FORMATION OF A FORERUNNER IN THE PASSAGE OF THE FRONT OF A LIGHT PULSE THROUGH A VACUUM-MEDIUM INTERFACE

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Analytic expressions are obtained for the forerunner field. Spectral and energy estimates are made. The possibility of its experimental observation is discussed.

1. The first to consider in detail transient processes in the optical range were Sommerfeld and Brillouin[1,2], who showed that the principles of special relativity theory are not violated in the anomalous-dispersion region. Subsequently, this question was raised many times in connection with an analysis of the concepts of phase and group velocity[3,4] and the propagation of radio pulses in the ionosphere[5]. It was shown that when the front of the light wave enters a dispersive medium, a transient process takes place, which leads to the formation of a "forerunner" propagating ahead of the stationary signal. Estimates of the duration and energy of the forerunner have shown that the registration of the forerunner lies beyond the capabilities of the then existing experimental techniques. However, recently developed experimental methods make it possible to register time intervals down to 10^{-14} sec[6,7] and to generate powerful ultrashort light pulses with steep fronts. In this connection, it is timely to raise the question of the possibility of experimentally studying transient processes in the optical frequency range.

To this end, we consider the character of the transient processes that develop upon reflection and refraction of a light pulse by a plane boundary.

2. We specify an electric field incident on a plane interface y = 0 between vacuum and a medium, in terms of a unit-amplitude wave:

$$E_0(r,t)=E_z=\begin{cases}\exp\left\{-i\omega_0\left(t-\dfrac{x\sin\alpha+y\cos\alpha}{c}\right)\right\} & t>\dfrac{x\sin\alpha+y\cos\alpha}{c}\\ 0, & t<\dfrac{x\sin\alpha+y\cos\alpha}{c},\end{cases}\tag{1}$$

the front of which has the surface

$$x\sin\alpha+y\cos\alpha=ct.\tag{2}$$

Let us write down the spectral components of the incident (E_0^ω), reflected (E_r^ω), and refracted (E_d^ω) waves in the form

$$E_0^\omega=F(\omega)\exp(-i\omega t+i\mathbf{k}_0\mathbf{r}),\tag{3a}$$

$$E_r^\omega=F(\omega)f(\alpha,n(\omega))\exp(-i\omega t+i\mathbf{k}_2\mathbf{r}),\tag{3b}$$

$$E_d^\omega=F(\omega)g(\alpha,\,n(\omega))\exp(-i\omega t+i\mathbf{k}_1\mathbf{r}),\tag{3c}$$

where $n(\omega)$ is the refractive index of the medium;

$$\mathbf{k}_0=\left\{\frac{\omega}{c}\sin\alpha,\ \frac{\omega}{c}\cos\alpha,0\right\},\ \mathbf{k}_1=\left\{\frac{\omega}{c}n(\omega)\sin\beta,\ \frac{\omega}{c}n(\omega)\cos\beta,0\right\};$$

$$\mathbf{k}_2=\left\{\frac{\omega}{c}\sin\alpha,\ -\frac{\omega}{c}\cos\alpha,0\right\};\ F(\omega)=-\frac{1}{2\pi i(\omega-\omega_0)};$$

$$\sin\alpha=n(\omega)\sin\beta;$$

$$f(\alpha,n(\omega))=\frac{\cos\alpha-\sqrt{n^2(\omega)-\sin^2\alpha}}{\cos\alpha+\sqrt{n^2(\omega)-\sin^2\alpha}};$$

$$g(\alpha,n(\omega))=\frac{2\cos\alpha}{\cos\alpha+\sqrt{n^2(\omega)-\sin^2\alpha}}\tag{4}$$

are the Fresnel coefficients for the reflected and refracted waves. By summing the spectral components, we can easily find the total field.

The steady state of the process, established after the lapse of a time interval longer than the transient times characteristic of the medium, satisfies the extinction theorem of Ewald and Oseen[8,9]. In this case two waves are formed in the medium, a refracted wave whose phase velocity equals c/n, and a non-refracted wave propagating with velocity c. This second wave exactly cancels out the field of the incident wave in the medium. However, during a time interval comparable with the characteristic times, the refracted and non-refracted waves still do not have time to form. A forerunner is then produced in a region sufficiently close to the front and propagates with velocity c in the direction of the incident wave.

The field of the signal in the medium is obtained by summing its harmonics. Each harmonic is a stationary signal, which "knows nothing" of its origin from a limited wave train, and behaves as a plane wave in a dispersive medium. Its propagation is described by the stationary refractive index and by the boundary conditions (Fresnel formulas), which are also stationary. The optical characteristics of the medium are determined by the natural frequencies ω_e of the bound electrons and their relaxation times τ_e. During a time shorter than $1/\omega_e$ or τ_e from the instant of arrival of the front at the point under consideration, the excitation and relaxation processes play a secondary role. For the damped classical oscillator model, this means that the electron does not have time to acquire neither a velocity nor a displacement from the equilibrium position. Therefore, if we use for the stationary harmonics the plasma index

$$n^2(\omega)=1-\Omega^2/\omega^2,\tag{5}$$

where Ω is the electron plasma frequency, then the obtained expressions should describe accurately the part of the signal close to the front. It is utterly meaningless to speak of a refractive index for harmonics of too high a frequency (wavelength of the order of the interatomic distances).

We can now write expressions describing the first stage of the transient process, i.e., the forerunner, the field of which for the refracted and reflected wave is described, in accordance with the foregoing, by the expressions ($a \gtrsim 0$)

$$E_d = -\frac{1}{2\pi i}\int_{ia-\infty}^{ia+\infty}\frac{2\omega}{(\omega-\omega_0)(\omega+\sqrt{\omega^2-\Omega^2/\cos^2\alpha})}$$

$$\times\exp\left\{-i\omega\left(t-\frac{x\sin\alpha}{c}-\frac{y\cos\alpha}{c}\sqrt{\omega^2-\Omega^2/\cos^2\alpha}\right)\right\} \quad (6a)$$

$$E_r = -\frac{1}{2\pi i}\int_{ia-\infty}^{ai+\infty}\frac{\omega-\sqrt{\omega^2-\Omega^2/\cos^2\alpha}}{(\omega-\omega_0)(\omega+\sqrt{\omega^2-\Omega^2/\cos^2\alpha})}$$

$$\times\exp\left\{-i\omega\left(t-\frac{x\sin\alpha+y\cos\alpha}{c}\right)\right\}. \quad (6b)$$

To calculate these integrals we use the procedure employed by Denisov[10]. Namely, we make a successive change of variables: $\omega = ip$, $p = \Omega(1/z - z)/2\cos\alpha$, and then $z = -\gamma w$ in the integral (6a) and $z = -w$ in the integral (6b). The integration contour in the w plane is then a circle with center at the origin; this circle does not enclose any singularities, and is directed counterclockwise. Separating in the integrand the factor $\exp\{q(w - 1/w)/2\}$, which is the generating function for the Bessel functions

$$\frac{1}{2\pi i}\oint\exp\left\{q\left(w-\frac{1}{w}\right)\Big/2\right\}w^k dw = (-1)^{k+1}J_{k+1}(q) = J_{-k-1}(q),$$

and expanding the remaining part of the integrand in powers of w, we obtain the solution in the form of a series of Bessel functions

$$E_d = J_0(\mu)-\gamma^2J_2(\mu)+\sum_{k=1}^{\infty}\left[\frac{1}{(w')^k}+\frac{1}{(w'')^k}\right](-1)^k(J_k(\mu)-\gamma^2J_{k+2}(\mu)), \quad (7a)$$

where

$$w' = -i\frac{\omega_0}{2\gamma}\cos\alpha-\frac{1}{\gamma}\sqrt{1-\frac{\omega_0^2\cos^2\alpha}{\Omega^2}},\quad \gamma=\sqrt{\frac{ct-x\sin\alpha-y\cos\alpha}{ct-x\sin\alpha+y\cos\alpha}}$$

$$w'' = -i\frac{\omega_0}{2\gamma}\cos\alpha+\frac{1}{\gamma}\sqrt{1-\frac{\omega_0^2\cos^2\alpha}{\Omega^2}}$$

$$\mu = \frac{\Omega}{\cos\alpha}\sqrt{\left(t-\frac{x\sin\alpha}{c}\right)^2-\left(\frac{y}{c}\cos\alpha\right)^2}.$$

$$E_r = \frac{1}{2\sqrt{1-\frac{\omega_0^2\cos^2\alpha}{\Omega^2}}}\sum_{k=1}^{\infty}\left(\frac{1}{w_1^k}-\frac{1}{w_2^k}\right)(-1)^k(J_{k+1}(\chi)+J_{k+3}(\chi)), \quad (7b)$$

where

$$\chi=\frac{\Omega}{\cos\alpha}\left(t-\frac{x\sin\alpha+y\cos\alpha}{c}\right),\ w_1=-i\frac{\omega_0}{\Omega}\cos\alpha-\sqrt{1-\frac{\omega_0^2\cos^2\alpha}{\Omega^2}},$$

$$w_2=-i\frac{\omega_0}{\Omega}\cos\alpha+\sqrt{1-\frac{\omega_0^2\cos^2\alpha}{\Omega^2}}.$$

Near the wave front, μ and χ are small, and, as can be seen, we can retain in (7a) and (7b) only the first terms:

$$E_d \approx J_0(\mu);\qquad E_r\approx -\tfrac{1}{2}J_2(\chi).$$

Thus, for example, the field intensity of the forerunner in the passing wave oscillates rapidly and decreases like $J_0(\mu)$. On the other hand, in the reflected wave the forerunner is delayed by a time Δt, which is determined by the position of the first maximum of the function $J_2(\chi)$. For normal incidence, $\Delta t \sim 3/\Omega$. If we assume $\Omega \sim 10^{16}$, corresponding to an electron concentration 10^{22} cm^{-3}, then the depth of the region in which the forerunner of the reflected ray is produced is $\Delta l = c\Delta t/2 \approx 5\times10^{-6}$ cm, i.e., of the order of an optical wavelength or of the skin depth in metals.

The front $\mu = 0$ of the incident wave propagates in the medium without being refracted and is accompanied by a forerunner.

3. To obtain the spectral and energy estimates of the forerunner, it is convenient to consider the more realistic problem of the passage of a light pulse through a plane-parallel plate. The problem is formulated in the same manner as the preceding one, except that the expression for the spectral component E_t^{ω} of the transmitted wave contains an additional phase shift over the thickness d of the plate, and one more Fresnel coefficient $g(\beta, 1/n(\omega))$, corresponding to the emergence of the light from the plate into the vacuum (inasmuch as we are interested only in the transient process, multiple reflections in the plate are disregarded). In this case we have

$$E_t^{\omega} = F(\omega)g(\alpha,n(\omega))g(\beta,1/n(\omega))\exp\{-i\omega t+i\mathbf{k}_0\mathbf{r}+i(\mathbf{k}_1-\mathbf{k}_0)\delta\}. \quad (8)$$

Here

$$\delta=\{d\,\mathrm{tg}\,\beta, d, 0\},\quad g\left(\beta,\frac{1}{n(\omega)}\right)=\frac{2\cos\beta}{\cos\beta+\sqrt{1/n^2(\omega)-\cos^2\beta}}$$

$$E_t = -\frac{1}{2\pi i}\int_{ia-\infty}^{ia+\infty}\frac{4\omega\sqrt{\omega^2-\Omega^2/\cos^2\alpha}}{(\omega-\omega_0)(\omega+\sqrt{\omega^2-\Omega^2/\cos^2\alpha})}$$

$$\times\exp\{-i\omega T+i\zeta\sqrt{\omega^2-\Omega^2/\cos^2\alpha}\}, \quad (9)$$

where

$$\zeta=(d/c)\cos\alpha,\quad T=t-[x\sin\alpha+(y-d)\cos\alpha]/c.$$

Calculating the integral, we obtain

$$E_t=J_0(\eta)-\sigma^4J_4(\eta)+\sum_{k=1}^{\infty}\left(\frac{1}{w_a^k}+\frac{1}{w_b^k}\right)(-1)^k(J_k(\eta)-\sigma^4J_{k+4}(\eta)), \quad (10)$$

where

$$w_a=-i\frac{\omega_0}{\sigma\Omega}\cos\alpha-\frac{1}{\sigma}\sqrt{1-\frac{\omega_0^2\cos^2\alpha}{\Omega^2}},\quad \eta=\frac{\Omega}{\cos\alpha}\sqrt{T^2-\zeta^2},$$

$$w_b=-i\frac{\omega_0}{\sigma\Omega}\cos\alpha+\frac{1}{\sigma}\sqrt{1-\frac{\omega_0^2\cos^2\alpha}{\Omega^2}},\quad \sigma=\sqrt{\frac{T-\zeta}{T+\zeta}}.$$

Since the plasma is optically less dense than the vacuum, a monochromatic wave of frequency ω_0, emerging from the plate, can experience total reflection. To this end it is necessary to have $\alpha \ge \alpha_{cr}$ and $\sin\alpha_{cr} = n(\omega_0)$. For a flat plate, at $\alpha = \alpha_{cr}$, the Fresnel coefficient at the second boundary $g(\beta, 1/n(\omega_0)) = 0$ and there should be no stationary field behind the plate. Therefore the expression obtained from formula (10) with $\alpha = \alpha_{cr}$ should describe only the forerunner. Owing to the universality of the properties of the medium with respect to the forerunner, this conclusion is valid for any medium. In this case

$$E_t=J_0(\eta)-2\sigma^2J_2(\eta)+\sigma^4J_4(\eta)-2i(\sigma J_1(\eta)-\sigma^3J_3(\eta)). \quad (11)$$

The real part of (11) corresponds to a signal beginning with $t = 0$, since $\cos\omega t$ is imaginary, like $\sin\omega t$.

It is easy to see that if $\omega_0 = \Omega$, total reflection occurs already at normal incidence. Then

$\zeta = d/c, \quad \eta = \Omega\sqrt{(t-(y-d)/c)^2 - (d/c)^2}, \quad \sigma = \sqrt{\frac{t-y/c}{t-y/c+2d/c}}$

Near the front we have

$$\eta \approx \Omega\sqrt{2(t-y/c)d/c} = \Omega\sqrt{2\tau d/c}.$$

The distances between the zeroes of the Bessel functions are approximately equal to π. Therefore, the distance between the m-th and (m + 1)-st zeroes of the forerunner is $\Delta\tau_m \approx \pi^2 mc/\Omega^2 d$, and consequently the "running" frequency of the forerunner is $\omega_m = 2\pi/2\Delta\tau_m = \Omega^2 d/\pi mc$. Since $\Omega\sqrt{2\tau_m d/c} \gtrsim \pi m$, we have

$$\omega_m \gtrsim \Omega\sqrt{d/2c\tau}.$$

It is seen from the last formula that the thicker the plate, the closer the zeroes at the front of the signal emerging from the plate.

Since the parameter σ is small at the front, and the values of the Bessel functions decrease rapidly away from the front, we can confine ourselves in the calculation of the total spectrum of the signal behind the plate only to $J_0(\eta)$. A simple calculation shows that the maximum of the total spectrum lies, as expected, at the plasma Langmuir frequency Ω.

Let us estimate the energy corresponding to the transient. For a cosinusoidal signal, the energy flux through a unit area on the plane y = d equals

$$W_t^{\cos} = \frac{c}{8\pi}\int_{d/c}^{3d/c} \{J_0(\xi) - 2\rho^2 J_2(\xi) + \rho^4 J_4(\xi)\}^2 dt, \tag{12}$$

and for a sinusoidal signal

$$W_t^{\sin} = \frac{c}{8\pi} 4 \int_{d/c}^{3d/c} \{\rho J_1(\xi) - \rho^3 J_3(\xi)\}^2 dt, \tag{13}$$

where

$$\xi = \Omega\sqrt{t^2 - (d/c)^2}, \quad \rho = \sqrt{\frac{t-d/c}{t+d/c}}.$$

The upper limit of the integral, 3d/c, is chosen such as to cut off all the multiply reflected rays. Using the theorem of the mean, noting that $\Omega d/c \sim 10^5$ when d = 1 cm, and using the asymptotic expression for Bessel functions, we obtain

$$W_t^{\sin},\ W_t^{\cos} \approx c/4\pi\Omega \text{ и } W_t/W_0 \sim \Omega^{-1}.$$

The presently attained laser powers and optical-radiation receiver sensitivities make a direct observation of the forerunner possible.

In the foregoing approximation, the forerunner propagates in the medium without damping. In an ideal plasma, damping can be due only to collisions. The refractive index with allowance for collisions[5] is

$$n(\omega) = \left(1 - \frac{\Omega^2}{\omega^2}\frac{1}{1 - i\nu/\omega}\right)^{1/2}, \tag{14}$$

where $\tau = 1/\nu$ is the average time between collisions. The argument of the exponential factor in the integral of (9), which contains the refractive index, reduces in this case to the form

$$\Phi = i\zeta\omega n(\omega) = -\zeta\frac{\Omega}{2}\left(\frac{1}{z} + z\right)\left\{\frac{1 - 2\nu(1/z - z)/\Omega(1/z + z)^2}{1 - 2\nu/\Omega(1/z - z)}\right\}^{1/2}.$$

Since the integration circle on the z-plane can have an arbitrarily small radius, the expression in the curly brackets can be expanded in a series:

$$\Phi \approx -\zeta\frac{\Omega}{2}\left(\frac{1}{z} + z\right) - 2\nu\zeta z^2.$$

This produces under the integral sign an additional damped exponential factor $\exp(-2\nu\zeta z^2) \approx 1 - 2\nu\zeta z^2$, which leads to the appearance of a series of additional terms in (10), starting with J_2. For the case of total internal reflection, formula (11) assumes the form

$$E_t \approx J_0 + 2\sigma^2(1 - \nu\zeta)J_2 - \sigma^4(1 + 4\nu\zeta)J_4 + 2i(\sigma J_1 - \sigma^3(1 + 2\nu\zeta)J_3).$$

For not too thick plates, the parameter $\nu\zeta = \nu d/c$ is small and the contribution of the collisions is accordingly small.

The refractive index for a magnetoactive medium

$$n(\omega) = \left(1 - \frac{\Omega^2}{\omega^2}\frac{1}{1 \pm \omega_H/\omega}\right)^{1/2}$$

has the same form as (14), from which we see that the optical activity of the medium has practically no effect on the forerunner.

For an experimental observation of the forerunner, the most convenient schemes are those permitting separation of the forerunner from the stationary signal. It is possible to use for this purpose a total-internal-reflection prism, crossed Nicol prisms, a Kerr cell, etc., which transmit only the forerunner in the forward direction. It should be borne in mind here that the maximum of the forerunner intensity does not correspond to the main signal frequency, and is shifted towards frequencies closer to the Langmuir frequency of the given medium.

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