THE NATURE OF MULTIMODE GENERATION WHEN QUASI-EQUILIBRIUM IS DISTURBED

V. S. MASHKEVICH

Institute of Physics, Ukrainian Academy of Sciences

Submitted April 12, 1967

Zh. Eksp. Teor. Fiz. 53, 1003–1005 (September, 1967)

The effect of violation of quasi-equilibrium in the active medium on laser radiation is investigated. Violation of quasi-equilibrium is considered with respect to energy (spectral inhomogeneity) and coordinates (spatial inhomogeneity). The problem concerning the classification of multimode generation (MG) and the dependence of the type of MG on the type of inhomogeneity is formulated and solved. It is shown that the MG due to spectral inhomogeneity does not occur immediately above threshold and possesses a discrete spectrum; the MG due to spatial inhomogeneity commences immediately above threshold and possesses a continuous spectrum. It is found that in the case of spectral inhomogeneity the number of generation lines can increase as a result of appearance of new lines as well as splitting of old ones.

BEFORE the onset of laser generation, the state of the medium can be regarded as in quasi-equilibrium: the total number $\mathfrak{N}$ of elementary excitation of the electronic subsystem (excited centers, electron-hole pairs) is determined by the pumping N and is not in equilibrium, but the distribution of the excitations over the states depends only on $\mathfrak{N}$. At the threshold value $N = N_1$, lasing sets in. Further increase of N leads to violation of the quasi-equilibrium. We shall consider this violation for the spatial and energy distributions of the excitations, that is, the spatial and spectral inhomogeneity is due to the inhomogeneity of the field of the generating modes and to the finite velocity of excitation migration, while the cause of the spectral inhomogeneity is the presence of generating modes and the inhomogeneity of the spectral-line broadening. In quasi-equilibrium, the stationary generation is of the single-mode type. Sufficiently strong violation of the quasi-equilibrium leads to multimode generation (MG).

The present communication contains a formulation and a solution of the fundamental problem concerning the classification of stationary MG and the dependence of the type of MG on the type of inhomogeneity.

The following classification is heuristic: let the type of the modes that can generate be fixed. Then these modes are numbered in accordance with the frequency ω in discrete steps of $\Delta\omega$. We raise the following two questions:

1. Does the multimode generation set in in the case of sufficiently small $\Delta\omega$ (i) practically immediately above threshold or (ii) when the pump N_2 exceeds N_1 by a finite amount, that is, (i) is $\lim_{\Delta\omega \to 0} (N_2 - N_1)$ equal to zero or (ii) different from zero?

2. If $\Delta\omega$ is sufficiently small and $N > N_2$, does generation occur (C) in a series of modes that are adjacent to each other, or in non-neighboring modes (D), that is, is the MG spectrum continuous (C) or discrete (D) as $\Delta\omega \to 0$?

We shall classify the MG in accordance with these attributes. Four variants are conceivable (iC), (iiC), (iD), (iiD). We emphasize that the classification is based on the behavior as $\Delta\omega \to 0$. It is physically required that $\Delta\omega$ be small compared with the width of the levels of the elementary excitations, that is, with the magnitude of the homogeneous broadening.

The central problem is the establishment of the dependence of the character of the MG on the character of violation of the quasi-equilibrium. We shall prove the following cardinal statements: 1) The MG due to the spectral inhomogeneity is of the type (iiD); 2) the MG due to the spatial inhomogeneity is of the type (iC). The proof is based on the generation condition at the frequency ω:

$$F(\omega) \equiv B(\omega) - \gamma(\omega) = 0, \tag{1}$$

where B is the gain and γ the losses.

Let us prove the first statement. In the case of spectral inhomogeneity we have

$$B(\omega) = \int b(\omega, \omega')\rho(\omega')\,d\omega',$$

where b is determined by the interaction between the mode and the excitations, and ρ is a function of the spectral density of the excitations. The function b is analytic with respect to both variables (on the real axis), ρ is integrable, and γ is analytic. Consequently F is an analytic function.[1] Therefore the condition (1) can be satisfied only at individual points[2]—property (D). When $N = N_1$, a first-order tangency takes place between curve F and the axis at one point, and generation occurs at the frequency of this point. For MG to begin, it is necessary to have either a third-order tangency or tangency at several points. But F is an analytic function of ω and a continuous function of N. Therefore it is necessary that N change by a finite amount when the character of the tangency changes—property (ii).

Let us classify (D)-generation in accordance with the character of its occurrence. If the MG occurs as a result of a third-order tangency, then it begins in the form of a continuous splitting of one line. This case is possible only if the curve $F(\omega)$ is symmetrical. If the

MG is the result of a first-order tangency at several points, it begins in the form of several lines, which are immediately separated by finite frequency intervals. Both cases were observed experimentally in spectrally inhomogeneous systems.[3] We note that in gas lasers, where $\Delta\omega$ is larger than the homogeneous broadening, the condition that $\Delta\omega$ be small is not satisfied. In this case neighboring modes can take part in the generation.

We proceed now to prove the second statement. Let the spatial distribution of the excitations be determined by the parameter $\eta = \eta(\mathbf{r})$. The quantity F is a functional of $\eta(\mathbf{r})$. Putting

$$\eta(\mathbf{r}) = \overline{\eta} + \Delta\eta(\mathbf{r}), \quad \overline{\Delta\eta} = 0,$$

where the bar denotes spatial averaging, and expanding in powers of $\Delta\eta$ in the kinetic equations, we can obtain in first approximation

$$F(\omega) = \bar{F}(\omega)\theta[-\bar{F}(\omega)] = \begin{cases} \bar{F}(\omega), & \bar{F}(\omega) < 0 \\ 0, & \bar{F}(\omega) > 0. \end{cases} \quad (2)$$

Here $\bar{F}$ is obtained from F by replacing η by $\bar{\eta}$. The function $F(\omega)$ is not analytic at the points ω satisfying the condition $\bar{F}(\omega) = 0$. It follows from (2) that generation takes place in the frequency region determined by the condition $\bar{F}(\omega) > 0$; this is MG of the type (C). Further, $\bar{F} = f[\omega, \bar{\eta}(N)]$ is a continuous increasing function of N (for fixed ω), from which we get the property (i). MG of type (iC) is observed experimentally in a ruby laser, where only spatial inhomogeneity is present.

Physically the difference in the manifestations of the spectral and spatial inhomogeneities is connected with the following. In the case of spectral inhomogeneity, the modes that are close to each other in frequency are "fed" by the same excitation in nearly equal amounts. In the case of spatial inhomogeneities these amounts may differ greatly from each other, owing to the fact that some of the antinodes of the neighboring modes may not coincide. In any case, generation begins at a frequency ω_m corresponding to the maximum of $F(\omega)$. With further increase of the pump, the generation inhibits the growth of the amplification. In the case of spectral inhomogeneity, this hindrance is manifest most strongly at frequencies adjacent to ω_m. Therefore the new generation frequencies are "repelled" from ω_m and the resultant generation spectrum is discrete. In the case of spatial inhomogeneity, the decelerating action of the generation is manifest more or less equally at all frequencies. Therefore new generation frequencies are adjacent to ω_m and the generation spectrum turns out to be continuous.

The foregoing results can serve as a criterion for the establishment of the character of the inhomogeneity from the character of the experimentally observed MG.

---

[1] G. M. Fikhtengol'ts, Kurs differentsial'nogo i integral'nogo ischisleniya (Course of Differential and Integral Calculus) vol. 2, Gostekhizdat, 1948.

[2] M. A. Lavrent'ev and B. V. Shabat, Metody teorii funktsiĭ kompleksnogo peremennogo (Methods of the Theory of Functions of Complex Variable) Fizmatgiz, 1958.

[3] W. H. Keene and J. A. Weiss, Appl. Optics 3, 545 (1964); V. S. Mashkevich and M. S. Soskin, ZhETF Pis. Red. 5, 456 (1967) [JETP Lett. 5, 369 (1967)].

Translated by J. G. Adashko
114