$$

$$\tau(t) = \prod_j\left(1 + \frac{\nu_j^2}{t_j^2}\right)\frac{(t-t_j)^2}{(t-t_j)^2 + \nu_j^2}. \tag{A.14}$$

The function $\hat{\tilde{K}}u(\xi)$ differs significantly from $u_0\operatorname{sign}\xi$ when $|\xi| \lesssim \epsilon$, making a contribution of the order ϵ to I_1. In addition, when $|\xi| \gg \epsilon$ this quantity contains small but slowly attenuating terms of order $\nu_j|t_j|^{-1}\exp\{-\nu_j|\xi|\} \times \sin(|t_j|\xi)$. These terms make a contribution of order $\Sigma\nu_j/t_j^2$ to I_1. Thus,

$$I_1 \sim \varepsilon + \sum_j \frac{\nu_j}{t_j^2}.$$

The functional I_2 is best calculated in the Fourier representation:

$$I_2 = \frac{1}{4\pi} \int_{-\infty}^{\infty} \left(|u(t)|^2 - \frac{4u_0^2}{t^2} \right) dt.$$

The contribution from the regions remote from the points $t = t_j$ are of the order of

$$\int_1^{\infty} \frac{\varepsilon^2}{\operatorname{sh}^2 \varepsilon t} t^3 \, dt \sim \frac{1}{\varepsilon^2}.$$

The contribution from the vicinities of these points is of the order of

$$\sum_j \frac{\varepsilon^2}{\operatorname{sh}^2 \varepsilon t_j} |t_j|^3 \frac{1}{\nu_j}.$$

We thus obtain

$$\chi I_1 + I_2 \sim \chi \left(\varepsilon + \sum_j \frac{\nu_j}{t_j^2} \right) + \frac{1}{\varepsilon^2} + \sum_j \frac{\varepsilon^2}{\operatorname{sh}^2 \varepsilon t_j} |t_j|^3 \frac{1}{\nu_j}. \quad \text{(A.15)}$$

Varying ν_j at fixed ϵ, we obtain

$$\nu_j \sim \chi^{-1/2} \frac{\varepsilon}{\operatorname{sh} \varepsilon |t_j|} |t_j|^{5/2}.$$

Substituting this result in (A.15) we get

$$\chi I_1 + I_2 \sim \chi\varepsilon + \chi^{1/2} / \varepsilon^{1/2} + 1/\varepsilon^2.$$

This expression reaches a maximum (of order $\chi^{2/3}$) at $\epsilon \sim \chi^{-1/3}$, thus confirming the estimate given in Sec. 3 of this paper.

The quantities ν_j are of order $\chi^{-1/2}$ at small values of j; with increasing number they increase, reaching at $j \sim 1/\epsilon$ values on the order of unity, and then decreasing exponentially. When $|\xi| \ll \epsilon$, the function $u(\xi)$ determined by formulas (A.12) and (A.14) increases like $\xi/\epsilon^{5/2}$. It then decreases to a value of the order of unity when $1 \gg |\xi| \gg \epsilon$. The approach to the asymptotic values $\pm u_0$ is slow, owing to the presence in $u(\xi)$ of terms proportional to

$$\frac{\varepsilon}{\operatorname{sh} \varepsilon |t_j|} |t_j|^{1/2} \exp\{-\nu_j |\xi|\} \sin(|t_j| \xi).$$

This character of the function $u(\xi)$ contradicts the exact equation (A.8); this is not connected with the concrete choice of the functions $g(\xi)$ and $\tau(t)$. This shows that actually the structure of the solution is more complicated. Apparently, however, the relation $\Delta \sim \chi^{2/3}$ obtained by us by a variational method is correct.

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