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SCATTERING OF ELECTROMAGNETIC WAVES BY TURBULENT PLASMA FLUCTUATIONS

Yu. G. KALININ, D. N. LIN, V. D. RYUTOV and V. A. SKORYUPIN

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We have investigated combination (Raman) scattering of externally generated electromagnetic waves on turbulent plasma oscillations. The characteristic radiation spectrum of the plasma in the centimeter range has been obtained. Estimates are given of the energy density of the electron plasma waves and the ion-acoustic waves. It is shown that under the conditions in the present experiment the characteristic wave vector associated with the scattering oscillations is of the same order as the reciprocal Debye length. The level of plasma turbulence is found to be 0.02.

1. INTRODUCTION

THE results of an investigation of turbulent heating of a plasma by current have been reported in[1,2]. The anomalous resistance, intense electromagnetic radiation, and electron heating observed in these experiments have been attributed to an instability associated with the current in the plasma.

The results of experiments on noise in a turbulent plasma and the transmission of a weak RF electromagnetic probing signal through a turbulent plasma-carrying current have been presented in[3,4]. It has been established in these experiments that at the time the longitudinal current is turned on the radiation spectrum is bounded from above by twice the plasma frequency. As the plasma is heated the conditions favorable for excitation of the two-stream instability are violated and the radiation of electromagnetic energy is terminated, but the plasma heating continues and the rate of heating can even increase. On the basis of these experiments and theoretical considerations with regard to the mechanism by which the plasma is heated by a current,[5] it has been proposed that under these experimental conditions the plasma is subject to the development of an ion-acoustic instability. The level of plasma turbulence W/nT_e as estimated by the transmission of the RF probing signal[4] and from measurements of the electromagnetic radiation and plasma diamagnetism[3], is found to be 0.06–0.1.

In the present work we have undertaken an investiga- of the RF properties of a plasma through which a current flows. We have investigated the characteristic electromagnetic radiation and the combination (Raman) scattering of externally generated electromagnetic waves on the turbulent fluctuations.

It is known that if a plasma supports strong plasma waves and ion-acoustic waves its characteristic radiation can be due to the nonlinear interaction of these waves. The nonlinear interaction of plasma waves leads to the appearance of radiation at frequencies $\sim 2\omega_{pe}$.[6,7] In the nonlinear interaction of plasma waves and ion-acoustic waves radiation will be found in the region of the plasma frequency ω_{pe}.[8,9] Radiation at frequencies of the order of ω_{pe} and $2\omega_{pe}$ has been observed experimentally in laboratory plasma experiments.[9,3]

Combination scattering of an external electromagnetic wave at a frequency ω_0 by turbulent plasma oscillations leads to the appearance of satellite lines which are separated from ω_0 by the frequency of the characteristic plasma oscillations. From a technical point of view it is simplest to investigate the scattering oscillations at the highest possible characteristic frequency (for example, the plasma frequency) so that a small signal at the combination frequency can be easily distinguished from a large signal at the frequency of the external electromagnetic wave. In most cases the ratio of incident power to the power in the scattered radiation is more than 10^7–10^8. In experiments, one detects most frequently the "violet" satellite at the sum frequency. This signal is easily distinguished from the signal at the basic frequency by means of a waveguide filter. A large number of experiments have verified the theory of combination scattering for both bounded and infinite plasmas. In some experiments[10] the scattering oscillations were produced by an external source while in others[11,12] scattering was observed at the plasma frequency in a turbulent plasma. A paper by Demidov and Fanchenko[13] was the first to report estimates of the level of turbulence and of the characteristic wave vector associated with the turbulent oscillations in experiments on combination scattering and radiation in a bounded plasma.

We now present a summary of the basic theoretical relations. Assume that a) the plasma is infinite, that is to say the condition $d \gg c/\omega_{pe}$ is satisfied, where d is the characteristic dimension of the plasma volume; b) the energy of the oscillations is uniformly distributed with respect to wave vector from $k = 0$ to $k = k_0$. Then we can write[7,14,15]

$$\mathcal{P}_{\omega_{pe}} \sim 10\omega_{pe}\left(\frac{\omega_{pe}}{ck_0}\right)W_i\frac{W_e}{mnc^2}, \tag{1}$$

$$\mathcal{P}_{2\omega_{pe}} \sim 30\omega_{pe}\left(\frac{\omega_{pe}}{ck_0}\right)^3 W_e\frac{W_e}{mnc^2}, \tag{2}$$

$$\mathcal{P}_{\omega_0+\omega_{pe}} \sim S_0\frac{\omega_{pe}}{c}\left(\frac{\omega_0}{\omega_{pe}}\right)^2\left(\frac{\omega_{pe}}{ck_0}\right)^3\frac{W_e}{mnc^2}, \tag{3}$$

where $\mathcal{P}_{\omega_{pe}}$ and $\mathcal{P}_{2\omega_{pe}}$ are the powers radiated from the unit volume in all directions at the plasma frequency and at twice the plasma frequency respectively; $\mathcal{P}_{\omega_0+\omega_{pe}}$ is the scattered power from a unit volume in all directions; W_e is the energy density in the plasma waves; W_i is the energy density of the ion-acoustic waves; S_0 is the energy flux in the incident wave; the remaining notation is conventional. It is evident that in principle the experimentally determined quantities $\mathcal{P}_{\omega_{pe}}$,

$\mathcal{P}_{2\omega_{pe}}$ and $\mathcal{P}_{\omega_0+\omega_{pe}}$ can give the quantities W_e, W_i and k_0.

2. EXPERIMENTAL APPARATUS AND METHOD

The apparatus on which these experiments has been carried out is an open-ended trap with a mirror ratio of 2. In these experiments the magnetic field at the center of the trap is 2.5×10^3 Oe (rise time 750 μsec and decay time 6 msec). The diameter of the vacuum chamber at the center is 15 cm and the length is 0.7 m; the chamber is pumped to a pressure of 2×10^{-7} mm Hg and then filled with a hydrogen plasma produced by two titanium injectors. The injectors are connected, through a controlled discharge switch, to a capacity of 0.2 μF. This capacitor is charged to 12 kV. In these experiments, in contrast with experiments on turbulent heating of a plasma by a current,[3] we were forced to reduce the voltage applied to the discharge gap since the energy density of the noise radiated by the plasma at large discharge voltages is considerably greater than the energy density of the scattered radiation. An electrical diagram of the apparatus is given in[1,2]. In these experiments we measure the current and the voltage of the direct discharge, the plasma density, the diamagnetism, the electromagnetic radiation, and the scattered microwave signal at λ = 3 cm.

The pressure of the hot plasma is determined from the displaced magnetic flux. In Fig. 1 we show oscillograms of the diamagnetic signal and the current. The quantity nT_e is computed from the relation

$$nT_e = 6 \cdot 10^{19} \frac{e_0 \tau H}{4\pi^2 R_0^2 K} \text{ eV-cm}^{-3},$$

where e_0(V) is the voltage applied to the plates of the oscilloscope, τ(sec) is the time constant of the integrating circuit, H(Oe) is the magnetic field, R_0(cm) is the plasma radius and K is the gain of the diamagnetic signal amplifier. Under the conditions of the present experiment $nT_e \approx 10^{14}$ eV·cm^{-3}.

The density is monitored by the transmission of microwave signals through the plasma at wavelengths λ = 3 cm and λ = 10 cm. It has been established that a pulse from a magnetron at λ = 3 cm is transmitted through the plasma[1)] while the signal from a klystron at λ = 10 cm is cut off. Thus, in the present experiments the density range is 1.2×10^{11} cm^{-3} $< n <$ 1.3×10^{12} cm^{-3}.

3. INVESTIGATION OF CHARACTERISTIC RADIATION

The investigation of the characteristic plasma radiation has been carried out by means of a modified wavemeter at wavelengths of 4, 6, 8, 10, and 12 cm. The receiving antenna for the microwave radiation is a symmetric antenna. The antenna accepts the electromagnetic-radiation component that is polarized along the confinement magnetic field. After being detected the signal is applied to a wideband amplifier and an oscilloscope. In order to make absolute measurements of the microwave power in the plasma radiation we have carried out several calibration experiments.[2)] First of all

[1)]The signal transmitted through the plasma is detected by the horn 4 (cf. Fig. 4).

[2)]The calibration (Fig. 3) is carried out by means of a microwave generator and half-wave transmitting antenna 10 which produces a known power flux at the receiving antenna 11.

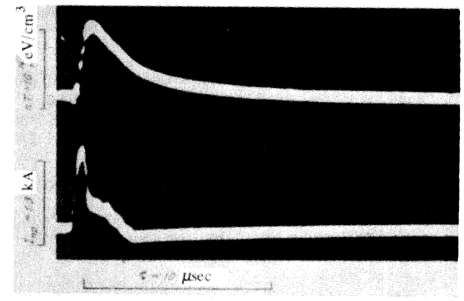

FIG. 1. Diamagnetic probe signal (upper trace) and direct discharge current. The maximum current corresponds to 1.2 kA. The capacitor for the direct discharge is charged to 12 kV and the quantity nT_e in the plasma is 10^{14} eV × cm^{-3}.

we obtain the dependence of detector sensitivity (antenna and wavemeter) on wavelength. Then the power calibration of the detector is carried out.

In these experiments it is also important to know how the resonance properties of the detector are affected by wavelength. It is evident that the smaller the quality factor of the resonator in the detector the larger will be the power accepted by the detection apparatus from a wide spectrum of characteristic plasma radiation. We note that in the present case the width of the line radiated by the generator was narrower than the resonance characteristics of the detection system.

Consider the absolute calibration of the detector. Let $F(\lambda - \lambda_0)$ be a function which characterizes the resonance properties of the detector, λ_0 the wavelength to which the detector is tuned, and P_λ the spectral density of the power flux incident on the receiving antenna. Then the power at the input of the detector is

$$P = \int P_\lambda F(\lambda - \lambda_0)\, d\lambda. \tag{4}$$

When the external radiation is the characteristic radiation of the plasma, the spectral width of which is considerably greater than the resonance width of the wavemeter, the quantity P_λ in Eq. (4) can be assumed to be a constant. In this case

$$P_1 \approx P_{\lambda_0} \int F(\lambda - \lambda_0)\, d\lambda. \tag{5}$$

However, if the incident radiation is the radiation of the calibrating klystron, in which the linewidth is much smaller than the linewidth of the detector, then the quantity $F(\lambda - \lambda_0)$ becomes a constant. In this case Eq. (4) can be written

$$P_2 \approx F(\lambda_1 - \lambda_0) \int P_\lambda\, d\lambda = P_0 F(\lambda_1 - \lambda_0), \tag{6}$$

where P_0 is the known power flux from a generator and λ_1 is the generator wavelength. Integrating Eq. (6) we have

$$\int P_2\, d\lambda_1 = P_0 \int F(\lambda_1 - \lambda_0)\, d\lambda_1. \tag{7}$$

Using Eqs. (5) and (7) we can easily obtain an expression for the spectral density of the power flux of the characteristic microwave radiation from the plasma:

$$P_{\lambda_0} = P_0 P_1 \Big/ \int P_2\, d\lambda_1. \tag{8}$$

The integral in the denominator of Eq. (8) is nothing more than the area of the resonance characteristic of the detector. The conversion from the known voltage on the plates of the oscilloscope to the power (P_1 and P_2) at the input of the detector can be easily carried out from the detector characteristics. The quantity P_0 is determined from the relation $P_0 = P_{gen}D/4\pi r^2$ where P_{gen} is the power from the generator, D = 1.64 is a coefficient associated with the directivity pattern of the half-wave transmitting antenna, and r is the distance between the transmitting antenna and the receiving antenna. Thus, if P_0, P_1 and $\int P_2 d\lambda_1$ are known we can find P_{λ_0}.

Let us now consider the relation between the spectral density of the power flux P_λ and the spectral density of the power Q_λ radiated from a unit volume of the turbulent plasma in all directions. If the length of the plasma is much greater than its diameter the radiation from the ends of the plasma can be neglected and we can assume that the radiation exhibits cylindrical symmetry. Let L be the length of the plasma, R_0 its radius and R the distance from the center of the plasma to the receiving antenna. The spectral density of the power radiated from the entire volume occupied by the plasma is $\pi R_0^2 L Q_\lambda$. In this case we assume that the quantity Q_λ is uniform over the plasma cross section. With the assumptions made above the spectral density of the power should coincide with the spectral density of the power passing through the surface of a cylinder of radius R, that is to say,

$$\pi R_0^2 L Q_\lambda = 2\pi R L P_\lambda$$

or

$$Q_\lambda = 2RP_\lambda / R_0^2. \tag{9}$$

For the present experimental conditions the plasma radius R_0 = 5 cm and the distance R = 8 cm. From Eq. (9) we then find

$$Q_\lambda = 0.64 P_\lambda \,\text{eV-cm}^{-1}.$$

In Fig. 2 we show the variation of the quantity Q_λ in the wavelength range from λ = 4 cm to λ = 12 cm. Two radiation lines are visible; one of these ($\lambda \sim 10$ cm) can be identified with the radiation at the frequency ω_{pe}. The second radiation line ($\lambda \sim 4.5$ cm) corresponds to radiation at the frequency $2\omega_{pe}$. The finite width of these lines is associated with the inhomogeneous density distribution in the plasma. As the plasma density is increased the spectrum of characteristic radiation is displaced in the direction of shorter wavelengths.

In the calculations carried out below it will be necessary to know the total powers $\mathcal{P}_{\omega_{pe}}$ and $\mathcal{P}_{2\omega_{pe}}$ radiated in each of the lines. The quantities $\mathcal{P}_{\omega_{pe}}$ and $\mathcal{P}_{2\omega_{pe}}$ are determined by the areas of the cross-hatched regions in Fig. 2. A simple calculation shows that $\mathcal{P}_{\omega_{pe}} = 16.3 \times 10^{-3}\ \text{W}\cdot\text{cm}^{-3}$ and $\mathcal{P}_{2\omega_{pe}} = 0.95 \times 10^{-3}\ \text{W}\cdot\text{cm}^{-3}$.

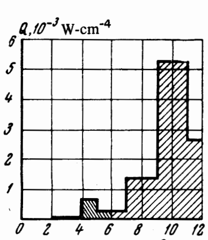

FIG. 2. The quantity Q_λ, the spectral density of the power radiated per unit volume of the plasma in all directions.

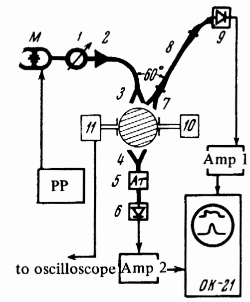

FIG. 3. Diagram of the experiment for investigating combination scattering.

4. INVESTIGATION OF COMBINATION SCATTERING

A diagram of the experiment is shown in Fig. 3. A microwave magnetron produces a signal at λ = 3 cm which goes through a variable attenuator 1 and an isolator 2 to the transmission antenna 3. (In this experiment all antennas are polarized so that **E** ∥ **H** and **k** ⊥ **H** where **E** and **k** are respectively the electric field and the wave vector associated with the electromagnetic wave while **H** is the confinement magnetic field.) At an angle $\theta = 60°$ with respect to the transmitting antenna there is a receiving antenna 7. The signal at frequency ω_0 is cut off by the waveguide beyond cutoff 8 while the signal at the combination frequency $\omega_0 + \omega_{pe}$ passes through the waveguide beyond the cutoff to the 3-centimeter detection head with short-circuiting plunger 9. The plunger is fixed in a position such that the detector is located at the node of the standing wave corresponding to the second harmonic $2\omega_0$ of the magnetron. When the waveguide beyond cutoff is used with the detector directly at the output of the magnetron no signal output is observed in the detector. The spectrum of scattered radiation extends from λ = 2 cm to λ = 2.4 cm. The signal from the detection head is applied to a wideband amplifier 11 and then to the plates in the oscilloscope. As we have indicated above, the density is monitored by transmitting a magnetron pulse through the plasma. The pulse is rectangular and the pulse length can vary from 1 to 3 μsec. The oscillograms in Fig. 4 show the following: the magnetron pulse, the scattered signal, the radiation in the absence of the magnetron pulse. The calculation of the power flux incident on the receiving antenna and the power scattered per unit volume in all directions is carried out in the preceding section. Evidently we can only obtain an estimate of this quantity since the volume in which the scattering occurs cannot

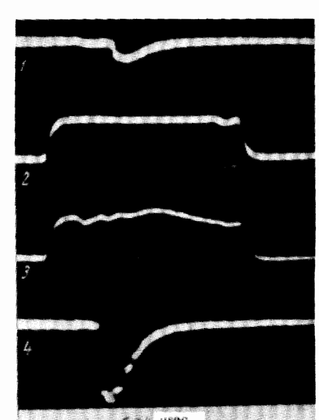

FIG. 4. Oscillogram showing the following: 1—emission from the plasma in the absence of external microwave radiation, 2—magnetron pulse (λ = 3 cm), 3—microwave radiation of the magnetron transmitted through the plasma, 4—signal picked up by the detector in the presence of the magnetron radiation.

be regarded as being of infinite extent. An estimate of the quantity $\mathscr{P}_{\omega_0+\omega_{pe}}$ yields the value $\mathscr{P}_{\omega_0+\omega_{pe}} = 70 \times 10^{-6}$ W/cm^3.

It has been established in control experiments that the power of the scattered radiation increases linearly with increasing magnetron power. This indicates that the incident electromagnetic wave does not perturb the plasma under the present experimental conditions.

5. DISCUSSION OF THE EXPERIMENTAL RESULTS

Knowing the absolute value of the noise power radiated by the plasma in the region corresponding to twice the plasma frequency and knowing the power of the scattered radiation $\mathscr{P}_{\omega_0+\omega_{pe}}$ we can estimate the spectral density of the energy in the plasma waves and find the characteristic wave vector. It is assumed that the radiation at frequency ω_{pe} is due to the interaction of the plasma waves with ion-acoustic waves; this is the basis for the estimate of the energy density of the ion-acoustic waves. This point is important for making a comparison between the present work and the theory of the ion-acoustic current-driven instability and turbulent heating.

Using Eqs. (2) and (3) we can estimate W_e:

$$W_e \sim \frac{\mathscr{P}_{2\omega_{pe}}}{\mathscr{P}_{\omega_0+\omega_{pe}}} \frac{S_0}{c} \frac{\lambda_{pe}^2}{30\lambda^2}. \qquad (10)$$

In the present case we find $\mathscr{P}_{2\omega_{pe}} \approx 9.50 \times 10^{-4}$ W/cm^3, $\mathscr{P}_{\omega_0+\omega_{pe}} \approx 7.0 \times 10^{-5}$ W/cm^3; $\lambda_{pe} = 10$ cm, $\lambda = 3$ cm; $S_0 = 1.2 \times 10^3$ W/cm^2. Substituting these quantities in Eq. (4) we find $W_e \sim 10^{12}$ eV$\cdot$cm^{-3}.

Using Eq. (2) we can find the characteristic wave vector k_0. Some simple calculations show that $k_0 \sim 9$ cm^{-1}. In the present experiment the quantity T_e as measured from the plasma diamagnetism is found to be 8×10^2 eV ($n = 1.2 \times 10^{11}$ cm^{-3}). It is then evident that the value of k_0 that has been found is of the same order of magnitude as the Debye wave number $k_0 = \omega_{pe}/v_{Te}$.

In determining W_i we have used Eq. (1):

$$W_i \sim \frac{\mathscr{P}_{\omega_{pe}}}{10W_e} \frac{mnc^2}{(\omega_{pe}/ck_0)\,\omega_{pe}} \sim 5 \cdot 10^{11} \text{ eV-cm}^{-3}.$$

The estimates made above indicate that under the conditions of the present experiment it is possible to establish a uniform distribution of energy over the degrees of freedom of the various turbulence oscillations ($W_e \sim W_i$). The ratio of the energy in the waves to the thermal energy of the electrons is found to be $(W_e + W_i)/nT_e \sim 0.02$.

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