SOVIET PHYSICS JETP VOLUME 28, NUMBER 2 FEBRUARY, 1969

STABLE AND UNSTABLE STATES IN THE $V\theta$ SECTOR OF THE LEE MODEL

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Submitted November 30, 1967

Zh. Eksp. Teor. Fiz. 55, 506-511 (August, 1968)

The behavior of the propagator of the $V\theta$ bound state in the Lee model is investigated with the aid of a known exact solution. It is shown that a unique bound state and resonance arise simultaneously in elastic $V\theta$ scattering in the case of strong coupling. This result is a characteristic field effect and does not exist in the case of potential interaction. Certain other features of $V\theta$ scattering are also indicated.

1. The Lee model was considered by many authors essentially in the sense of its field-theoretical interpretation, determination of the masses of physical particles, and renormalization of the charge. In many papers, this analysis was limited to the region of stable V-particles (see, for example,[1,2]). In papers[3-5] devoted to unstable particles, only general-theoretical questions of introduction and interpretation of unstable states were considered in general, as well as the determination of their masses and the lifetimes. In all these papers, only the $N\theta$ sector of the model was considered, and it is tacitly assumed that there can exist either one bound state or one resonance in $N\theta$ scattering (see, for example[6]). In the former case this state was identified with a stable V particle, and in the latter the resonance was interpreted as an unstable physical V particle. However, it was made clear in[7] that in the $N\theta$ sector, when traditional form factors are used, there can exist simultaneously both stable and resonant states. In accordance with the behavior of these states as functions of the coupling constant and of the renormalized mass $m_V(0)$, it was proposed to interpret these states as being stable and unstable V particles and bound $N\theta$ states (the latter—as complex systems).

In the present paper we present a similar investigation of the possible states in the $V\theta$ sector of the Lee model, exact solutions for which were obtained relatively recently[8-10]. A number of nontrivial singularities which appear in the $V\theta$ sector are explained: the simultaneous appearance (at a certain value of the coupling constant g) of stable and unstable states, the finite interval of variation of the energy of the stable state when g is varied, and certain other features which do not take place in the case of the ordinary potential scattering and in the $N\theta$ sector of the Lee model.

2. The Hamiltonian of the Lee model is written as follows:

$$H = H_0 + H_I, \tag{1}$$

$$H_0 = m_V\psi_V^+\psi_V + \delta m\psi_V^+\psi_V + m_N\psi_N^+\psi_N + \int \omega(\mathbf{q})a^+(\mathbf{q})a(\mathbf{q})\,d^3q, \tag{2}$$

$$H_I = -\frac{g}{(2\pi)^{3/2}}\left\{\psi_V^+\psi_N\int\frac{f(\omega)}{\sqrt{2\omega}}a(\mathbf{q})\,d^3q + \text{h. c.}\right\}, \tag{3}$$

where $\psi_V(\psi_V^+)$ and $\psi_N(\psi_N^+)$—renormalized fermion operators of annihilation (creation) of the V and N particles, satisfying the canonical anticommutation relations; $a(q)(a^+(q))$—boson operator of annihilation (creation) of the θ particle, satisfying the canonical commutation relations: $\omega(q) \equiv \omega = \sqrt{q^2+\mu^2}$, μ—mass of θ particle, g—renormalized coupling constant,

$$g^2 = Zg_0^2, \tag{4}$$

where g_0—nonrenormalized coupling constant and

$$Z^{-1} = 1 + g^2\int\frac{f^2(\omega)}{2\omega}\frac{d^3k}{(\omega-b)^2}, \qquad b = m_V - m_N, \tag{5}$$

$f(\omega)$—real cutoff function, such that

$$f(\mu) = 1, \quad \lim_{\omega\to\infty} f(\omega) = 0, \qquad f(\omega) \neq 0 \quad \text{for } \omega \geqslant \mu; \tag{7}$$

$m_V = m_V^{(0)} + \delta m$, $m_V^{(0)}$--bare mass of the V particle,

$$\delta m = -g^2\int\frac{f^2(\omega)}{2\omega}\frac{d^3k}{\omega-b}. \tag{8}$$

For the investigation of the $V\theta$ sector of the Lee model, we shall need a number of results of the lower $N\theta$ sector. In this sector, the propagator of $S_V'(\omega)$ the physical V particle[11] has the following form (see, for example,[4]):

$$[S_V'(\omega)]^{-1} \equiv \left\langle 0 \left| \psi_V\frac{1}{E+i\varepsilon-H}\psi_V^+ \right| 0 \right\rangle$$

$$= (\omega-b)\left[1 + 4\pi g^2(\omega-b)\int_\mu^\infty\frac{f^2(\omega')\sqrt{\omega'^2-\mu^2}\,\omega'\,d\omega'}{(\omega'-b)^2(\omega'-\omega-i\varepsilon)}\right], \tag{9}$$

where $|0\rangle$ is the vacuum state and $E = m_N + \omega$ is the energy of the V particle. The $N\theta$-scattering matrix $S \equiv S(\omega)$ is connected with the propagator (9) by the relation

$$S(\omega) = [S_V'^*(\omega)]^{-1}/[S_V'(\omega)]^{-1}. \tag{10}$$

The poles of the S matrix give stable states at $E < m_N + \mu$, i.e., when $\omega < \mu$. The energy of these states can be found from the condition

$$[S_V'(\omega)]^{-1} = 0. \tag{11}$$

The resonances of the amplitude of $N\theta$ scattering are obtained at

$$\operatorname{Re}[S_V'(\omega)]^{-1} = 0 \quad (\omega > \mu). \tag{12}$$

We note, incidentally, that the third of the conditions (7) exclude the possibility of appearance of false S-

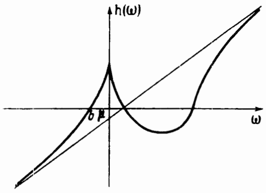

FIG. 1. Behavior of the function $h(\omega)$ for a form factor in the form $f(k) = M^2[M^2 + k^2]^{-1}$, where M determines the character of the cutoff.

matrix poles connected with the form factor $f(\omega)$.

The behavior of the V-particle propagator was investigated in detail in[7]. It was shown that $[S_V'(\omega)]^{-1}$ is analytic everywhere except for a cut on the real axis. The typical behavior of the function

$$h(\omega) \equiv [S_V'(\mu)]^{-1} = (\omega - b)\Big[1 + 4\pi g^2(\omega - b) \times \int\limits_{\mu}^{\infty} \frac{f^2(\omega')\sqrt{\omega'^2 - \mu^2}\,\omega'\,d\omega'}{(\omega' - b)^2(\omega' - \omega - i\varepsilon)}\Big] \quad (9')$$

is shown in Fig. 1. In considering the $V\theta$ sector we shall henceforth assume that the V particle is stable, i.e., we confine ourselves to the quantities $b < \mu$ and to simplify the notation we put $m_N = 0$. To avoid the appearance of ghost states (see[2]) we assume that $g < g_{cr}$, so that $0 < Z \le 1$.

3. We proceed to $V\theta$ sector of the model. The propagator describing the elastic $V\theta$ scattering is of the form

$$[S_{V\theta}'(\omega)]^{-1} \equiv \left\langle 0 \left| \psi_V a \frac{1}{E + i\varepsilon - H} \psi_V^+ a^+ \right| 0 \right\rangle = \left[\frac{1 + h(\omega)A(\omega)}{1 - h(\omega)A(\omega)}\,\frac{1}{h(\omega)}\right]^{-1}, \quad (13)$$

where now $E = m_V + \omega$, and

$$A(\omega) = \frac{1}{\pi}\int\limits_{\mu}^{\infty} \frac{d\omega'}{h(\omega + b - \omega' + i\varepsilon)}\,\mathrm{Im}\,\frac{1}{h(\omega')}. \quad (14)$$

Using for this sector general relations of the type (10)–(12), we find that the resonances and the bound states are the roots of the equation

$$1 - h(\omega)A(\omega) = 0 \quad (15)$$

for $\omega < \mu$ (bound states) and

$$1 - \mathrm{Re}\,[h(\omega)A(\omega)] = 0 \quad (16)$$

for $\omega \ge \mu$ (resonances in elastic $V\theta$ scattering). The imaginary part $\mathrm{Im}[h(\omega)A(\omega)]$ makes it possible to determine the width of the resonance (see[4]). Varying the form factor, we can make the product $\mathrm{Im}\,h(\omega)\,\mathrm{Im}\,A(\omega)$ in the relation

$$\mathrm{Re}[h(\omega)A(\omega)] = \mathrm{Re}\,h(\omega)\,\mathrm{Re}\,A(\omega) - \mathrm{Im}\,h(\omega)\,\mathrm{Im}\,A(\omega) \quad (17)$$

quite small, and assume approximately that the roots of (16) coincide with the roots of the equation

$$1 - \mathrm{Re}\,h(\omega)\,\mathrm{Re}\,A(\omega) = 0. \quad (18)$$

The resultant error causes no fundamental changes.

Let us examine the function $A(\omega)$ in greater detail. It can be written in the form

$$A(\omega) = -\frac{1}{\pi}\int\limits_{\mu}^{\infty} \frac{d\omega'}{[\mathrm{Re}\,h(\omega + b - \omega') + i\,\mathrm{Im}\,h(\omega + b - \omega')]}\,\frac{\mathrm{Im}\,h(\omega')}{|h(\omega')|^2}, \quad (19)$$

where $\mathrm{Im}\,h(\omega + b - \omega') \ne 0$ only when $\omega + b - \omega' > \mu$, i.e., the point $\omega'_{cr} = \omega + b - \mu$ will be a branch point. We consider two cases.

1. $\omega'_{cr} < \mu$, i.e., $\omega < 2\mu - b$. Then $\mathrm{Im}\,h(\omega + b - \omega') \equiv 0$ and we have

$$\mathrm{Re}\,A(\omega) = -\frac{1}{\pi}\,\mathrm{P}\int\limits_{\mu}^{\infty} \frac{d\omega'}{\mathrm{Re}\,h(\omega + b - \omega')}\,\frac{\mathrm{Im}\,h(\omega')}{|h(\omega')|^2}, \quad (20)$$

$$\mathrm{Im}\,A(\omega) = -\mathrm{Im}\,\frac{1}{h(\omega)} \quad (21)$$

(P denotes the integral in the sense of the principal value).

2. $\omega'_{cr} > \mu$, i.e., $\omega > 2\mu - b$. Then $\mathrm{Im}\,h(\omega + b - \omega')$ appears and

$$\mathrm{Re}\,A(\omega) = -\frac{1}{\pi}\int\limits_{\mu}^{\infty} \frac{\mathrm{Re}\,h(\omega + b - \omega')}{|h(\omega + b - \omega')|^2}\,\frac{\mathrm{Im}\,h(\omega')}{|h(\omega')|^2}\,d\omega', \quad (22)$$

$$\mathrm{Im}\,A(\omega) = \frac{1}{\pi}\int\limits_{\mu}^{\omega + b - \mu} \frac{\mathrm{Im}\,h(\omega + b - \omega')}{|h(\omega + b - \omega')|^2}\,\frac{\mathrm{Im}\,h(\omega')}{|h(\omega')|^2}\,d\omega'. \quad (23)$$

Thus, the form of the characteristic function

$$G(\omega) = \mathrm{Re}\,h(\omega)\,\mathrm{Re}\,A(\omega) \quad (24)$$

is given by the relations (9′), (20), and (22). Let us note a number of properties of the functions $h(\omega) \equiv (\omega - b)H(\omega)$ and $A(\omega)$. Both $h(\omega)$ and $A(\omega)$ are real when $\omega < \mu$. From (9′) we can readily notice that $H(b) = 1$, i.e., $h(b) = 0$, and consequently $G(b) = 0$, and that

$$\lim_{|\omega| \to \infty} H(\omega) = Z. \quad (25)$$

Further, $h(\mu) > 0$, and consequently $A(\mu) > 0$. Since $h(\omega)$ is continuous, it has only one root at $\omega < \mu$, and decreases monotonically[7], it follows that

$$\lim_{\omega \to -\infty} h(\omega)A(\omega) = C < 0. \quad (26)$$

In addition, since the factor $\mathrm{Im}[h(\omega)]^{-1}$ is proportional to g^2, we can expect the equality (18) to be satisfied at certain $g < g_{cr}$. Indeed, on going over beyond the critical value of the renormalized interaction constant, the integral in $A(\omega)$ begins to diverge, for in this case $h(\omega + b - \omega')$ has a pole (ghost state) at $\omega + b - \omega' < 0$, i.e., at $\omega' > \omega + b$, and this point always falls in the region of integration, and consequently it is possible to obtain any final state for $G(\omega)$ when $0\ g < g_{cr}$. A plot of $G(\omega)$ is shown in Fig. 2.

It is clear that when g is small $G(\omega$ can be arbitrarily small and positive. When g increases from

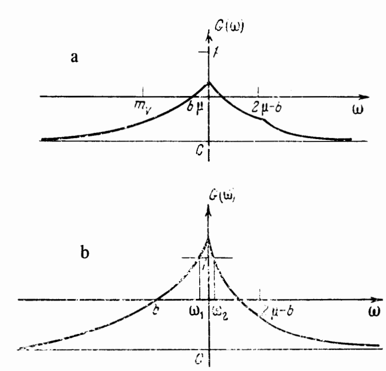

FIG. 2. Form of the function $G(\omega)$: a – weak coupling, $G(\omega) < 1$ (no single particle states); b – strong coupling. The energies ω_1 and ω_2 at which $G(\omega) = 1$ correspond to the bound state and the resonance in $V\theta$ scattering respectively.

zero to a certain g', determined from the condition[12]

$$Z(g') = 1/2, \tag{27}$$

$G(\mu)$ increases from zero to 1. In this case neither resonances nor bound states are observed (weak interaction). Only starting with $g > g'$ do both a bound state and a resonance arise simultaneously (strong interaction).

4. Let us list the characteristic features of the behavior of the function $G(\omega)$:

a) When $\omega < \mu$ the function $G(\omega)$ increases monotonically to the first threshold;

b) the function $G(\omega)$ decreases monotonically when $\mu < \omega < 2\mu - b$ ($b < \mu$), and the derivative experiences a jump at the point μ;

c) when $\omega > 2\mu - b$, the function $G(\omega)$ continues to decrease monotonically (with a jump of the derivative at the point $\omega = 2\mu - b$), and the rate of decrease increases immediately after the jump;

d) $\lim G(\omega) = C < 0$ as $\omega \to \infty$.

Thus, the monotonic decrease of the function $G(\omega)$ and its negative asymptotic value as $\omega \to +\infty$ ensure the existence of a single resonance, which appears simultaneously with a single bound state. We note that simultaneous appearance of both a stationary and a quasi-stationary state has no analog in the case of ordinary potential scattering.The simultaneous appearance of a resonance and a bound state is observed also in the model with the bilinear interaction[13]. We can therefore conclude that such an effect is a multiple one, i.e., it is typical of an interaction with two intermediate particles (the two-meson cloud around an N particle). In addition, the threshold of Nθ production (inelastic channel at $E > 2\mu - b$) is characterized by a unique behavior of the cross section of elastic Vθ scattering, in that there is no infinite jump of the derivative, such as in the behavior of the cross section in the case of potential interaction[14]. We note also that the natural limitation $g < g_{cr}$ (corresponding to a real non-renormalized coupling constant) is the cause of the existence of a lower energy limit for the bound state (cs.[15], where this result was obtained from the approximation solution of the Schrodinger equation), which is not observed in the Nθ sector of the Lee model[7], where the energies of the bound states and of the resonances can be arbitrary when the coupling constant is varied (with the same limitations) $(-\infty < E < +\infty)$.

Thus, an example of a simple field model makes it possible to reveal very interesting features of the field-theoretical description of an unstable state, having no analog in the potential scattering and in the lower sector of the Lee model.

In conclusion, the author thanks V. B. Gostev and A. R. Frenkin for useful discussions.

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Translated by J. G. Adashko
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