SPIN WAVES IN A DEGENERATE ELECTRON FLUID

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The conditions for existence of weakly decaying spin waves in an electron fluid with an anisotropic Fermi surface are found for a simple model which takes into account interelectron correlations.

THE discovery of spin waves in nonferromagnetic metals,[1,2] which was predicted by the theory of degenerate electron fluid,[3] raised the problem of the construction of a theory of waves in a Fermi liquid which takes into account the anisotropy of the Fermi surface of the conduction electrons.[4] In the present communication, we have set as our aim the elucidation of the condition for the existence of weakly decaying spin waves which are determined by the anisotropies of the Fermi surface. In this connection, we take the simplest model for account of the interelectron correlations, which allows us to separate the role of the effect of Fermi-liquid interaction and the effect of the anisotropy of the Fermi surface. That is, we assume that the spin part of the energy of the electron in a Fermi liquid has the form[5,6]

$$\delta\varepsilon_2(\mathbf{p}) = -\mu_0\mathbf{B} + \int d\mathbf{p}'\,\psi\delta\sigma(\mathbf{p}'), \tag{1}$$

where $\mathbf{B}$ is the magnetic induction, μ_0 the magnetic moment of the electron, $\delta\sigma$ the increase in the spin density in phase space and, finally, ψ is a constant which characterizes the correlation of the electrons. The assumption on the independence of ψ of the momentum is the essential simplification which makes it possible to go comparatively far in the analysis of the possibility of propagation of the spin waves. At the same time, we can suppose that the approximation (1) correctly takes into account the qualitative effect of the production of spin waves, although the description of certain quantitative effects here can be shown to be incomplete.

Before we consider the oscillations of the magnetization of the electron fluid, we note that in a constant and homogeneous magnetic field $\mathbf{H}_0$, in correspondence with (1), the spin density of the equilibrium state has the form

$$\sigma = -\gamma\frac{\partial f_0}{\partial\varepsilon}\mathbf{H}_0. \tag{2}$$

Here $f_0(\epsilon)$ is the Fermi distribution and

$$\frac{\partial f_0}{\partial\varepsilon} = -\frac{2}{(2\pi\hbar)^3}\delta(\varepsilon-\varepsilon_0), \tag{3}$$

and, finally,

$$\gamma = \mu_0/(1+\beta), \tag{4}$$

where

$$\beta = \frac{2\psi}{(2\pi\hbar)^3}\int\frac{dS}{v}. \tag{5}$$

In Eq. (5), the integration is carried out over the Fermi surface, while v is the velocity of the electron.

According to (2), the static paramagnetic susceptibility χ of the electron fluid has the form[6]

$$\chi = \mu_0\gamma\int\frac{dS}{v}\frac{2}{(2\pi\hbar)^3}. \tag{6}$$

Using Eq. (2), which describes the equilibrium state of the electron fluid in a constant magnetic field, we can consider small oscillations of the spin density in phase space. In accord with the existing experimental situation, we take $\delta\sigma$ perpendicular to the direction of the constant field $\mathbf{H}_0$. Then the kinetic equation for the spin density can be written in the form[6]

$$\frac{\partial\delta\sigma}{\partial t} + \left(\mathbf{v}\frac{\partial}{\partial\mathbf{r}} + \frac{e}{c}[\mathbf{vH}_0]\frac{\partial}{\partial\mathbf{p}}\right)\left(\delta\sigma - \frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_2\right)$$
$$-\frac{2\gamma}{\hbar}\left[\mathbf{H}_0, \delta\sigma - \frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_2\right] = -\left(\frac{1}{\tau}+\frac{1}{\tau_2}\right)\left(\delta\sigma - \frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_2\right)$$
$$+\frac{1}{\tau}\frac{\partial f_0}{\partial\varepsilon}\left[\int d\mathbf{p}\frac{\partial f_0}{\partial\varepsilon}\right]^{-1}\int d\mathbf{p}\left(\delta\sigma - \frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_2\right). \tag{7}^*$$

Here we must use Eq. (1) for $\delta\epsilon_2$, in which we employ in place of $\mathbf{B}$ the variable magnetic induction $\mathbf{b}$. Furthermore, e is the negative change of the electron, τ the characteristic relaxation time of the momentum of the electron, and τ_2 the time for spin transfer. We shall be interested in the case of infrequent collections, which justifies the use of the model collision integral on the right side of the kinetic equation (7).

Solving Eq. (7), we can find the nonequilibrium density of magnetization of the electron fluid:

$$\mathbf{m}(\mathbf{r}, t) = \mu_0\int d\mathbf{p}\,\delta\sigma(\mathbf{p}, \mathbf{r}, t). \tag{8}$$

Orienting the z axis along the magnetic field and denoting $m_\pm = m_x \pm im_y$, $b_\pm = b_x \pm ib_y$, after simple calculations, we find

$$m_\pm = b_\pm\frac{\chi_\pm}{1+4\pi\chi_\pm}, \tag{9}$$

where the high frequency magnetic susceptibility has the form

$$\chi(\omega,\mathbf{k}) = \chi\frac{1-(1+i/\omega\tau)X}{1-[\beta/(1+\beta)+i/\omega\tau]X - 4\pi\chi[1-(1+i/\omega\tau)X]} \tag{10}$$

and

$$X(\omega,\mathbf{k}) = \left[\int\frac{dS}{v}\right]^{-1}\int\frac{dS}{v}\frac{i\omega}{\Omega}\int_{\infty\cdot\mathrm{sign}\,\Omega} d\varphi'$$
$$\times\exp\left\{\frac{i}{\Omega}\int_{\varphi}^{\varphi'} d\varphi''\left[\omega \pm \Omega_0 + \frac{i}{\tau} + \frac{i}{\tau_2} - \mathbf{kv}(\varphi'')\right]\right\}. \tag{11}$$

Here $\Omega_0 = 2\gamma H_0/\hbar = 2\mu_0 H_0/\hbar(1+\beta) = \omega_s/(1+\beta)$, ω_s is the Bloch frequency of spin resonance, Ω the cyclotron frequency of the gyroscopic rotation of the electron, determined by the formula

$$\Omega = 2\pi\frac{eH_0}{c}\left(\frac{\partial S}{\partial\varepsilon}\right)^{-1} < 0, \tag{12}$$

*$[\mathbf{vH}_0] \equiv \mathbf{v}\times\mathbf{H}_0$.

where $S = S(\epsilon, p_z)$ is the cross sectional area of the surface $\epsilon(p) = \epsilon_0$ in the plane $p \cdot H_0 = p_z H_0 = \text{const}$, which represents the trajectory of the electron in the momentum space. Finally, φ is the angle determining the position of the electron on its trajectory; it changes from zero to 2π, assuming that the trajectory is closed;[7] in this case, in particular,

$$\frac{dp_y}{d\varphi} = \frac{eH_0}{c\Omega} v_x. \tag{13}$$

The dispersion equation of spin waves corresponds to the vanishing of the high frequency magnetic susceptibility:[6]

$$\mu_\pm(\omega, \mathbf{k}) = 1 + 4\pi\chi_\pm(\omega, \mathbf{k}) = 0. \tag{14}$$

Therefore, the spectrum of the spin waves is determined by the relation

$$1 = \left(\frac{\beta}{1+\beta} + \frac{i}{\omega\tau}\right) X(\omega, \mathbf{k}). \tag{15}$$

Considering the consequences following from this dispersion relation, we return first to the case of long waves, much longer than the radius of the gyroscopic rotation of the electron, when the right side of Eq. (15) can be expanded in a power series in the wave vector. With accuracy up to terms of order k^2, we have

$$\omega \pm \omega_s + \frac{i}{\tau_2} = -2\omega\left(\beta + i\frac{1+\beta}{\omega\tau}\right)\left[\int \frac{dS}{v}\right]^{-1} \times \int \frac{dS}{v} \sum_{n=-\infty}^{+\infty} \frac{|\mathbf{k}\mathbf{v}_n(\varepsilon, p_z)|^2}{n^2\Omega^2(\varepsilon, p_z) - (\omega \pm \Omega_0 + i/\tau + i/\tau_2)^2}, \tag{16}$$

where we use the expansion

$$\mathbf{v}(\varepsilon, p_z, \varphi) = \sum_{n=-\infty}^{+\infty} \mathbf{v}_n(\varepsilon, p_z) e^{in\varphi}. \tag{17}$$

for the electron velocity vector. For an anisotropic Fermi surface, this series is generally infinite. In the vicinity of the usual spin resonance, Eq. (16) gives

$$\omega = \mp\omega_s - \frac{i}{\tau_2} \pm 2\omega_s\left(\beta \mp i\frac{1+\beta}{\omega_s\tau}\right)\left[\int \frac{dS}{v}\right]^{-1} \times \int \frac{dS}{v} \sum_{n=-\infty}^{+\infty} \frac{|\mathbf{k}\mathbf{v}_n(\varepsilon, p_z)|^2}{n^2\Omega^2(\varepsilon, p_z) - \omega_s^2[\beta/(1+\beta) \pm i\nu/\omega_s]^2}. \tag{18}$$

Here we have used the notation $\nu = 1/\tau + 1/\tau_2$. It is obvious that the spin waves, which occur near the ordinary spin resonance $\omega = \omega_s$ can be shown to be weakly damped only for

$$\omega_s \gg 1/\tau \quad (\gg 1/\tau_2). \tag{19}$$

Under such conditions of the smallness of the effects of collisions, Eq. (18) has the form $(\omega > 0)$

$$\omega = \omega_s - 2\omega_s\beta(1+\beta)^2\left[\int \frac{dS}{v}\right]^{-1} \int \frac{dS}{v} \sum_{n=-\infty}^{+\infty} \frac{|\mathbf{k}\mathbf{v}_n(\varepsilon, p_z)|^2}{n^2\Omega^2(\varepsilon, p_z)(1+\beta)^2 - \beta^2\omega_s^2}. \tag{20}$$

In order that collision-free Landau damping not take place, the following conditions must be satisfied (see[8])

$$\omega \pm \Omega_0 = n\Omega(\varepsilon, p_z). \tag{21}$$

It is evident that such a requirement always holds when the absolute value of the left side of the inequality (21) is less than the smallest value of the cyclotron frequency

$$|\omega \pm \Omega_0| < \Omega_{minimorum}. \tag{22}$$

In the opposite case, thanks to the dependence of the cyclotron frequency on p_z, the damping decrement of the spin waves that is characteristic for the anisotropic Fermi surface [according to Eq. (20)] is generally comparable with the difference in the frequencies of the spin wave and the frequency of the electron spin resonance. On the other hand, as also follows from Eq. (20), even if the equality (31) is allowed, collision-free damping can be comparatively small for comparatively small β and in such directions of propagation of the spin wave for which the quantity

$$\int \frac{dS}{v} |\mathbf{k}\mathbf{v}_0(\varepsilon, p_z)|^2 \tag{23}$$

does not become small.

A definite simplification of the general theory occurs for the case of propagation of spin waves transverse to a constant field. By orienting the x axis along the wave vector $\mathbf{k}$, and taking the relation (13) into account, we can describe Eq. (11) in the form

$$X(\omega, \mathbf{k}) = -\left[\int_{\varepsilon=\varepsilon_0} \frac{dp_z}{\Omega}\right]^{-1} \int_{\varepsilon=\varepsilon_0} \frac{dp_z}{\Omega} \frac{i\omega}{2\pi\Omega} \times \left[1 - \exp\left\{2\pi i\left(\frac{\omega \pm \Omega_0}{\Omega} + \frac{i}{\Omega\tau} + \frac{i}{\Omega\tau_2}\right)\right\}\right]^{-1} \times \int_0^{2\pi} d\varphi \exp\left\{-i\frac{\omega \pm \Omega_0}{\Omega}\varphi + \left(\frac{1}{\Omega\tau} + \frac{1}{\Omega\tau_2}\right)\varphi + \frac{ikc}{eH_0} p_y(\varphi)\right\} \times \int_\varphi^{\varphi+2\pi} d\varphi' \exp\left\{i\frac{\omega \pm \Omega_0}{\Omega}\varphi' - \left(\frac{1}{\Omega\tau} + \frac{1}{\Omega\tau_2}\right)\varphi' - \frac{ikc}{eH_0} p_y(\varphi')\right\}. \tag{24}$$

Equation (24) in the case of an anisotopic Fermi surface possesses resonance properties for

$$\omega \pm \Omega_0 = n\Omega_{extr}, \tag{25}$$

where Ω_{extr} represents the extremal values of Ω as a function of p_z (Ω_{max} or Ω_{min}) on the Fermi surface, i.e., for $\epsilon(p) = \epsilon_0$. Leaving only the resonance part of Eq. (24) in the vicinity of the resonance (25), we can write down the following dispersion relation, which determines the spectrum of the spin waves:

$$1 = \left(\frac{\beta}{1+\beta} + \frac{i}{\omega\tau}\right)\left[\int m^* dp_z\right]^{-1} \int_{\varepsilon=\varepsilon_0} \frac{m^* dp_z \omega}{\omega \pm \Omega_0 - n\Omega(p_z) + i/\tau + i/\tau_2} \times \frac{1}{2\pi}\int_0^{2\pi} d\varphi \exp\left\{-i\frac{\omega \pm \Omega_0}{\Omega}\varphi + \left(\frac{1}{\Omega\tau} + \frac{1}{\Omega\tau_2}\right)\varphi + \frac{ikc}{eH_0} p_y(\varphi)\right\} \times \frac{1}{2\pi}\int_0^{2\pi} d\varphi' \exp\left\{i\frac{\omega \pm \Omega_0}{\Omega}\varphi' - \left(\frac{1}{\Omega\tau} + \frac{1}{\Omega\tau_2}\right)\varphi' - \frac{ikc}{eH_0} p_y(\varphi')\right\}. \tag{26}$$

Here $m^* = eH_0/c\Omega$ is the effective mass of the electron on the Fermi surface corresponding to the given value of p_z.

The spin waves, according to Eq. (26), can be weakly damped if

$$|\omega \pm \Omega_0 - n\Omega_{extr}| \gg \frac{1}{\tau} + \frac{1}{\tau_2}. \tag{27}$$

Neglecting collisions in such a limit, we can write Eq. (26) in the form

$$1 + \frac{1}{\beta} = \left[\int m^* dp_z\right]^{-1} \int m^* dp_z \frac{\omega |w_n(p_z)|^2}{\omega \pm \Omega_0 - n\Omega(p_z)}, \tag{28}$$

where

$$w_n(p_z) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \exp\left\{in\varphi - \frac{ikc}{eH_0} p_y(\varphi, p_z)\right\}. \tag{29}$$

In the case of an isotropic Fermi surface, the cyclotron frequency Ω does not depend on p_z, which allows us to obtain Eqs. (2.42)–(4.15) of the Appendix from the dispersion equation (28) of the book of Akhiezer, Bar'yakhtar, and Peletminskiĭ[6].

Proceeding to consideration of the consequences which follow from the dispersion equation (28) for the case of essentially nonisotropic Fermi surface, we first note that for long waves this dispersion equation can be used only for small values of β. It is further necessary to emphasize that the sign of β determines the region of frequencies in which the dispersion equation (28) has solutions. Thus, for $\beta > 0$ or correspondingly $1 \pm \Omega_0/\omega > 0$, we have $|\omega \pm \Omega_0| > |n||\Omega|_{max}$, while for $1 \pm \Omega_0/\omega < 0$, we have $|\omega \pm \Omega_0| < |n||\Omega|_{min}$. On the other hand, for $\beta < 0$ or for $1 \pm \Omega_0/\omega < 0$, we have $|\omega \pm \Omega_0| > |n||\Omega|_{max}$ while for $1 \pm \Omega_0/\omega > 0$ we have $|\omega \pm \Omega_0| < |n||\Omega|_{min}$. These relations determine the limiting values of the spin wave frequencies which are characteristic for the short-wave portion of the spectrum and for arbitrary values of β, which do not violate the condition of the stability of the ground state $(\beta > -1)$.

In the limit of wavelengths less than the Larmor radius of the electron, we can write down the following asymptotic expression for w_n:

$$w_n(p_z) = \sqrt{\frac{|\Omega|}{2\pi k}} \sum_j |v_x'(\varphi_j)|^{-1/2} \exp\left\{in\varphi_j - \frac{ikc}{eH_0} p_y(\varphi_j) + i\frac{\pi}{4}\,\mathrm{sign}\, v_x'(\varphi_j)\right\}, \tag{30}$$

where $v_x' = \partial v_x/\partial\varphi$, while φ_j are the points at which $v_x(\varphi_j) = 0$. By assuming that there are two such points: φ_{max} and φ_{min}, corresponding to the maximum $(p_{y\,max})$ and minimum $(p_{y\,min})$ values of the projection of the momentum of the electron on the y axis, we have

$$|w_n(p_z)|^2 = \frac{|\Omega|}{\pi k}\left\{\frac{1}{2|v_x'(\varphi_{max})|} + \frac{1}{2|v_x'(\varphi_{min})|} + \frac{\cos[n\varphi(p_z) + kD(p_z) + \pi/2]}{|v_x'(\varphi_{max})v_x'(\varphi_{min})|^{1/2}}\right\}. \tag{31}$$

Here

$$\varphi(p_z) = \varphi_{max} - \varphi_{min}, \quad D(p_z) = c[p_{y\,max} - p_{y\,min}]/|e|H_0.$$

In the case $n = 0$, Eq. (28) obviously has the following solution:

$$\omega = \mp\Omega_0\left\{1 - \frac{\beta}{1+\beta}\left[\int m^*\,dp_z\right]^{-1} \int_{\varepsilon=\varepsilon_0} m^*\,dp_z |w_0(p_z)|^2\right\}. \tag{32}$$

Keeping Eq. (31) in mind, we can show that in the limit of short waves, the frequency of the spin wave tends toward Ω_0 for $n = 0$.

For $n \neq 0$, the extremal values of the cyclotron frequency play an important role. Here, we have used the results of the work of Kaner and Skobov[8] as a control; in that case, the integral, similar to that appearing on the right side of our dispersion equation (28), was studied in application to the problem of cyclotron waves in an electron gas. We turn to the case of a central cross section $(p_z = 0)$, in which the cyclotron frequency, which is an even function of p_z, takes on an extremal value. Also, $D(p_z)$ reaches an extremum on the central cross section. In the vicinity of the central cross section, Eq. (31) takes the form (see[8])

$$|w_n(p_z)|^2 = \frac{|\Omega|}{\pi k|v_x'(\varphi_{max})|}\{1 - \sin[kD(p_z) + n\pi]\}. \tag{33}$$

Then, under the assumption of the inequalities

$$|kD''| \ll \frac{n\Omega''_{extr}}{n\Omega_{extr} - (\omega \pm \Omega_0)} > 0 \tag{34}$$

the dispersion equation (28) can be written in the form

$$1 + \frac{1}{\beta} = -\frac{\sqrt{2}\,|m^*\Omega|}{k|v_x'|\int m^*\,dp_z}\left[\frac{n\Omega''_{extr}}{n\Omega_{extr} - (\omega \pm \Omega_0)}\right]^{1/2} \times \frac{n\Omega_{extr} \mp \Omega_0}{n\Omega''_{extr}}\{1 - \sin[kD + \pi n]\}. \tag{35}$$

Here the inequality

$$\beta n[n\Omega_{extr} \mp \Omega_0]\Omega''_{extr} < 0, \tag{36}$$

should be satisfied; this determines the distribution of the spectra of spin waves about the extremal values of the cyclotron frequencies.

The dispersion equation (35) evidently yields

$$\omega = (n\Omega_{extr} \mp \Omega_0)\left\{1 - \frac{2\beta^2}{(1+\beta)^2}\left(\frac{eH_0}{ck}\right)^2 \frac{n\Omega_{extr} \mp \Omega_0}{n\Omega''_{extr}} \times \frac{[1 - \sin(kD + \pi n)]^2}{\left(v_x'\int m^*\,dp_z\right)^2}\right\}. \tag{37}$$

The identical satisfaction of the right side of the inequality (34) then follows, in particular, and also the possibility of satisfaction of the left side of this inequality in the short-wave limit. Account of the inequality (36) allows us to write down the solution (37) in the form of two expressions:

$$\omega = \mp\Omega_0 + n|\Omega|_{max} - \frac{a^2}{n(kD)^2}[1 - \sin(kD + \pi n)]^2 \qquad \omega\beta n > 0, \tag{38}$$

$$\omega = \mp\Omega_0 + n|\Omega|_{min} + \frac{a^2}{n(kD)^2}[1 - \sin(kD - \pi n)]^2, \qquad \omega\beta n < 0, \tag{39}$$

where $a^2 \sim |\Omega|\beta^2(1+\beta)^2$. Equation (38) holds when the cyclotron frequency is maximal on the central cross section, while Eq. (39) holds for a minimum value of the cyclotron frequency.

We note that if the positions of the extrema D and Ω are not identical, then the relative amplitude of oscillations of the frequency of the spin wave as a function of the wave number is less than unity (cf.[8]).

The limitation of our consideration to the short-wave limit leads to the result that the same frequency Ω_0 appears in different resonances. A later, more general theory of an anisotropic electron fluid should, in particular, solve the problem of the spectrum of such frequencies which, as may be thought, differ somewhat from one another.

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