

EPR OF Nd^{3+} IONS IN FLUORITE

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The cubic EPR spectrum of the Nd^{3+} ion in CaF_2 is investigated. The value of the parameter X which defines the wave functions of the Γ_8 quadruplet is -0.5 . The presence of rhombic and trigonal distortions in the cubic potential explains a number of features of the spectrum such as the shape of the lines, their broadening with decrease of the high frequency quantum, and the absence of transitions between the levels of various Kramers doublets. The EPR spectra and spin-lattice relaxation of Nd^{3+} ions involved in the complex Nd^{3+} - TR^{3+} centers in CaF_2 crystals are investigated. It is found that the symmetry of the crystal field in which the Nd^{3+} ion is located in Nd^{3+} - La^{3+} and Nd^{3+} - Pr^{3+} pairs differs from the field symmetry in the similar Nd^{3+} - Nd^{3+} pair.

It is common opinion that in fluorite crystals grown in a fluorinating atmosphere trivalent rare earth impurity ions are found in crystalline fields of cubic and tetragonal symmetry, and that the probability of formation of other centers is very small for impurity concentrations of less than one percent. Investigations of EPR spectra have not, by and large, contradicted these suppositions. A certain exception is the neodymium ion, for which, on the one hand, there are no reliable reports of an EPR spectrum of ions in a field of cubic symmetry, and on the other hand, pairs of interacting Nd^{3+} ions in fields of rhombic symmetry are observed^[1] in the fluorite EPR spectrum. Moreover, the probability of formation of pair centers is comparable with the probability of formation of tetragonal centers for impurity concentrations as small as 0.1%. For other rare earth ions, pair centers are not observed at all. In the present paper, EPR results are presented for Nd^{3+} ions in a field of cubic symmetry, and for Nd^{3+} ions which form pair centers with other rare earth ions (Nd^{3+} - TR^{3+}).

1. EPR OF Nd^{3+} IONS IN THE CUBIC FIELD OF A CaF_2 CRYSTAL

The problem of the wave functions and energy levels (within the multiplets having $L \neq 0$ and $J > 5/2$) of rare earth ions in a crystalline field of cubic symmetry was investigated by Lea, Leask, and Wolf.^[2] These authors calculated the dependence of the wave functions and relative splittings, and determined the dependence of the ground state upon the parameter X , which characterizes the ratio of the fourth to the sixth degree terms in the expansion of the crystalline field potential in spherical harmonics. According to them, the ground state of Nd^{3+} in a field of cubic symmetry is a Γ_8 quadruplet. Figure 1 shows the X -dependence of the G -factors that characterize the splitting of the levels in the Γ_8 quadruplet, in a magnetic field directed along a [001] crystalline axis. The G -factors were calculated from the wave functions of^[2]. The continuous and dashed lines of Fig. 1 denote level splittings, the transitions between which are, respectively, allowed ($\Delta m_J = \pm 1$) or forbidden.

Vincow and Low^[3] reported that they observed in heat-treated crystals of CaF_2 with a magnetic field along a [001] axis the following lines of Nd^{3+} , which are related to Γ_8 : one with $G = 2.26 \pm 0.02$, and another, of smaller intensity, with $G = 1.1 \pm 0.05$. They deduced^[3] that terms of sixth order in the expansion of the potential could be neglected ($X = -1$), which is not in accordance with results obtained for other rare earth ions. In particular, it has been shown^[4] that $X = -0.55$ for Pr^{2+} , which is isoelectronic with Nd^{3+} . At the same time, Ranon^[5], citing^[3], uses $X = -0.61$. As is seen in Fig. 1, for this value of the parameter there exist in fact splittings between the levels, corresponding to the transitions observed in^[3]. But the most intense transition, at $G = 2.26$, ought to be forbidden.

In the present work, the spectrum of Nd^{3+} ions in a cubic field was detected in crystals grown by the Stockbarger method in a fluorinating atmosphere. The most intense spectrum was in crystals with Na impurity, which clearly is related to the decrease in the number of interstitial F^- ions. In some crystals the spectrum was not seen at all.

The measurements were made at a frequency of about 38 GHz and a temperature of 4.2°K . The lines had unusual shapes (see Fig. 2), and broadened upon passage to lower frequencies. The spectrum could not be detected in the 3-cm range of frequencies. Two lines

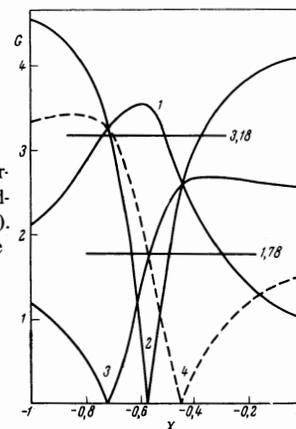


FIG. 1. G -factor values, characterizing the level splitting in the Γ_8 quadruplet in a magnetic field ($H \parallel [001]$). Curve 1 corresponds to the difference of the energy eigenstates $|+3/3\rangle$ and $|-3/2\rangle$; 2 to $|+1/2\rangle$ and $|-1/2\rangle$; 3 to $|+1/2\rangle$ and $|+3/2\rangle$; 4 to $|+3/2\rangle$ and $|-1/2\rangle$. The horizontal lines are the observed values of the G -factors.

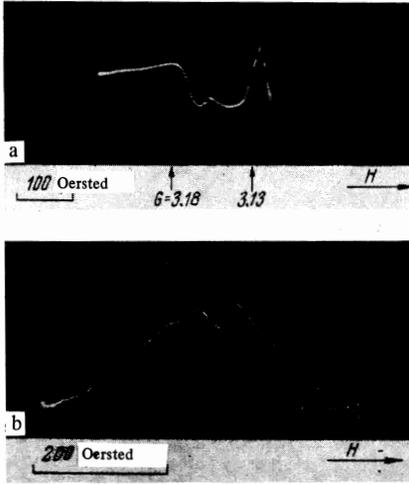


FIG. 2. Line shapes in the cubic EPR spectrum of Nd^{3+} in CaF_2 .
 a) $G = 3.18$; also observed in the photograph is a symmetrical line with a g-factor of 3.13 (belonging to the $\text{Nd}^{3+} - \text{Na}^+$ center);
 b) $G = 1.78$.

were observed with the magnetic field along [100]: one with a G-factor of 3.18 ± 0.04 , having a sharp fall-off on the high-field side (Fig. 2a), and the other with $G = 1.78 \pm 0.02$, having the shape represented in Fig. 2b. As is seen from Fig. 1, in which the horizontal lines correspond to the experimental values of G, the most appropriate value of the parameter X is -0.5 , and the allowed transitions 1 and 2 are observed. For $X = -0.5$, transition 3 with $G \approx 2.4$ is also allowed, but, as will be seen below, the transition is not observed because of its large breadth, induced by random splittings of the quadruplets into two doublets as a result of the distortion of the cubic symmetry of the crystalline field. The eigenstates of the basic quadruplet and the corresponding energy levels W_i in a magnetic field are given in the table.

The spin Hamiltonian for Γ_8 , following Bleaney,^[6] is

$$\hat{H} = g\beta(H_x\hat{S}_x + H_y\hat{S}_y + H_z\hat{S}_z) + f\beta(H_x\hat{S}_x^3 + H_y\hat{S}_y^3 + H_z\hat{S}_z^3), \quad (1)$$

where β is the Bohr magneton, and g and f are constants characterizing the level splitting in a magnetic field. The effective spin S is $3/2$. In our case $g = -2.13$ and $f = 1.42$. The angular dependence of the energy levels can be written as^[6]

$$(W/\beta H)^2 = 1/4(5\gamma^2 + 3\delta^2) \pm \gamma\{\gamma^2 + 1/2\delta^2[9(l^2 + m^2 + n^2) - 3]\}^{1/2}, \quad (2)$$

where $\gamma = g + 7/4f$, $\delta^2 = f(g + 5/2f)$, and l , m , and n are the direction cosines.

In Figs. 3 and 4 the angular dependence of the resonance transitions, both experimental and calculated

Energy level	Wave function in the representation of	
	total angular momentum $J = 9/2$	effective spin $S = 3/2$
$W_1 = 1,59 \beta H$	$0,964 +^3/2 \rangle + 0,102 -^3/2 \rangle$	$ +^3/2 \rangle$
$W_2 = 0,89 \beta H$	$0,62 +^3/2 \rangle - 0,62 +^1/2 \rangle + 0,48 -^7/2 \rangle$	$ -^1/2 \rangle$
$W_3 = -0,89 \beta H$	$0,62 -^3/2 \rangle - 0,62 -^1/2 \rangle + 0,48 +^7/2 \rangle$	$ +^1/2 \rangle$
$W_4 = -1,59 \beta H$	$0,964 -^3/2 \rangle + 0,102 +^3/2 \rangle$	$ -^3/2 \rangle$

from Eq. (2), are depicted as functions of the orientation of the magnetic field in the (001) and (110) planes.

The unusual line shapes, the absence of the transition at $G \approx 2.4$ (for $H \parallel [001]$), and the broadening of the lines with decrease in frequency can be explained if one assumes that random distortions of the cubic potential of the lattice are induced by the displacements of the neighboring F^- ions. The cause of the displacements of these ions can be the relatively large ionic radius of Nd^{3+} .^[1] Since one or two F^- ions will most likely be displaced, the distortion must have rhombic or trigonal character. Considering only the most important terms,^[8] one can compare, respectively, the following Hamiltonians for such distortions:

$$V^{\text{rh}} = A_1[3\hat{J}_z^2 - J(J+1) + \hat{J}_+^2 + \hat{J}_-^2], \quad (3)$$

$$V^{\text{tr}} = A_2[3\hat{J}_z^2 - J(J+1)] + A_3[\hat{J}_z(\hat{J}_+^3 + \hat{J}_-^3) + (\hat{J}_+^3 + \hat{J}_-^3)\hat{J}_z]. \quad (4)$$

The values of the constants A, which characterize the magnitude of the distortion, change from one cell with a Nd^{3+} ion to another. The operators (3) and (4) have the following nonvanishing matrix elements between quadruplet states:

$$\begin{aligned} V^{\text{rh}}_{3/2, 3/2} &= V^{\text{rh}}_{-3/2, -3/2}, & V^{\text{rh}}_{3/2, 1/2} &= V^{\text{rh}}_{-1/2, -1/2}, & V^{\text{rh}}_{3/2, -1/2} &= V^{\text{rh}}_{-1/2, 1/2}, \\ V^{\text{tr}}_{3/2, 3/2} &= V^{\text{tr}}_{-3/2, -3/2}, & V^{\text{tr}}_{3/2, 1/2} &= V^{\text{tr}}_{-1/2, -1/2}, & V^{\text{tr}}_{3/2, -1/2} &= -V^{\text{tr}}_{-1/2, 1/2}. \end{aligned} \quad (5)$$

In first order perturbation theory, the distortion of the crystalline field leads to the splitting of the Γ_8 quadruplet into two doublets. The magnitude of the splitting is

$$\Delta E = (V^{\text{rh}} + V^{\text{tr}})_{3/2, 3/2} - (V^{\text{rh}} + V^{\text{tr}})_{1/2, 1/2}. \quad (6)$$

In second order perturbation theory, we find that for a magnetic field directed along a [001] axis, the corrections to the energy levels W_i are

$$\begin{aligned} \varepsilon_1 &= \frac{|V^{\text{rh}}_{3/2, -1/2}|^2}{0,7\beta H} + \frac{|V^{\text{tr}}_{3/2, 1/2}|^2}{2,48\beta H}, & \varepsilon_2 &= -\frac{|V^{\text{rh}}_{3/2, -1/2}|^2}{0,7\beta H} + \frac{|V^{\text{tr}}_{3/2, -1/2}|^2}{2,48\beta H}, \\ \varepsilon_3 &= \frac{|V^{\text{rh}}_{-3/2, 1/2}|^2}{0,7\beta H} - \frac{|V^{\text{tr}}_{3/2, 1/2}|^2}{2,48\beta H}, & \varepsilon_4 &= -\frac{|V^{\text{rh}}_{-3/2, 1/2}|^2}{0,7\beta H} - \frac{|V^{\text{tr}}_{3/2, -1/2}|^2}{2,48\beta H}. \end{aligned} \quad (7)$$

From (7) it follows that for the levels W_1 and W_4 of the quadruplet, the transition between which is observed at a field corresponding to $G = 3.18$, both types of distortion of the crystalline field produce an increase in the splitting ($\varepsilon_1 - \varepsilon_4 > 0$), and consequently the absorption line must have a small slope on the low-field side, as is indeed observed (see Fig. 2a). For the W_2 and W_3 levels, the corrections from the rhombic and trigonal distortions have different signs, and lead, respectively, to converging and diverging of the levels. Thus the line at $G = 1.78$ will have small slopes on both sides (see Fig. 2b): on the high-field side owing to the distortion of rhombic character, and on the low-field side owing to the trigonal distortion. The fact that the line has two peaks can be explained by assuming that the distortion of the crystalline field distribution function is unsymmetrical. The asymmetry is due to the fact that distortions of one sign predominate, and that the maximum of the distribution function occurs at nonvanishing distor-

¹⁾ For the Dy^{3+} ion, the ground states of which are also Γ_8 in the cubic field of a CaF_2 crystal, but whose ionic radius is smaller than that of Nd^{3+} , the effects enumerated above are not observed [7].

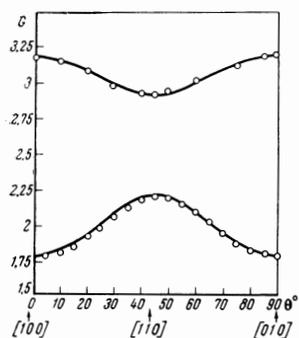


FIG. 3. Angular dependences of the resonance transitions on the orientation of the magnetic field in the (001) plane.

tions. The relatively large ionic radius of Nd³⁺ is the cause for the distortion which produces such a distribution function.

With decrease in magnetic field, the magnitude of the corrections (7) increases, which explains the line broadening with decrease of the radio-frequency quantum.

From the shape of the line at $G = 1.78$, it is possible to estimate the values of the matrix elements in Eqs. (7) separately for the distortions of rhombic and trigonal character. Considering that at the frequency of about 38 GHz the line width at half-intensity is about 150 Oe, one can obtain

$$|V_{\frac{1}{2}, -\frac{1}{2}}^{\text{rh}}| \approx 0.05 \text{ cm}^{-1}, \quad |V_{\frac{1}{2}, \frac{1}{2}}^{\text{tr}}| \approx 0.1 \text{ cm}^{-1}. \quad (8)$$

The initial splitting ΔE , Eq. (6), has the same order of magnitude, and thus the transition near $G = 2.4$ is not observed due to its very large width (~ 1000 Oe).

2. EPR INVESTIGATION OF Nd³⁺-TR³⁺ CENTERS IN FLUORITE

In [9] it was shown that the Nd³⁺ ion can form complex centers with any other rare earth ion (TR³⁺) in a crystal of CaF₂. In such centers the Nd³⁺ and TR³⁺ ions are located in adjacent cells, in which they replace the Ca²⁺ ions. For the same ions, the formation of several types of complex centers is possible, differing in the method of compensation of the excess positive charge. Fairly strong shifts of the Stark components in the absorption spectrum of the Nd³⁺ ions were observed, [9] which can hardly be explained only by a change in the radius of the neighboring ion upon replacement of one rare earth ion by another. Most likely, there occurs a displacement of the F⁻ ions which enter into the immediate surroundings of both the Nd³⁺ ion and the neighboring TR³⁺ ion. An EPR investigation of the complex

centers is thus of interest, since it can yield directly the symmetry of the immediate surroundings of the paramagnetic ions.

In the present work, an investigation of the EPR spectra of Nd³⁺ ions appearing in complex centers was made. The investigation was carried out at frequencies of 9.14 and 38 GHz at a temperature of 4.2° K, on the same crystals as in [9]. In the crystals, an impurity of one of the following trivalent ions existed along with the neodymium impurity: La³⁺, Ce³⁺, Pr³⁺, Sm³⁺, Tb³⁺, Dy³⁺, Ho³⁺, Er³⁺, Tm³⁺, Lu³⁺, and Y³⁺. In spite of the fact that in [9] the formation of complex centers of Nd³⁺ with all these ions was detected, new EPR spectra of Nd³⁺ ions, corresponding to Nd³⁺-TR³⁺ pairs, were observed only for the first four ions of the above series. We remark that in all crystals, spectra of Nd³⁺ ions occurring in crystalline fields of tetragonal and rhombic symmetry were observed. In some crystals the above-described spectrum of Nd³⁺ in cubic surroundings was observed. Furthermore, the following previously studied (see [10]) spectra of ions of the additional impurities were observed: Ce³⁺-tetragonal spectrum; Tb³⁺-tetragonal spectrum; Dy³⁺-cubic spectrum; Er³⁺-cubic, tetragonal, and trigonal spectra.

We shall discuss in more detail the EPR spectra of Nd³⁺ ion pairs, since it is natural to assume that Nd³⁺-TR³⁺ pair centers are analogous to Nd³⁺-Nd³⁺ centers in their manner of formation. It was shown in [1] and [11] that the spectrum of rhombic symmetry belongs to Nd³⁺ ions located in neighboring cells. Six inequivalent positions of the pairs are observed. The orthogonal symmetry axes of the EPR spectrum coincide with the crystalline directions $[\bar{1}\bar{1}0]$, $[001]$, and $[110]$. For pairs arranged along $[110]$, these directions are designated in [11] as the x, y, and z axes, respectively. The principal values of the g-tensor are $g_x \approx 2.03$, $g_y \approx 3.43$, $g_z \approx 2.13$.

According to the results of Voron'ko, Kaminskiĭ, and Osiko, [12] optical spectra of three types of centers are observed in crystals of CaF₂ with Nd³⁺ impurity: L, M, and N (we note that the introduction of the additional impurities, [9] while in no way influencing the spectral character of the L centers, leads to a splitting of the lines belonging to the M and N centers; thus the M and N centers are complex centers). Comparison of the distances to the excited levels, determined from optical [12] and relaxation (by EPR methods) [11] measurements, permits one to infer that the tetragonal EPR spectrum corresponds to the L center. Regarding the M and N centers, the accuracy of the relaxation measurements is insufficient, because of the proximity of the energy of the first excited levels of the Nd³⁺ ions in these centers, to establish that the rhombic EPR spectrum of the ion pairs is associated with one of the centers. However, taking into account the change in the optical and EPR spectra upon γ -irradiation of the crystals, it is possible to infer that the rhombic EPR spectrum corresponds to the M center. On the one hand, it is established in [13] that upon γ -irradiation approximately one-half of the M centers is destroyed but that the number of N centers is unchanged, while on the other hand it is demonstrated in [14] that the intensity of the rhombic spectrum of the Nd³⁺ ion pairs decreases by one-half upon γ -irradiation.

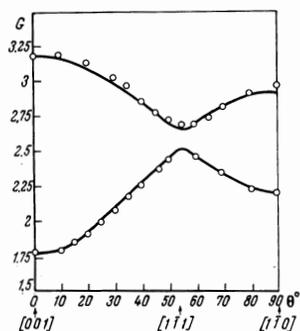


FIG. 4. Angular dependences of the resonance transitions on the orientation of the magnetic field in the (110) plane.

CaF₂: (Nd³⁺ + La³⁺). A rhombic EPR spectrum of Nd³⁺ ions was detected and investigated in crystals with impurities of Nd³⁺ and La³⁺ ions. The spectrum is characterized by a g-tensor with the principal values:

$$g_x = 1.50 \pm 0.02; \quad g_y = 3.53 \pm 0.05; \quad g_z = 2.13 \pm 0.01. \quad (9)$$

The z-axis coincides with the crystalline [110] direction, and the x- and y-axes are displaced in the (110) plane by an angle of $\pm(20 \pm 1)^\circ$ from the $[1\bar{1}0]$ and $[001]$ directions, respectively. Twelve inequivalent ions are observed.

CaF₂: (Nd³⁺ + Pr³⁺). A spectrum of ions was detected in crystals with impurities of Nd³⁺ and Pr³⁺ ions, quite similar to the spectrum detected in crystals with La³⁺ impurity. The principal values of the g-tensor for this spectrum are

$$g_x = 1.54 \pm 0.05; \quad g_y = 3.5 \pm 0.05; \quad g_z = 2.05 \pm 0.01. \quad (10)$$

The angle through which the x- and y-axes are displaced from the $[1\bar{1}0]$ and $[001]$ directions are $\pm(20 \pm 1)^\circ$. In spite of the fact that the spectra (9) and (10) are very similar, they are not one and the same spectrum (compare g_z in (9) and (10)).

In crystals with an additional impurity of Ce³⁺ or Sm³⁺ ions, the investigation of the spectra is complicated by the spin-spin interactions, in the complex centers, of these ions with the Nd³⁺ ion: The angular dependence of the EPR lines does not follow the relation

$$g^2 = g_x^2 l^2 + g_y^2 m^2 + g_z^2 n^2,$$

where l , m , and n are the direction cosines; furthermore, the intensities of the lines decrease as a result of their splitting. Therefore, although lines with behavior characteristic of spectral pairs were detected in the EPR spectra, a detailed investigation for the case of Ce³⁺ and Sm³⁺ ions was not possible. We remark that for Nd³⁺-Sm³⁺ pairs, a shift in the x- and y-axes in the (110) plane also occurs.

The rhombic spectrum of Nd³⁺ ions corresponds to the M center, as already stated above. Since the axes of the g-tensor characterizing the rhombic spectrum of the Nd³⁺-Nd³⁺ pairs do not coincide with the axes of the g-tensors which characterize the EPR spectra of the Nd³⁺-Pr³⁺ and Nd³⁺-La³⁺ pairs, one can then naturally assume that in these cases spectra of the Nd³⁺ ions in pairs corresponding to the N center are observed. In order to clarify this question, a measurement and comparison of the temperature dependence of the spin-lattice relaxation times were carried out, in a region where the relaxation is governed by two-phonon processes, for the Nd³⁺-Nd³⁺ pair (M center) and the Nd³⁺-TR³⁺ pairs. Of particular interest is a comparison of the relaxation of the M center with the relaxation of the Nd³⁺-Pr³⁺ pairs. According to [9], when the additional impurity is the Pr³⁺ ion, the radius of which is the closest to the ionic radius of Nd³⁺, the line splitting in the M and N centers is practically nonobservable. Thus the energy levels and eigenstates of the Nd³⁺ ion when a Pr³⁺ ion is located in an adjacent cell, are the same as in the case of Nd³⁺ ion pairs in the respective M and N centers. Consequently, the temperature dependences of the relaxation times must be identical, if the Nd³⁺-Nd³⁺ and Nd³⁺-La³⁺ pairs are similar in the manner of compensation of excess charge.

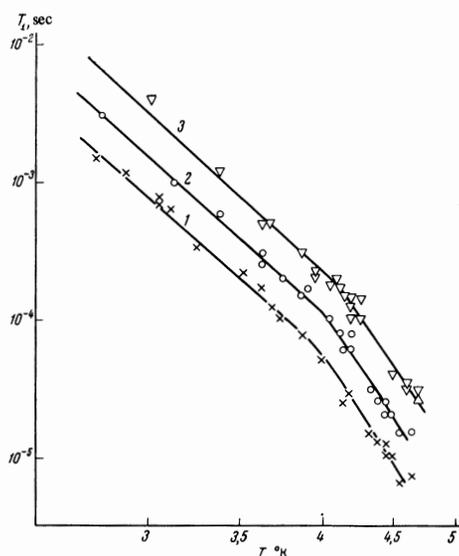


FIG. 5. Temperature dependences of the time T_1 of the Nd³⁺ ions in pair centers: 1 corresponds to the Nd³⁺-Nd³⁺ pair, 2 to the Nd³⁺-Pr³⁺ pair, and 3 to the Nd³⁺-La³⁺ pair.

The dependence of T_1 of the Nd³⁺ ions in the Nd³⁺-Nd³⁺, Nd³⁺-Pr³⁺ and Nd³⁺-La³⁺ pairs, is shown in Fig. 5 on a log-log plot for temperatures less than 4.5°K. The relaxation time measurements were made by the pulse method. For the temperature range shown in Fig. 5, the relaxation time is essentially determined by the two-phonon processes of combination scattering (the Raman process, $T_1 \propto T^{-9}$). As follows from the figure, the relaxation times of the neodymium ions in different pairs are very close to one another over the entire range, which argues for an identical method of formation of the pairs. A confirmation of this deduction is the fact that the intensity of the new EPR spectra is decreased upon γ -irradiation, just as it is for the spectra of the Nd³⁺ ion pairs.

Thus, from the relaxation measurements, it follows that the observed EPR spectra belong to the neodymium ions in the Nd³⁺-TR³⁺ pairs, which arise from the replacement of one of the Nd³⁺ ions in an M center by a rare earth ion of the additional impurity. Nevertheless, the symmetry of the Nd³⁺-TR³⁺ centers differs from the symmetry of the M centers through a difference in direction of the g-tensor axes. The change in the symmetry of the crystalline field at the site of the Nd³⁺ ion originates, possibly, as a result of a shift of one of the two F⁻ ions found in the immediate surroundings both of the Nd³⁺ ion and its neighboring rare earth ion. The displacement of the F⁻ ions can be caused by the differences in the ionic radii of Nd³⁺ and TR³⁺. The shift, observed in [14], in the g-tensor axes of the rhombic spectrum of an isolated Nd³⁺ ion, which originates from the rhombic spectrum of the Nd³⁺ ion pairs (M center) upon γ -irradiation, can be explained similarly. In this case, with a valence change of one of the Nd³⁺ ion pairs as a result of the capture of a slow electron, an increase in its ionic radius results, which leads to the displacement of the F⁻ ion and to the change in the symmetry of the field at the site of the neighboring Nd³⁺ ion. The explanation of this shift given in [14] is incorrect.

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